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Research Article

HYDROMAGNETIC BOUNDARY LAYER FLOW OF ROTATING DUSTY FLUID UNDER VARYING PRESSURE GRADIENT

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ABSTRACT: An asymptotic analysis of a hydromagnetic boundary layer flow of an incompressible viscous conducting dusty fluid bounded by semi-infinite plate is considered. The flow is due to the influence of time dependent pressure gradient and uniform magnetic field. The analytical solution of the boundary layer equations are obtained by asymptotic behaviour of Laplace transform treatment. The solutions for small times, shown that the general features of hydromagnetic boundary layer flow is unaffected by the dusty parameter as well as rotation and magnetic parameter. In subsequent large times, the structure of velocity distribution and the associated boundary layer is investigated i.e., the effect of magnetic parameter, Ekman parameter and Hall current parameter are depicted graphically.

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Keywords: Pressure gradient, Boundary layer, Rotating dusty fluid, Ekman number, Hall parameter.

1. INTRODUCTION

Saffman [1] has initiated and investigated the effect of dusty particles on the stability of the laminar flow of an incompressible fluid with constant mass concentration of dust particles. Michael and Miller [2] have discussed the motion of dusty gas occupying the semi infinite space above a rigid plane boundary. This interest stems from the fact that the magnetohydrodynamic boundary layer flow finds its applications in wide range of science and technology like MHD power generation, cooling of nuclear reactors and in several astrophysical situations. The theory of rotating fluids is highly important due to its occurrence in various natural phenomena and for its applications in various technological situations which are directly governed by the action of Coriolis force. The broad subjects of oceanography, Meteorology, Atmospheric science and Limnology all contain some important and essential features of rotating fluids.

Further the fluid flow problems in rotating medium have attracted many scholars and there appeared a number of studies in literature viz. Tiwari and Kamal Singh [3] have obtained solution for an asymptotic analysis of an unsteady hydromagnetic boundary layer flow generated impulsively incompressible viscous conducting fluid with uniform distribution of dust particle bounded by semi-infinite plate. Prasada Rao and Krishna [4] have studied Hall effect on unsteady hydromagnetic flow. Kanch, Jana [5] investigated Hall effects on unsteady hydromagnetic flow past a rotating disk when the fluid at infinity rotates about non-coincident axes. Ghosh, Anwar Beg and Zueco [7] have studied the hydromagnetic natural convection boundary layer flow past an infinite vertical flat plate under the influence of a transverse magnetic field with magnetic induction effects. Debnath [8] studied the effect of hall current on unsteady hydromagnetic flow past a porous plate in a rotating fluid system.

Aim of this paper is to study the effect of Hall current on hydromagnetic flow on an oscillating plate in a rotating fluid with uniform distribution of dust particles in the presence of time dependent pressure gradient. Laplace Transform technique is employed to obtain the solution. But its exact inversion would be extremely difficult, so the asymptotic behavior of the solution has been analyzed for both small and large time to highlight the transient approach to the steady flow and other physical process involved in it.

2. MATHEMATICAL FORMULATION

Consider an unsteady flow induced in a semi-infinite plate of an electrically conducting incompressible viscous fluid with uniform distribution of dust particles bounded by an infinite plate at $z = 0$. A uniform magnetic field

B_0 is acting normal to plate. The fluid as well as the plate is in a state of solid body rotation with constant angular velocity Ω about the z-axis normal to the plate and additionally, non-torsional oscillation of frequency ω_1 is imposed on the plate in its own plane.

An unsteady hydromagnetic dusty fluid flow in a rotating co-ordinate system is governed by the following equations [1]:

For fluid phase:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} + 2\vec{\Omega} \times \vec{u} = -\nabla p_1 + \frac{1}{\rho}(\vec{J} \times \vec{B}) + \nu \nabla^2 \vec{u} + \frac{KN}{\rho}(\vec{v} - \vec{u}), \tag{2.1}$$

$$\nabla \cdot \vec{u} = 0, \tag{2.2}$$

For dust phase:

$$m\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} + 2\vec{\Omega} \times \vec{v}\right] = K(\vec{u} - \vec{v}), \tag{2.3}$$

$$\frac{\partial N}{\partial t} + \nabla \cdot \vec{v} = 0, \tag{2.4}$$

we have the following nomenclature:

$\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$ are the velocities of fluid and dust phase respectively, p1- pressure field including the centrifugal term, $\vec{J}$-electric current density, $\vec{B}$-total magnetic field, N-number density of dust particles, m-mass of the dust particle, K- Stokes-co-efficient of resistance, ρ-density, ν-kinematic viscosity of the fluid, μ_e magnetic permeability and t-time.

Assuming that the magnetic Reynolds number to be small, we neglect the induced magnetic field in comparison with the applied magnetic field. The generalized Ohm's law, in the absence of the electric field is

$$\vec{J} + \frac{\omega_e \tau_e}{B_0}(\vec{J} \times \vec{B}) = \sigma\left[\vec{u} \times \vec{B} + \frac{1}{en_e}\nabla p_e\right], \tag{2.5}$$

where ω_e, τ_e, σ, e, p_e and n_e are respectively the cyclotron frequency of electrons, the electron collision time, the electrical conductivity, the electron charge, the electron pressure and the number density of the electron. The ion-slip and thermoelectric effects are not included in equation (2.5). Further, it is assumed that $\omega_e \tau_e \sim O(1)$ and $\omega_i \tau_i \ll 1$, where ω_i and τ_i are cyclotron frequency and collision time for ions respectively.

Now assume that the velocity field depends on z and t only, so that

$$u(z, t) = [u_1(z, t),\ u_2(z, t),\ u_3(z, t)], \tag{2.6}$$

$$v(z, t) = [v_1(z, t),\ v_2(z, t),\ v_3(z, t)]. \tag{2.7}$$

For the present problem

$$u_3(z, t) = 0,\ v_3(z, t) = 0 \quad \text{and} \quad N = N_0 \text{ (constant)}. \tag{2.8}$$

The equations of motion (2.1) and (2.3) takes the form

$$\frac{\partial u_1}{\partial t} - 2\Omega u_2 = \nu \frac{\partial^2 u_1}{\partial z^2} + \frac{\sigma B_0^2}{\rho(1+m^2)}(mu_2 - u_1) - \frac{l}{\tau}(u_1 - v_1), \tag{2.9}$$

$$\frac{\partial u_2}{\partial t} - 2\Omega u_1 = \nu \frac{\partial^2 u_2}{\partial z^2} + \frac{\sigma B_0^2}{\rho(1+m^2)}(mu_1 - u_2) - \frac{l}{\tau}(u_2 - v_2), \tag{2.10}$$

$$\frac{\partial v_1}{\partial t} - 2\Omega v_2 = \frac{1}{\tau}(u_1 - v_1), \tag{2.11}$$

$$\frac{\partial v_2}{\partial t} - 2\Omega v_1 = \frac{1}{\tau}(u_2 - v_2). \tag{2.12}$$

where $l = \frac{mN_0}{\rho}$ (mass concentration) and $\tau = \frac{m}{K}$ (relaxation time).

Introducing the notation $p = u_1 + iu_2$ and $q = v_1 + iv_2$ in the equations (2.9) to (2.12), we get

$$\frac{\partial v_2}{\partial t} - 2\Omega v_1 = C + A\cos(Bt) + \nu \frac{\partial^2 p}{\partial z^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}(1+im)\,p + \frac{l}{\tau}(q-p), \tag{2.13}$$

$$\frac{\partial v_2}{\partial t} - 2\Omega v_1 = \frac{1}{\tau}(p-q). \tag{2.14}$$

In view of the imposed oscillation on the plate, equations (2.13) and (2.14) have to be solved subject to a non-slip boundary condition at the plate and no disturbance at infinity as

$$p(z,t) = p_0 + p_0^*(a_1 e^{i\omega_1 t} + b_1 e^{-i\omega_1 t}), \tag{2.15}$$

$$q(z,t) = q_0 + p_0^*(a_1 e^{i\omega_1 t} + b_1 e^{-i\omega_1 t}) \quad \text{on} \quad z = 0,\ t > 0, \tag{2.16}$$

$$p(z,t),\quad q(z,t) \to 0 \ \text{ as } \ z \to \infty,\ t > 0, \tag{2.17}$$

where p_0, p_0^*, q_0 are constants with the dimention of velocities, and a, b, a_1 and b_1 are complex constants, so that the real and imaginary parts of $p(z,t)$ and $q(z,t)$ become real on the plate.

The initial conditions of the problem are

$$p(z,t) = q(z,t) = 0 \quad \text{at} \quad t \le 0 \ \text{ for all } \ z. \tag{2.18}$$

3. SOLUTION OF THE PROBLEM

To make the above system dimensionless, introduce the following non-dimensional variables

$$z = \frac{zU^*}{\nu}, \quad t' = \Omega t, \quad p' = \frac{p}{U^*}, \quad q' = \frac{q}{U^*}, \quad C' = \frac{\nu}{U^{*3}}.$$

and the non-dimensional parameter

$$E = \frac{2\Omega\nu}{U^{*2}}, \qquad \text{the Ekman number,}$$

$$m = \omega_e \tau_e, \qquad \text{the Hall parameter,}$$

$$M = \left(\frac{\sigma\nu B_0^2}{\rho U^{*2}}\right)^{\frac{1}{2}}, \qquad \text{the magnetic parameter,}$$

also $\omega = \omega_1/\Omega$ is the non-dimensional frequency of oscillation and $\tau' = \tau U^{*2}/\nu = mU^{*2}/k\nu$.

After non-dimensionalizing equations (2.13) and (2.14) and the boundary and initial conditions (2.15)-(2.18) can be written as follows:

$$\frac{\partial^2 p}{\partial z^2} - \frac{E}{2}\frac{\partial p}{\partial t} - \left[iE + \frac{M^2}{(1-im)} \right] p + C + A\cos(Bt) - \frac{l}{\tau}(p-q) = 0\,, \tag{3.1}$$

$$\frac{E}{2}\frac{\partial q}{\partial t} + iEq - \frac{l}{\tau}(p-q) = 0, \qquad z > 0\,, \tag{3.2}$$

$$p = \frac{p_0}{U^*} + ae^{i\omega t} + be^{i\omega t}\,, \tag{3.3}$$

$$q = \frac{q_0}{U^*} + a_1 e^{i\omega t} + b_1 e^{-i\omega t} \qquad \text{on} \qquad z = 0, \quad t > 0\,, \tag{3.4}$$

$$p, q \to 0 \quad \text{as} \quad z \to \infty, \quad t > 0, \tag{3.5}$$

$$p, q = 0 \quad \text{at} \quad t — 0 \quad \text{for all} \quad z. \tag{3.6}$$

Case 1: In this case the constant pressure gradient is considered i.e.,

$$-\frac{1}{\rho}\nabla p_1 = C \text{ (constant)}. \tag{3.7}$$

To solve the initial value problem, we introduce the Laplace transforms for fluid and dust velocities of $p(z, t)$ and $q(z, t)$ respectively as,

$$\overline{p}(z,s) = \int_0^\infty e^{-st} p(z,t)\,dt \qquad \text{and} \qquad \overline{q}(z,s) = \int_0^\infty e^{-st} q(z,t)\,dt\,. \tag{3.8}$$

On applying the Laplace transform, the equations (3.1) and (3.2) reduces second order differential equations and are solved by using the transformed boundary conditions. The solutions for $p(z, s)$ and $q(z, s)$ are obtained as

$$\overline{p} = \left[\frac{p_0}{U^* s} + \frac{a}{s-i\omega} + \frac{b}{s+i\omega} - \frac{C}{Ks} \right] e^{-z\sqrt{K}} + \frac{C}{Ks}, \tag{3.9}$$

$$\overline{q} = \left[\frac{q_0}{U^* s} + \frac{a_1}{s-i\omega} + \frac{b_1}{s+i\omega} - \frac{C}{Ks} \right] e^{-z\sqrt{K}} + \frac{C}{Ks}. \tag{3.10}$$

3.1 Solutions for Small Times

The nature of the flow fields $p(z, t)$ and $q(z, t)$ for small times can be determined by the asymptotic behavior of their Laplace transforms for the large value of $|s|$ are given by

$$\overline{p} = \left[\frac{p_0}{U^* s} + \frac{a}{s-i\omega} + \frac{b}{s+i\omega} - \frac{C}{\frac{E}{2}s^2} \right] e^{-z\sqrt{\frac{E}{2}s}} + \frac{C}{s\sqrt{\frac{E}{2}s}}, \tag{3.11}$$

$$\overline{q} = \left[\frac{q_0}{U^* s} + \frac{a_1}{s-i\omega} + \frac{b_1}{s+i\omega} - \frac{2C}{E^2\tau s^2(s+D)} \right] e^{-z\sqrt{\frac{E}{2}s}} + \frac{2C}{s^2E^2\tau(s+D)}. \tag{3.12}$$

Taking inverse Laplace transform to equations (3.18) and (3.19) one can get

$$p(z,t) \sim \frac{ae^{i\omega t}}{2}\left[e^{z\sqrt{\frac{iE\omega}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{i\omega t}\right\}+e^{-z\sqrt{\frac{iE\omega}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}-\sqrt{i\omega t}\right\}\right]$$

$$+\frac{be^{-i\omega t}}{2}\left[e^{z\sqrt{\frac{-iE\omega}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{-i\omega t}\right\}+e^{-z\sqrt{\frac{-iE\omega}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}-\sqrt{-i\omega t}\right\}\right]$$

$$+\frac{p_0}{U^*}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{E}{2t}}\right\}-\frac{2C}{E}\left[\left(t+\frac{z^2E}{4}\right) erfc\left\{z\frac{1}{2}\sqrt{\frac{E}{2t}}\right\}-ze^{\frac{-z^2E}{8t}}\sqrt{\frac{E}{2\pi}}\right]+\frac{2Ct}{E},$$

$$q(z,t) \sim \frac{a_1e^{i\omega t}}{2}\left[e^{z\sqrt{\frac{iE\omega}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{i\omega t}\right\}+e^{-z\sqrt{\frac{iE\omega}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}-\sqrt{i\omega t}\right\}\right]$$

$$+\frac{b_1e^{-i\omega t}}{2}\left[e^{z\sqrt{\frac{-iE\omega}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{-i\omega t}\right\}+e^{-z\sqrt{\frac{-iE\omega}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}-\sqrt{-i\omega t}\right\}\right]$$

$$+\frac{q_0}{U^*}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{E}{2t}}\right\}-\frac{4C}{E^2\tau}\left[\frac{e^{-Dt}}{2D^2}\left[e^{z\sqrt{\frac{-ED}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{-Dt}\right\}\right.\right.$$

$$\left.\left.-\frac{1}{D^2}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}\right\}+\frac{1}{D}\left[\left(t+\frac{z^2E}{4}\right) erfc\left\{\frac{z}{2}\sqrt{\frac{E}{2t}}\right\}-ze^{\frac{-z^2E}{8t}}\sqrt{\frac{E}{2\pi}}\right]\right]\right]$$

$$+\frac{2C}{E^2\tau}\left[\frac{1}{D^2}(e^{-Dt}-1)+\frac{t}{D}\right].$$

From the above solution one can see that immediately after the pulsatile motion is imposed on the plate, an unsteady boundary layer flow builds up in the vicinity of the plate. Further the solution consists of Stokes layer of thickness of order $\sqrt{\frac{\nu}{\omega}}$ and the Rayleigh layer of order $\sqrt{\nu t}$. Also one can observe that the solution is remains unaffected by the dusty parameter as well as rotation and magnetic term. Similar discussion is true for $q(z, t)$ also.

3.2 Solutions for Large Times

Solutions $p(z, t)$ and $q(z, t)$ for large times can be determined by the asymptotic behavior of their Laplace transforms for the small value of |s| are given by

$$\bar{p}=\left[\frac{p_0}{U^*s}+\frac{a}{s-i\omega}+\frac{b}{s+i\omega}-\frac{C\delta}{\gamma Ls(s+\frac{1}{L})}\right]e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{s+\frac{1}{L}}}+\frac{C\delta}{\gamma Ls(s+\frac{1}{L})} \tag{3.13}$$

$$\bar{q}=\left[\frac{q_0}{U^*s}+\frac{a_1}{s-i\omega}+\frac{b_1}{s+i\omega}-\frac{C\delta}{\gamma LE\tau(s+D)s(s+\frac{1}{L})}\right]e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{s+\frac{1}{L}}}+\frac{C\delta}{\gamma L\tau E(s+D)s(s+\frac{1}{L})} \tag{3.14}$$

Taking inverse Laplace transform of equations (3.20) and (3.20) we get

$$p(z,t) \sim \frac{ae^{i\omega t}}{2}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}+i\omega\right)t}\right\}\right.$$

$$\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}+i\omega\right)t}\right\}\right]$$

$$+\frac{be^{-i\omega t}}{2}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}-i\omega\right)t}\right\}\right.$$

$$\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}-i\omega\right)t}\right\}\right]$$

$$+\frac{p_0}{2U^*}\left[e^{z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\frac{t}{L}}\right\}+e^{-z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\frac{t}{L}}\right\}\right]$$

$$-\frac{\delta C}{\gamma}\left[\frac{1}{2}\left[e^{z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\frac{t}{L}}\right\}+e^{-z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\frac{t}{L}}\right\}\right]\right.$$

$$\left.-e^{\frac{-t}{L}}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}\right\}\right]+\frac{\delta C}{\gamma}[1-e^{\frac{-t}{L}}],$$

$$q(z,t) \sim \frac{a_1 e^{i\omega t}}{2}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}+i\omega\right)t}\right\}\right.$$

$$\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}+i\omega\right)t}\right\}\right]$$

$$+\frac{b_1 e^{-i\omega t}}{2}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}-i\omega\right)t}\right\}\right.$$

$$\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}-i\omega\right)t}\right\}\right]$$

$$+\frac{q_0}{2U^*}\left[e^{z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\frac{t}{L}}\right\}+e^{-z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\frac{t}{L}}\right\}\right]$$

$$-\frac{2C\delta}{\gamma E\tau}\left\{\frac{1}{D}\left[e^{z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{\frac{z}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\frac{t}{L}}\right\}+e^{-z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{\frac{z}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\frac{t}{L}}\right\}\right]\right.$$

$$-\frac{(L)}{(DL-1)}e^{-\frac{t}{L}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}\right\}$$

$$+\frac{e^{-Dt}}{2(D^2L-D)}\left[e^{z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}+D}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}+D\right)t}\right\}\right.$$

$$\left.+e^{-z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}+D}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}+D\right)t}\right\}\right]\Bigg\}$$

$$+\frac{C\delta}{\gamma E\tau}\left[\frac{1}{D}-\frac{Le^{\frac{-t}{L}}}{(DL-1)}-\frac{e^{-Dt}}{(D^2L-D)}\right].$$

Case 2: Here the periodic pressure gradient is considered i.e.,

$$-\frac{1}{\rho}\nabla p_1 = A\cos(Bt). \tag{3.15}$$

Applying the same procedure as in the case-1, the solutions for $\overline{p}\,(z, s)$, and $\overline{q}\,(z, s)$ are obtained as

$$\overline{p}=\left[\frac{p_0}{U^*s}+\frac{a}{s-i\omega}+\frac{b}{s+i\omega}-\frac{1}{K}\left(\frac{As}{s^2+B^2}\right)\right]e^{-z\sqrt{K}}+\frac{1}{K}\left(\frac{As}{s^2+B^2}\right), \tag{3.16}$$

$$\overline{q}=\left[\frac{p_0}{U^*s}+\frac{a_1}{s-i\omega}+\frac{b_1}{s+i\omega}-\frac{1}{K}\left(\frac{As}{s^2+B^2}\right)\right]e^{-z\sqrt{K}}+\frac{1}{K}\left(\frac{As}{s^2+B^2}\right). \tag{3.17}$$

3.3 Solutions for Small Times

The nature of the flow fields $p\,(z, t)$ and $q\,(z, t)$ for small times can be determined by the asymptotic behavior of their Laplace transforms for the large value of $|s|$ are given by

$$\overline{p}=\left[\frac{p_0}{U^*s}+\frac{a}{s-i\omega}+\frac{b}{s+i\omega}-\frac{2}{E}\left(\frac{A}{s^2+B^2}\right)\right]e^{-z\sqrt{\frac{E}{2}s}}+\frac{2}{E}\left(\frac{A}{s^2+B^2}\right), \tag{3.18}$$

$$\overline{q}=\left[\frac{q_0}{U^*s}+\frac{a_1}{s-i\omega}+\frac{b_1}{s+i\omega}-\frac{2}{E^2\tau}\left(\frac{A}{(s+D)(s^2+B^2)}\right)\right]e^{-z\sqrt{\frac{E}{2}s}}+\frac{2}{E^2\tau}\left(\frac{A}{(s+D)(s^2+B^2)}\right). \tag{3.19}$$

Taking inverse Laplace transform to equations (3.18) and (3.19) one can get

$$p(z,t)\sim\frac{ae^{i\omega t}}{2}\left[e^{z\sqrt{\frac{iE\omega}{2}}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{i\omega t}\right\}+e^{-z\sqrt{\frac{iE\omega}{2}}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}-\sqrt{i\omega t}\right\}\right]$$

$$+\frac{be^{-i\omega t}}{2}\left[e^{z\sqrt{\frac{-iE\omega}{2}}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{-i\omega t}\right\}+e^{-z\sqrt{\frac{-iE\omega}{2}}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}-\sqrt{-i\omega t}\right\}\right]$$

$$-\frac{Ae^{iwt}}{2EB^2}\left[e^{z\sqrt{\frac{iEB}{2}}}\, erfc\left\{\frac{z}{2}\sqrt{\frac{E}{2t}}+\sqrt{iBt}\right\}+e^{-z\sqrt{\frac{iEB}{2}}}\, erfc\left\{\frac{z}{2}\sqrt{\frac{E}{2t}}-\sqrt{iBt}\right\}\right]$$

$$-\frac{Ae^{iwt}}{2EB^2}\left[e^{z\sqrt{\frac{-iEB}{2}}}\, erfc\left\{\frac{z}{2}\sqrt{\frac{E}{2t}}+\sqrt{iBt}\right\}+e^{-z\sqrt{\frac{-iEB}{2}}}\, erfc\left\{\frac{z}{2}\sqrt{\frac{E}{2t}}-\sqrt{-iBt}\right\}\right]-\frac{A}{EB^2}(e^{iwt}+e^{-iwt}).$$

$$q(z,t)\sim\frac{a_1 e^{i\omega t}}{2}\left[e^{z\sqrt{\frac{iE\omega}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{i\omega t}\right\}+e^{-z\sqrt{\frac{iE\omega}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}-\sqrt{i\omega t}\right\}\right]$$

$$+\frac{b_1 e^{-i\omega t}}{2}\left[e^{z\sqrt{\frac{-iE\omega}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{-i\omega t}\right\}+e^{-z\sqrt{\frac{-iE\omega}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}-\sqrt{-i\omega t}\right\}\right]$$

$$+\frac{q_0}{U^*}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{E}{2t}}\right\}+\frac{4}{E^2\tau}\left[\frac{A}{DB^2}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}\right\}\right.$$

$$-\frac{Ae^{-Dt}}{2(D^2+B^2)}\left[e^{z\sqrt{\frac{-ED}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{-Dt}\right\}+e^{-z\sqrt{\frac{-ED}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}-\sqrt{-Dt}\right\}\right]$$

$$-\frac{Ae^{-iBt}}{4iB(D-iB)}\left[e^{z\sqrt{\frac{-iEB}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{-iBt}\right\}+e^{-z\sqrt{\frac{-iEB}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}-\sqrt{-iBt}\right\}\right]$$

$$\left.-\frac{Ae^{-iBt}}{4iB(D+iB)}\left[e^{z\sqrt{\frac{-iEB}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{-iBt}\right\}+e^{-z\sqrt{\frac{-iEB}{2}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}-\sqrt{-iBt}\right\}\right]\right]$$

$$+\frac{4}{E^2\tau}\left[+\frac{A}{D^2+B^2}e^{-Dt}+\frac{A}{2iB}\left(\frac{e^{iBt}}{D+iB}-\frac{e^{-iBt}}{D-iB}\right)\right].$$

3.4 Solutions for Large Times

Solutions $p(z,t)$ and $q(z,t)$ for large times can be determined by the asymptotic behavior of their Laplace transforms for the small value of $|s|$ are given by

$$\overline{p}=\left[\frac{p_0}{U^*s}+\frac{a}{s-i\omega}+\frac{b}{s+i\omega}-\frac{\delta}{\gamma L(s+\frac{1}{L})}\left(\frac{As}{s^2+B^2}\right)\right]e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{s+\frac{1}{L}}}+\frac{\delta}{\gamma L(s+\frac{1}{L})}\left(\frac{As}{s^2+B^2}\right),$$

$$\overline{q}=\left[\frac{q_0}{U^*s}+\frac{a_1}{s-i\omega}+\frac{b_1}{s+i\omega}-\frac{2\delta}{E\gamma\tau L(s+D)(s+\frac{1}{L})}\left(\frac{As}{s^2+B^2}\right)\right]e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{s+\frac{1}{L}}}$$

$$+\frac{2\delta}{E\gamma\tau L(s+\frac{1}{L})}\left(\frac{As}{s^2+B^2}\right). \tag{3.20}$$

Taking inverse Laplace transform of equations (3.20) and (3.20) we get

$$p(z,t) \sim \frac{ae^{i\omega t}}{2}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}+i\omega\right)t}\right\}\right.$$

$$\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}+i\omega\right)t}\right\}\right]$$

$$+\frac{be^{-i\omega t}}{2}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}-i\omega\right)t}\right\}\right.$$

$$\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}-i\omega\right)t}\right\}\right]$$

$$+\frac{p_0}{2U^*}\left[e^{z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\frac{t}{L}}\right\}+e^{-z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\frac{t}{L}}\right\}\right]$$

$$-\frac{\delta Ae^{-iBt}}{\gamma 4(1-iBL)}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-iB}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}-iB\right)t}\right\}\right.$$

$$\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-iB}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}-iB\right)t}\right\}\right]$$

$$-\frac{\delta Ae^{iBt}}{\gamma 4(1+iBL)}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+iB}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}+iB\right)t}\right\}\right.$$

$$\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+iB}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}+iB\right)t}\right\}\right]$$

$$+\frac{\delta A}{\gamma(1+B^2L^2)}e^{\frac{-t}{L}}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}\right\}+\frac{A\delta}{\gamma}\left[\frac{e^{iBt}}{2(1-iBL)}+\frac{e^{-iBt}}{2(1+iBL)}-\frac{e^{\frac{-t}{L}}}{1+B^2L^2}\right],$$

$$q(z,t) \sim \frac{a_1e^{i\omega t}}{2}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}+i\omega\right)t}\right\}\right.$$

$$\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}+i\omega\right)t}\right\}\right]$$

$$+\frac{b_1e^{-i\omega t}}{2}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}-i\omega\right)t}\right\}\right.$$

$$+ e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-i\omega}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}} - \sqrt{\left(\frac{1}{L}-i\omega\right)t}\right\}\Bigg]\Bigg]$$

$$+ \frac{q_0}{2U^*}\left[e^{z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{\frac{z}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}} + \sqrt{\frac{t}{L}}\right\} + e^{-z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{\frac{z}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}} - \sqrt{\frac{t}{L}}\right\}\right]$$

$$- \frac{2\delta}{\gamma E\tau}\left\{\frac{ADe^{-Dt}}{2(DL-1)}\left[e^{z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}-D}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}} + \sqrt{\left(\frac{1}{L}-D\right)t}\right\}\right.\right.$$

$$\left. + e^{-z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}-D}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}} - \sqrt{\left(\frac{1}{L}-D\right)t}\right\}\right] + \frac{AL}{(L-D)(1+B^2L^2)}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}\right\}$$

$$+ \frac{Ae^{-iBt}}{4(D-iB)(1-iLB)}\left[e^{z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}-iB}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}} + \sqrt{\left(\frac{1}{L}-iB\right)t}\right\}\right.$$

$$\left. + e^{z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}-iB}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}} + \sqrt{\left(\frac{1}{L}-iB\right)t}\right\}\right] + \frac{AL}{(L-D)(1+B^2L^2)}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}\right\}$$

$$+ \frac{Ae^{iBt}}{4(D+iB)(1+iLB)}\left[e^{z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}+iB}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}} + \sqrt{\left(\frac{1}{L}+iB\right)t}\right\}\right.$$

$$\left.\left. + e^{z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}+iB}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}} + \sqrt{\left(\frac{1}{L}+iB\right)t}\right\}\right]\right\}$$

$$+ \frac{2\delta}{\gamma E\tau}\left[\frac{Ae^{\frac{-t}{L}}}{(L-D)(1+B^2L^2)} + \frac{Ae^{-iBt}}{2(D-iB)(1-iBL)} + \frac{Ae^{iBt}}{2(D+iB)(1+iBL)}\right].$$

Case 3: By combining the case 1 and case 2, the pressure gradient becomes pulsatile, i.e.,

$$-\frac{1}{\rho}\nabla p_1 = C + A\cos(Bt). \tag{3.21}$$

Where C and A are constants, and B is the infrequency of oscillation. The solutions for $p(z, s)$, and $q(z, s)$ are obtained as

$$\overline{p} = \left[\frac{p_0}{U^*s} + \frac{a}{s-i\omega} + \frac{b}{s+i\omega} - \frac{1}{K}\left(\frac{C}{S} + \frac{As}{s^2+B^2}\right)\right]e^{-z\sqrt{K}} + \frac{1}{K}\left(\frac{C}{S} + \frac{As}{s^2+B^2}\right), \tag{3.22}$$

$$\overline{q} = \left[\frac{q_0}{U^*s} + \frac{a_1}{s-i\omega} + \frac{b_1}{s+i\omega} - \frac{1}{K}\left(\frac{C}{S} + \frac{As}{s^2+B^2}\right)\right]e^{-z\sqrt{K}} + \frac{1}{K}\left(\frac{C}{S} + \frac{As}{s^2+B^2}\right). \tag{3.23}$$

3.5 Solutions for Small Times

The nature of the flow fields $p(z, t)$ and $q(z, t)$ for small times can be determined by the asymptotic behavior of their Laplace transforms for the large value of $|s|$ are given by

$$\bar{p} = \left[\frac{p_0}{U^* s} + \frac{a}{s - i\omega} + \frac{b}{s + i\omega} - \frac{2}{E}\left(\frac{C}{S^2} + \frac{A}{s^2 + B^2}\right)\right] e^{-z\sqrt{\frac{E}{2}s}} + \frac{2}{E}\left(\frac{C}{S^2} + \frac{A}{s^2 + B^2}\right), \tag{3.24}$$

$$\bar{q} = \left[\frac{q_0}{U^* s} + \frac{a_1}{s - i\omega} + \frac{b_1}{s + i\omega} - \frac{2}{E^2\tau}\left(\frac{C}{S^2(s + D)} + \frac{A}{(s + D)(s^2 + B^2)}\right)\right] e^{-z\sqrt{\frac{E}{2}s}}$$
$$+ \frac{2}{E^2\tau}\left(\frac{C}{S^2(s + D)} + \frac{A}{(s + D)(s^2 + B^2)}\right). \tag{3.25}$$

Taking inverse Laplace transform to equations (3.24) and (3.25) one can get

$$p(z, t) \sim \frac{ae^{i\omega t}}{2}\left[e^{z\sqrt{\frac{iE\omega}{2}}}\, erfc\left\{\frac{1}{2} z\sqrt{\frac{E}{2t}} + \sqrt{i\omega t}\right\} + e^{-z\sqrt{\frac{iE\omega}{2}}}\, erfc\left\{\frac{1}{2} z\sqrt{\frac{E}{2t}} - \sqrt{i\omega t}\right\}\right]$$
$$+ \frac{be^{-i\omega t}}{2}\left[e^{z\sqrt{\frac{-iE\omega}{2}}}\, erfc\left\{\frac{1}{2} z\sqrt{\frac{E}{2t}} + \sqrt{-i\omega t}\right\} + e^{-z\sqrt{\frac{-iE\omega}{2}}}\, erfc\left\{\frac{1}{2} z\sqrt{\frac{E}{2t}} - \sqrt{-i\omega t}\right\}\right]$$
$$+ \frac{q_0}{U^*}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{E}{2t}}\right\} + \frac{2C}{E}\left[\left(t + \frac{z^2 E}{4}\right) erfc\left\{\frac{1}{2} z\sqrt{\frac{E}{2t}}\right\} - ze^{\frac{-z^2 E}{8t}}\sqrt{\frac{E}{2t}}\right] - \frac{Ae^{iwt}}{2EB^2}$$
$$\times\left[e^{z\sqrt{\frac{iEB}{2}}}\, erfc\left\{\frac{z}{2}\sqrt{\frac{E}{2t}} + \sqrt{iBt}\right\} + e^{-z\sqrt{\frac{iEB}{2}}}\, erfc\left\{\frac{z}{2}\sqrt{\frac{E}{2t}} - \sqrt{iBt}\right\}\right]$$
$$- \frac{Ae^{-iwt}}{2EB^2}\left[e^{z\sqrt{\frac{-iEB}{2}}}\, erfc\left\{\frac{z}{2}\sqrt{\frac{E}{2t}} + \sqrt{-iBt}\right\} + e^{-z\sqrt{\frac{iEB}{2}}} \times erfc\left\{\frac{z}{2}\sqrt{\frac{E}{2t}} - \sqrt{-iBt}\right\}\right]$$
$$+ \frac{2Ct}{E} - \frac{A}{EB^2}(e^{iwt} + e^{-iwt}),$$

$$q(z, t) \sim \frac{a_1 e^{i\omega t}}{2}\left[e^{z\sqrt{\frac{iE\omega}{2}}}\, erfc\left\{\frac{1}{2} z\sqrt{\frac{E}{2t}} + \sqrt{i\omega t}\right\} + e^{-z\sqrt{\frac{iE\omega}{2}}}\, erfc\left\{\frac{1}{2} z\sqrt{\frac{E}{2t}} - \sqrt{i\omega t}\right\}\right]$$
$$+ \frac{b_1 e^{-i\omega t}}{2}\left[e^{z\sqrt{\frac{-iE\omega}{2}}}\, erfc\left\{\frac{1}{2} z\sqrt{\frac{E}{2t}} + \sqrt{-i\omega t}\right\} + e^{-z\sqrt{\frac{-iE\omega}{2}}}\, erfc\left\{\frac{1}{2} z\sqrt{\frac{E}{2t}} - \sqrt{-i\omega t}\right\}\right]$$
$$+ \frac{q_0}{U^*}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{E}{2t}}\right\} - \frac{4}{E^2\tau}\left[\frac{Ce^{-Dt}}{2D^2}\left[e^{z\sqrt{\frac{-ED}{2}}}\, erfc\left\{\frac{1}{2} z\sqrt{\frac{E}{2t}} + \sqrt{-Dt}\right\}\right.\right.$$
$$\left.\left. + e^{-z\sqrt{\frac{-ED}{2}}}\, erfc\left\{\frac{1}{2} z\sqrt{\frac{E}{2t}} - \sqrt{-Dt}\right\}\right] + \frac{C}{D^2}\, erfc\left\{\frac{1}{2} z\sqrt{\frac{E}{2t}}\right\}\right.$$

$$+\frac{C}{D}\left[\left(t+\frac{z^2E}{4}\right)erfc\left\{\frac{z}{2}\sqrt{\frac{E}{2t}}\right\}-ze^{\frac{-z^2D}{8t}}\sqrt{\frac{E}{2t}}\right]$$

$$-\frac{Ae^{-Dt}}{2(D^2+B^2)}\left[e^{z\sqrt{\frac{-ED}{2}}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{-Dt}\right\}+e^{-z\sqrt{\frac{-ED}{2}}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}-\sqrt{-Dt}\right\}\right]$$

$$-\frac{Ae^{-iBt}}{4iB(D-iB)}\left[e^{z\sqrt{\frac{-iEB}{2}}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{-iBt}\right\}+e^{-z\sqrt{\frac{-iEB}{2}}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}-\sqrt{-iBt}\right\}\right]$$

$$-\frac{Ae^{-iBt}}{4iB(D+iB)}\times\left[e^{z\sqrt{\frac{-iEB}{2}}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}+\sqrt{-iBt}\right\}+e^{-z\sqrt{\frac{-iEB}{2}}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{E}{2t}}-\sqrt{-iBt}\right\}\right]\Bigg]$$

$$+\frac{4}{E^2\tau}\left[\frac{C}{D^2}(e^{-Dt}-1)\frac{Ct}{D}+\frac{A}{D^2+B^2}e^{-Dt}+\frac{A}{2iB}\left(\frac{e^{iBt}}{D+iB}-\frac{e^{-iBt}}{D-iB}\right)\right]\Bigg].$$

3.6 Solutions for Large Times

Solutions $p(z, t)$ and $q(z, t)$ for large times can be determined by the asymptotic behavior of their Laplace transforms for the small value of $|s|$ are given by

$$\overline{p}=\left[\frac{p_0}{U^*s}+\frac{a}{s-i\omega}+\frac{b}{s+i\omega}-\frac{\delta}{\gamma L(s+\frac{1}{L})}\left(\frac{C}{S}+\frac{As}{s^2+B^2}\right)\right]e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{s+\frac{1}{L}}}+\frac{\delta}{\gamma L(s+\frac{1}{L})}\left(\frac{C}{S}+\frac{As}{s^2+B^2}\right), \quad (3.26)$$

$$\overline{q}=\left[\frac{q_0}{U^*s}+\frac{a_1}{s-i\omega}+\frac{b_1}{s+i\omega}-\frac{2\delta}{E\gamma\tau L(s+D)(s+\frac{1}{L})}\left(\frac{C}{S}+\frac{As}{s^2+B^2}\right)\right]e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{s+\frac{1}{L}}}$$
$$+\frac{2\delta}{E\gamma\tau L(s+\frac{1}{L})}\left(\frac{C}{S}+\frac{As}{s^2+B^2}\right). \quad (3.27)$$

Taking inverse Laplace transform of equations (3.26) and (3.27) we get

$$p(z,t)\sim\frac{ae^{i\omega t}}{2}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+i\omega}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}+i\omega\right)t}\right\}\right.$$

$$\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+i\omega}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}+i\omega\right)t}\right\}\right]$$

$$+\frac{be^{-i\omega t}}{2}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-i\omega}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}-i\omega\right)t}\right\}\right.$$

$$\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-i\omega}}\,erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}-i\omega\right)t}\right\}\right]$$

$$+\frac{p_0}{2U^*}\left[e^{z\sqrt{\frac{\gamma}{\delta}}}\,erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\frac{t}{L}}\right\}+e^{-z\sqrt{\frac{\gamma}{\delta}}}\,erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\frac{t}{L}}\right\}\right]$$

$$
\begin{aligned}
&-\frac{\delta C}{\gamma}\left[\frac{1}{2}\left[e^{z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\frac{t}{L}}\right\}\right.\right.\\
&\left.\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\frac{t}{L}}\right\}\right]-e^{\frac{-t}{L}}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}\right\}\right]\\
&-\frac{\delta A e^{-iBt}}{\gamma 4(1-iBL)}\times\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-iB}}\; erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}-iB\right)t}\right\}\right.\\
&\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-iB}}\; erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}-iB\right)t}\right\}\right]\\
&-\frac{\delta A e^{iBt}}{\gamma 4(1+iBL)}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+iB}}\; erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}+iB\right)t}\right\}\right.\\
&\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+iB}}\; erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}+iB\right)t}\right\}\right]\\
&+\frac{\delta A}{\gamma(1+B^2L^2)}\, e^{\frac{-t}{L}}\, erfc\left\{z\frac{1}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}\right\}+\frac{C\delta}{\gamma}[1-e^{\frac{-t}{L}}]\\
&+\frac{A\delta}{\gamma}\left[\frac{e^{iBt}}{2(1-iBL)}+\frac{e^{-iBt}}{2(1+iBL)}-\frac{e^{\frac{-t}{L}}}{1+B^2L^2}\right],
\end{aligned}
$$

$$
\begin{aligned}
q(z,t)\sim\;&\frac{a_1 e^{i\omega t}}{2}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+i\omega}}\; erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}+i\omega\right)t}\right\}\right.\\
&\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}+i\omega}}\; erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}+i\omega\right)t}\right\}\right]\\
&+\frac{b_1 e^{-i\omega t}}{2}\left[e^{z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-i\omega}}\; erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}-i\omega\right)t}\right\}\right.\\
&\left.+e^{-z\sqrt{\frac{\gamma}{\delta}}\sqrt{L}\sqrt{\frac{1}{L}-i\omega}}\; erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}-i\omega\right)t}\right\}\right]\\
&+\frac{q_0}{2U^*}\left[e^{z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{\frac{z}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\frac{t}{L}}\right\}+e^{-z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{\frac{z}{2}\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\frac{t}{L}}\right\}\right]\\
&-\frac{2\delta}{\gamma E\tau}\left\{\frac{C}{2D}\left[e^{z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\frac{t}{L}}\right\}\right.\right.
\end{aligned}
$$

$$+e^{-z\sqrt{\frac{\gamma}{\delta}}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\frac{t}{L}}\right\}\Bigg]-\frac{CL}{(DL-1)}e^{-\frac{t}{L}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}\right\}$$

$$\frac{Ce^{-Dt}}{2(D^2L-D)}\left[e^{z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}-D}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}-D\right)t}\right\}\right.$$

$$\left.+e^{-z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}-D}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}-D\right)t}\right\}\right]$$

$$\frac{ADe^{-Dt}}{2(DL-1)}\left[e^{z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}-D}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}-D\right)t}\right\}\right.$$

$$\left.+e^{-z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}-D}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}-\sqrt{\left(\frac{1}{L}-D\right)t}\right\}\right]$$

$$+\frac{AL}{(L-D)(1+B^2L^2)}erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}\right\}$$

$$+\frac{Ae^{-iBt}}{4(D-iB)(1-iLB)}\left[e^{z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}-iB}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}-iB\right)t}\right\}\right.$$

$$\left.+e^{z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}-iB}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}-iB\right)t}\right\}\right]+\frac{AL}{(L-D)(1+B^2L^2)}erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}\right\}$$

$$+\frac{Ae^{iBt}}{4(D+iB)(1+iLB)}\left[e^{z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}+iB}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}+iB\right)t}\right\}\right.$$

$$\left.\left.+e^{z\sqrt{\frac{\gamma L}{\delta}}\sqrt{\frac{1}{L}+iB}}\, erfc\left\{\frac{1}{2}z\sqrt{\frac{\gamma}{\delta}}\sqrt{\frac{L}{t}}+\sqrt{\left(\frac{1}{L}+iB\right)t}\right\}\right]\right\}$$

$$+\frac{2\delta}{\gamma E\tau}\left[\frac{C}{D}-\frac{CLe^{\frac{-t}{L}}}{(DL-1)}+\frac{Ce^{-Dt}}{(D^2L-D)}+\frac{Ae^{\frac{-t}{L}}}{(L-D)(1+B^2L^2)}\right.$$

$$\left.+\frac{Ae^{-iBt}}{2(D-iB)(1-iBL)}+\frac{Ae^{iBt}}{2(D+iB)(1+iBL)}\right].$$

where

$$L=\frac{\beta\delta-\gamma\lambda}{2\gamma\delta},\qquad K=\frac{\alpha s^2+\beta s+\gamma}{\lambda s+\delta},\qquad \alpha=\frac{\tau^2E^2}{4}(1-im),$$

$$\beta=\tau^2E(1-im)\left(iE+\frac{1}{2\tau}\right)+\frac{E}{2}\tau(M^2\tau-l)$$

$$\gamma = -E\tau^2(1-im)\left(E-\frac{i}{\tau}\right)+im^2\tau^2E+M^2\tau+ilE\tau(1-im)$$

$$\lambda=(1-im)\,\tau\frac{E}{2},\qquad \delta=\tau\,(1-im)\,(iE\tau+1),\qquad D=2i+\frac{2}{M\tau}.$$

4. RESULTS AND DISCUSSION

The present investigation deals with the study of an unsteady hydromagnetic boundary layer flow in rotating dusty fluid in the presence of hall current and time dependent pressure gradient. The governing equations are solved by applying the of Laplace transform technique. The effect of various physical parameters like Ekman number E, magnetic parameter M and Hall current parameter m are examined and depicted graphically. Further it is found that, the thickness of the boundary layer changes with the Ekman number, magnetic parameter and Hall current parameter. In fact, the boundary layer thickness increases with increase in Hall current parameter. Similar prediction for Hall current effect is also made by Debnath [8] and R. Tiwari and Kamal singh [3].

Figure 1, 2 and 3 depict the velocity profile for fluid and dust phase versus z, for different values of Ekman number E for three cases respectively, We infer from these figures that the fluid and dust velocities decreases with the increasing in Ekman number for both small and large times.

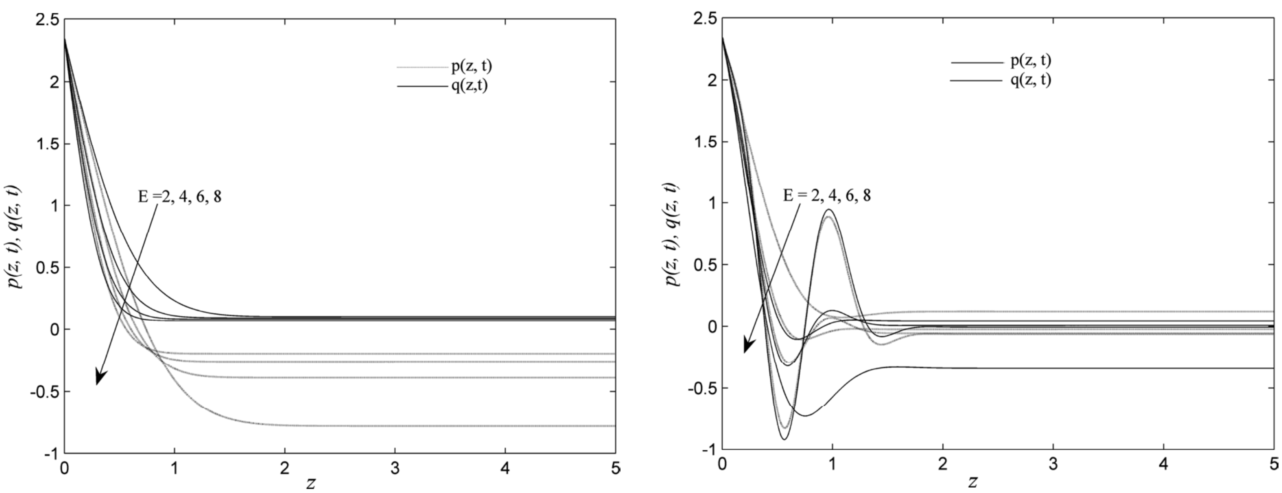


Figure 1: Effect of Ekman Number on Fluid and Dust Velocity for Small and Large Times (Case 1)

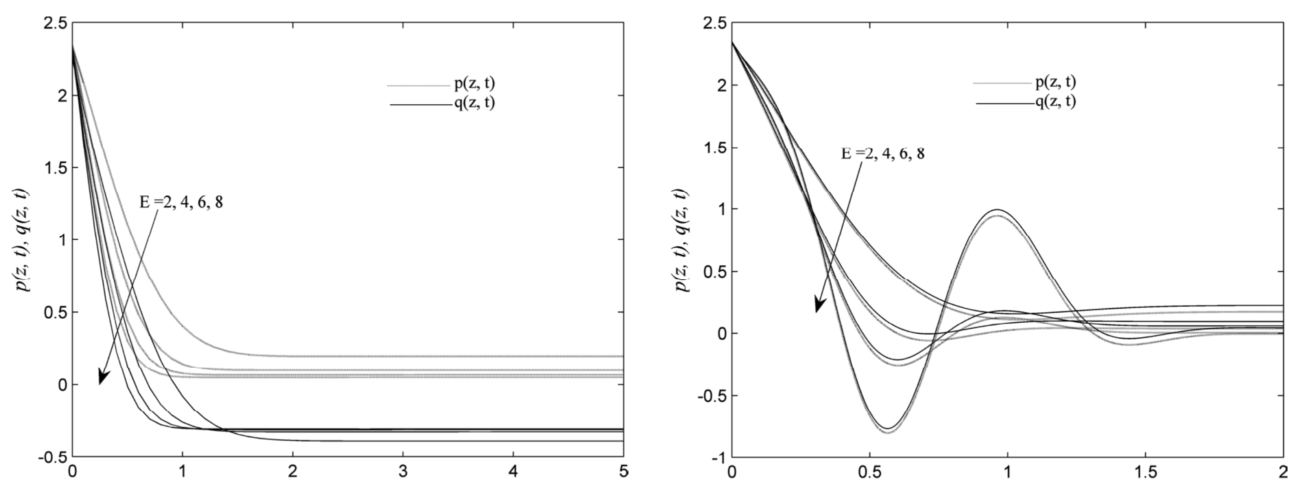


Figure 2: Effect of Ekman Number on Fluid and Dust Velocity for Small and Large Times (Case 2)

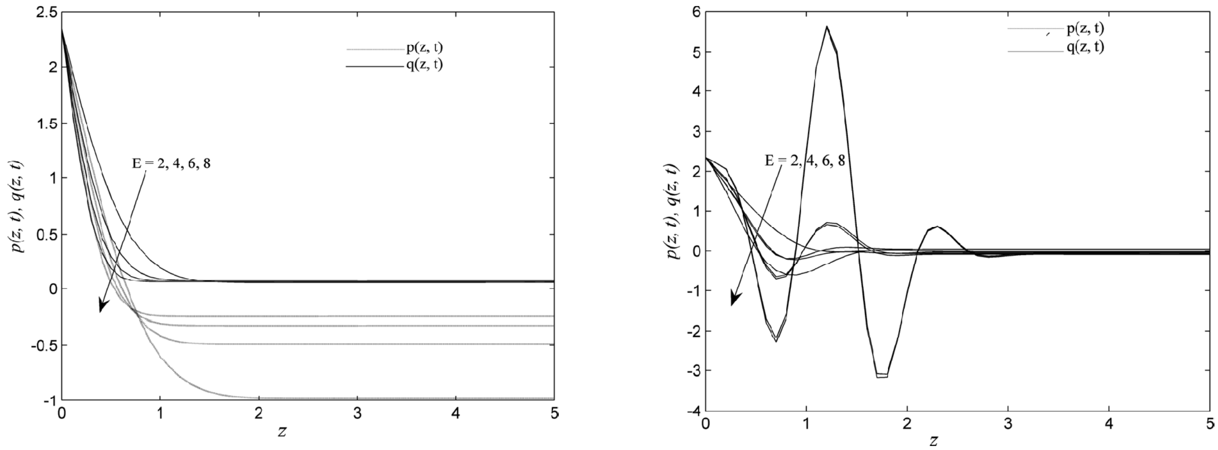
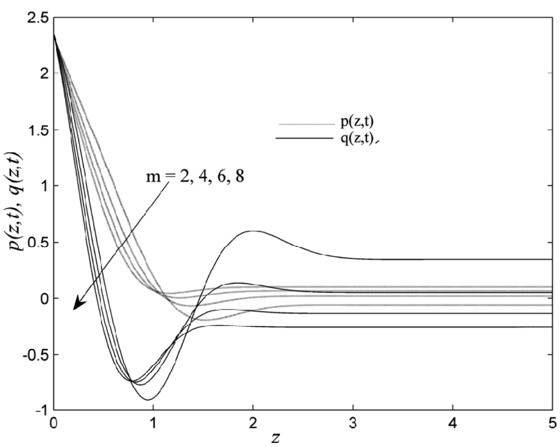
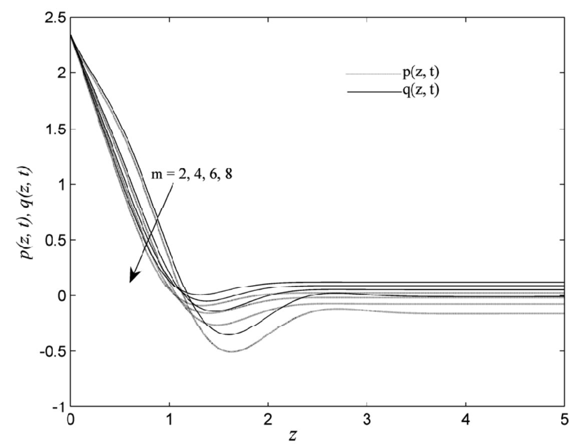


Figure 3: Effect of Ekman Number on Fluid and Dust Velocity for Small and Large Times (Case 3)

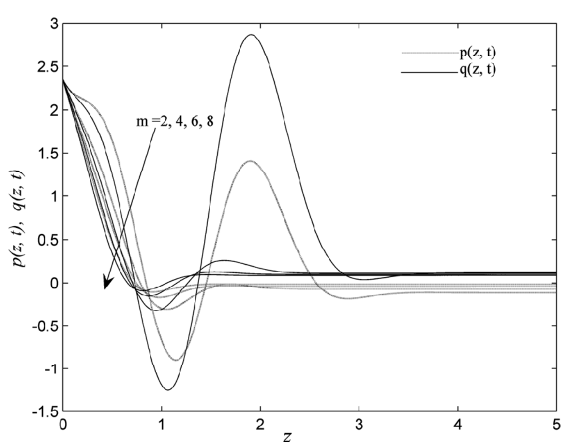
The velocity of fluid and dust remains unaffected by magnetic parameter and Hall parameter in small times solution, where as in large times both fluid and dust velocities are effected by the magnetic parameter and Hall parameter.



(Case 1)

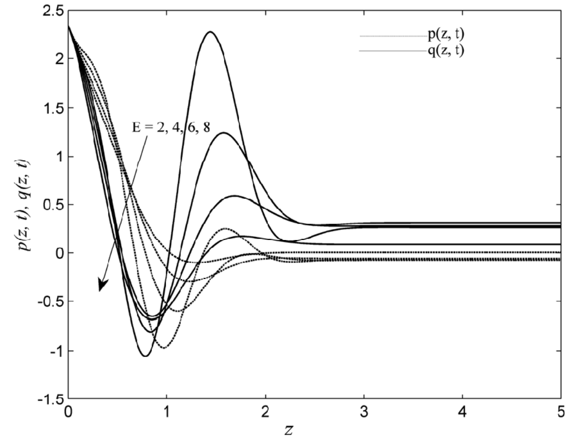


(Case 2)

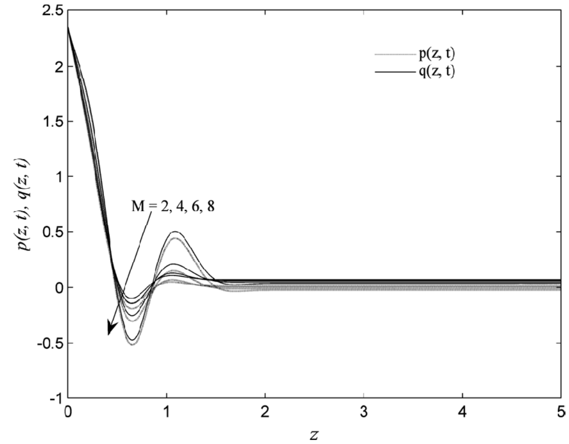


(Case 3)

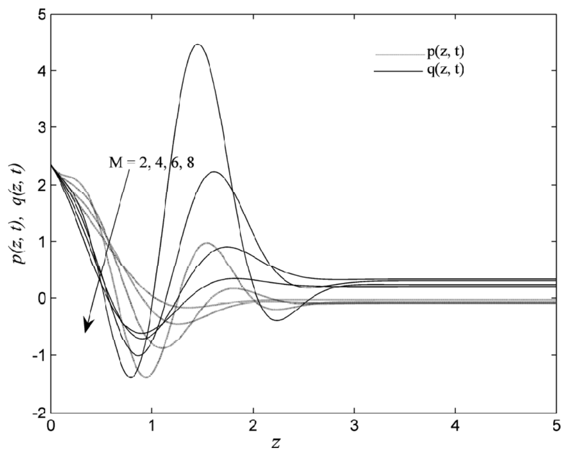
Figure 4: Effect of Hall Current Parameter on Fluid and Dust Velocity for Large Times



(Case 1)

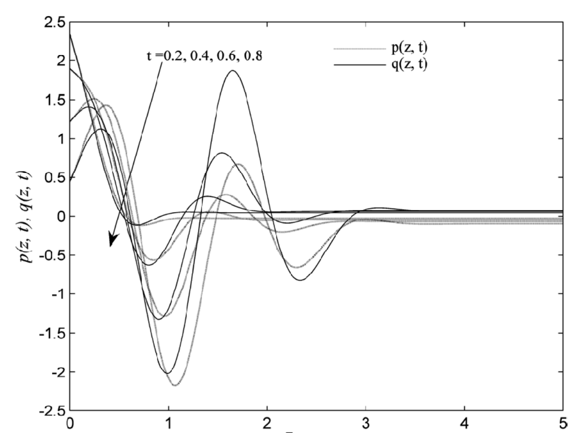


(Case 2)

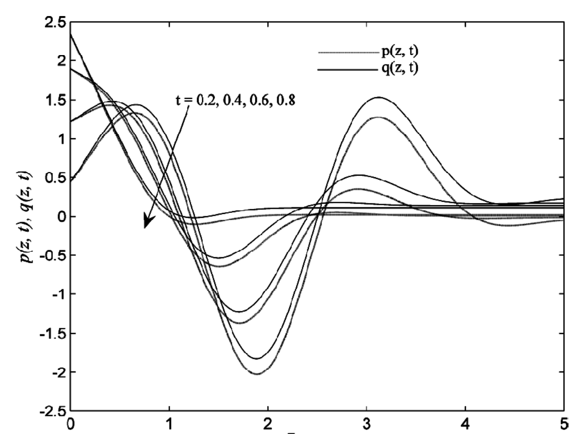


(Case 3)

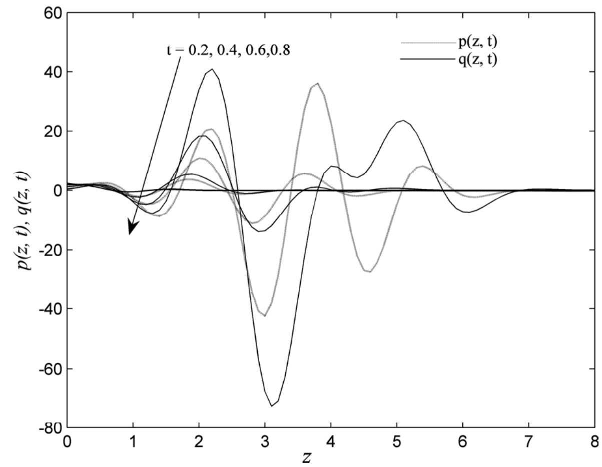
Figure 5: Effect of Magnetic Parameter on Fluid and Dust Velocity for Large Times



(Case 1)



(Case 2)



(Case 3)

Figure 5: Effect of Magnetic Parameter on Fluid and Dust Velocity for Large Times

Figure 4 represents the graph of fluid and dust velocities versus *z*, for different values of Hall current parameter m for all the three cases. It is evident from the figure that the fluid and dust velocity decreases with increase of *m*, i.e., the boundary layer thickness increases with increase in Hall current parameter.

Figure 5 indicates the fluid and dust velocity versus z. Here the effect of increasing values of a magnetic parameter M decreases the fluid and dust velocity. As *M* increases, the Lorentz force also increases. Hence it leads to deceleration of the flow.

Figure 6 reveals that the fluid and dust velocity decrease with the increase in the time for all the cases. It is interesting to note that the thickness of boundary decreases with increasing time for large times.

5. CONCLUSIONS

A mathematical analysis on boundary layer flow of an rotating dusty fluid in the presence of hall current with time dependent pressure gradient is examined. Some of the important observations of our analysis obtained by the graphical representation are listed below:

- The solution remains unaffected by magnetic parameter and Hall current parameter in small times, where as these effects in large times.
- The effect of Ekman number E decrease the fluid and dust partical velocity.
- The effect of magnetic parameter M and Hall current parameter m decrease the fluid and dust partical velocities.
- The effect of time decrease both the fluid and dust velocity.

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