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Glittering covariance estimators: Sophisticated but worthless

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Abstract: The present research seeks to estimate a set of variance-covariance approaches utilizing data from the Pakistan stock exchange's non-financial sector. The research spans ten years, from January 3, 2011, through December 31, 2021. The data is divided in two parts. The first part of the data is used to estimate covariance matrices, while the second part is used to evaluate the estimators' ex-post performance. The weekly market price of shares is used to create equally weighted indexes of ten sectors. The research assesses the accuracy and efficacy of twelve covariance estimators using two evaluation criteria, RMSE and MVP risk profile. The study's results, based on RMSE and MVP risk profile, show that applying more complex covariance estimators versus equally weighted variance-covariance estimators in portfolio creation provides no incremental advantage to portfolio managers .Hence, less complicated methods are preferable.

Keywords: Variance-covariance matrix, portfolio construction, Pakistan stock exchange.

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1. Introduction

The conventional portfolio optimization approach by Markowitz (1952)has contributed significantly to the modern theory of investment. Markowitz came up with the idea of mean variance (MV) and solved the optimization problem using quadratic equation. His quantitative method has been tested in number of studies and discussed widely in literature. The proposed method had a huge impact on the economics of the financial world and is considered as a milestone in modern finance. It eventually led to the well-known Capital Asset Pricing Model (CAPM), which was developed by Sharpe (1964). Theoretically, mean-variance (MV) approach suggests investor an optimal framework for the placement and management of investments based on relevant variance-covariance matrix and future return vectors parameters. However, when used practically in investment management environment, this approach produces multiple corner solutions (Broadie, 1993), having inherent issues of estimation errors (Michaud, 1989), the portfolio reallocations result in significant transaction costs (Best & Grauer, 1991), and in many cases the results of out-of-sample asset allocation are extremely poor. The issue aggravates specially at the when the size of portfolio increases and it suggests more problematic solutions.

Traditional mean-variance optimization issues may be looked at in two ways. The theoretical approach concentrates on the assumptions and hypothetical elements of the mean-variance paradigm, while the implementation method examines how investors might estimate the anticipated return vectors and covariance matrix of investment vehicles in order to properly apply the framework. The current research focuses on the implementation side of mean-variance optimization. Elton and Gruber (1973) and DeMiguel, Garlappi, and Uppal (2007) underscore the key role played by the covariance estimator in effectively applying mean-variance optimization techniques. The estimation of variance-covariance matrix is said to be the most challenging and difficult aspect of the framework Ledoit and Wolf (2003b).

2. Literature review

The method of sample covariance matrix is a rational approach for estimating covariances of asset classes pair-wise using data from previous covariances. Researches criticized this method for number of reasons (Pafka & Kondor, 2004; Saghir, 2020). It is particularly prone to mistakes when the quantity of underpinning asset classes exceeds the sample size. This phenomenon is referred to as error maximization by (Michaud, 1989). Sharpe (1963) presents a simple technique to explain covariances by only a single element, the market factor. Researchers such as Vasicek (1973), Blume (1971) and King (1966) tried to enhance the estimator by taking into consideration the betas' mean-reverting tendency, modifying their variation, and incorporating additional variables other than the sole common factor, respectively.

Factors pertaining to historic sample covariances can also be identified using statistical and non-theory based methods like principal component analysis (PCA). Elton and Gruber (1973) recommend the use of average correlation covariance estimators to estimate covariance. While the research on covariance estimators is much too broad to cover in this study, however the typical approach of determining the covariance matrix is subject to either estimation or specification errors. In light of the inaccuracies and statistical instability of estimation methods, DeMiguel et al. (2007) come to the empirical conclusion that non-theory based methods of portfolio diversification outperform more complex asset allocation techniques in the long run.

The financial literature makes use of a basic statistical theory in order to achieve the best possible balance between estimate error and specification error. Bengtsson and Holst (2002), Chan, Karceski, and Lakonishok (1999), Jagannathan and Ma (2003),Ledoit and Wolf (2003a), Ledoit and Wolf (2003b) demonstrate experimentally that shrinkage-based estimators and the portfolio of estimators are the most appropriate methods for estimating covariance. According to decision theory, there is a moment when the specification error and estimate error are at their least significant differences. It has been proposed by Stein (1956) that the optimal point can be identified by taking a weighted average of the two estimators.

According to Ledoit and Wolf (2003a), the Bayesian shrinkage technique to optimization may be used in conjunction with the single-index covariance estimator and the sample variance-covariance estimator to improve performance. This procedure ensures that the estimation error in the sample covariance is reduced while without causing a significant amount of specification error. It produces a shrinkage matrix, in which all of the covariances (off-diagonal components) of the standard sample matrix are shrunk without affecting the diagonal members of the conventional sample matrix. Ledoit and Wolf (2003b)show that the sample covariance may be reduced to a constant correlation covariance estimator by shrinking the sample covariance. Jagannathan and Ma (2003) propose a portfolio of covariance estimators to compete with the more complex Ledoit and Wolf (2003a) estimator. The portfolio of covariance estimators includes the equally weighted average of the sample variance-covariance estimator with any other covariance estimator, as well as the sample covariance estimator plus any other covariance estimator.

Compared to the sample covariance estimator, both the shrinkage estimator and the equally weighted estimate are expected to perform much better. Shrinkage estimators are conceptually more sophisticated than the sample covariance estimator and equally weighted average of a portfolio of estimators. Benninga and Disatnik (2007) utilize data from the New York Stock Exchange to demonstrate that investors do not gain from employing shrinkage estimators over an equally weighted portfolio of covariance estimators when compared to an equally weighted portfolio of covariance estimators. When it comes to the relative benefits of sophisticated and simple estimators in the setting of equities markets of Asian nations, there is no clear agreement in the literature with regard to variance-covariance estimation.

Section 3 covers the data and research techniques employed, as well as the criteria used to compare the results. Section 4 presents the study's empirical findings, which are followed by discussion. Section 5 concludes this study.

3. Data Structure and Research methodology

The study uses official data portal of Pakistan stock exchange (PSX) for data-gathering of non-financial companies. Time period of the study ranges from January 3, 2011 to December 31, 2021. The data set is divided into two sections. The first half of the data is utilized to calculate covariance matrices, while the second is utilized to test the estimators' ex-post efficiency. Equally weighted indices of 10 sectors are constructed using weekly market price of shares while each index composed of companies with in that sector. The returns are estimated based on the continuous compounding assumption for each asset group using: $(R_{i,t})=ln(E_t/E_{t-1})$.Here. E_t and E_{t-1} represents the recent and previous price of share respectively.

3.1. The variance covariance matrix

The variance covariance matrix is a combination of diagonal and off-diagonal elements in a square. In the matrix, elements in diagonal places show variances while elements in off-diagonal places show covariances among all asset classes. The matrix can be written as follow:

$$\sum = \begin{bmatrix} \sum o_1^2/l & \sum o_1 o_2/l & \cdots & \sum o_1 o_i/l \\ \sum o_2 o_1/l & \sum o_2^2/l & \vdots & \sum o_2 o_i/l \\ \cdots & \cdots & \cdots & \cdots \\ \sum o_i o_1/l & \sum o_i o_2/l & & \sum o_i^2/l \end{bmatrix}$$

Where; the symbol of $\sum$ represents the matrix of variance-covariance of order ($o\times o$), l represents to the elements of data for each asset group whereas o_1 represents the mean deviation and o_1^2/lrepresents covariance of asset category of *i* and *j*.

The following table-1 presents list of covariance methods considered.

Table-1 Alternative covariance estimators

Sr.	Covariance Estimators	Signs
	Traditional variance-covariance estimators	
1	Diagonal matrix of variance-covariance	VCM-1
2	Sample matrix of variance-covariance	VCM-2
3	Constant correlation matrix of variance-covariance	VCM-3
	Factor approach of variance-covariance estimators	
4	Single index matrix of variance-covariance	VCM-4
	Portfolio of Estimators	
5	Portfolio composed of Sample, diagonal matrix	VCM-5
6	Portfolio composed of Sample, single index matrix	VCM-6
7	Portfolio composed of Sample, constant correlation covariance matrix	VCM-7
8	Portfolio composed of Sample, single index & constant correlation matrix	VCM-8
9	Portfolio composed of Sample, overall mean &single index matrix	VCM-9
	Shrinkage based variance-covariance estimators	
10	Shrinkage towards diagonal matrix	VCM-10
11	Shrinkage towards single index matrix	VCM-11
12	Shrinkage towards constant correlation matrix	VCM-12

3.1.1. Diagonal matrix of variance-covariance

It's a covariance vector field with sample variances in diagonal entries and Zeroes in off-diagonals. The matrix can be written as follow:

$$\Sigma^d = \begin{bmatrix} \sum o_1^2/l & 0 & \cdots & 0 \\ 0 & \sum o_2^2/l & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \sum o_i^2/l \end{bmatrix}$$

3.1.2. Sample matrix of variance-covariance

Suppose, Z are asset classes with N total observation for asset's returns. Where, p_{1t} represents return for i^{th}class of asset at the time t, $\overline{p_1}$is mean return of i^{th} class of asset as $\overline{p_1}$=$\sum_{t=1}^{N} \frac{p_{it}}{N}$, *i=1,2,3,...,N*. Let, M be the excess return of asset classes while the transpose of the matrix be M^T.

$$M = \begin{bmatrix} (p_{11} - \bar{p_1}) & (p_{21} - \bar{p_2}) & \cdots & (p_{Z1} - \bar{p_Y}) \\ (p_{12} - \bar{p_1}) & (p_{22} - \bar{p_2}) & \cdots & (p_{Z2} - \bar{p_Y}) \\ \cdots & \cdots & \cdots & \cdots \\ (p_{1N} - \bar{p_1}) & (p_{2N} - \bar{p_2}) & \cdots & (p_{YN} - \bar{p_Y}) \end{bmatrix}$$

$$\Sigma^s = \frac{M^T * M}{K-1} \text{ ...... } (ii)$$

The sample matrix of variance-covariance (Σ)is estimated using equation (*ii*) in this investigation.

3.1.3. Constant correlation matrix of variance-covariance

Elton and Gruber (1973)introduced the constant correlation matrix of variance-covariance. According to this model, co-movement of asset classes is driven by a constant correlation coefficient between them. In order to calculate this constant correlation coefficient, the mean among all the asset returns of correlation coefficients is taken into account. It can be referred to as the overall mean approach. The relationship between the correlation coefficient and the covariance may be expressed mathematically as $\sigma_{ps} = \rho_{\mathrm{ps}}\sigma_{\mathrm{p}}\sigma_{js}$,while covariances are calculated through the following method.

$$\sigma_{ik} = \begin{cases} \sigma_{pp} \quad = \sigma_{\mathrm{p}}^2 & \text{where } \mathrm{p = k} \\ \sigma_{ps} = \rho_{\mathrm{ps}}\sigma_{\mathrm{p}}\sigma_{js} & \text{where } \mathrm{p \neq k} \end{cases}$$

Following Chan et al. (1999), study uses the approach to estimate covariance matrix of asset returns. The average correlation coefficient is computed after estimating the correlation matrix among these asset classes. The estimated average correlation coefficient is utilised as the covariance among all asset classes in the calculation of the covariance matrix.

3.1.4. Single index matrix of variance-covariance

The method of single index, suggested by Sharpe (1963) assumes linear relation between asset return for single asset class and the benchmark return. The formula is considered as an attempt to reduce the complexity of sample covariance estimators. The mathematically expression can be written as:

$$y_{st} = \theta_s + \gamma_s r_t + \varepsilon_t$$

Here, y_{st} represents return for asset class*'s'*, r_t denotes to return of the market portfolio and ε_t shows the error term which is $E(\varepsilon_s \varepsilon_p) = 0$. The covariance matrix calculated using the single index approach is expressed as:

$$\sigma_{sj} = \begin{cases} \beta_{\mathrm{j}}\beta_{\mathrm{p}}\sigma_{\mathrm{r}}^2 & \text{where} \mathrm{s = p} \\ \sigma_{\mathrm{s}}^2 & \text{where } \mathrm{s \neq p} \end{cases}$$

Here, σ_{r}^2 represents variance for return of benchmark and β_{j} denotes to the slopes. The method for the estimation of covariance based on single index is relatively more robust to other covariance estimator in terms of estimation error. It has significantly less estimation error compared to the sample variance covariance estimator. This method, is however exposed to specification error due to assumption of single factor.

3.1.5. Shrinkage based variance-covariance estimators

Single index models, such as Market portfolio, are based on a single component, whereas sample covariance is referred to be X factor model. The sample covariance model and the single index base variance-covariance method are two opposite ends of the same road. Because sample covariance matrices are prone to estimate errors, while single index models are prone to specification errors, it may be argued that there is need to minimize both types of errors.

A reliable estimator could be an N factor model, in which case X>N>1. Stein (1956)presented the concept of minimization of the quadric loss function. Following the footsteps, Ledoit and Wolf (2003a)proposed combining two covariance matrices and determining the optimal weight. Sample covariance matrices have been criticized the primarily on the fact that it apply too little and very basic structure. The shrinkage covariance estimator can be calculated as:

$$\Sigma^{s} = \vartheta(Q) + (1 - \vartheta)\Sigma^{S}$$

Here, ϑ denotes to the weighs for the target covariance matrix, Σ^{S} be the sample covariance matrix while Σ^{s}represents the matrix of covariance after the shrinkage. Ledoit and Wolf (2003a) shrunk sample matrix to single index based matrix and Ledoit and Wolf (2003b) shrunk it to constant correlation based covariance matrix and Kwan (2011) shrunk the sample covariance matrix to the diagonal covariance matrix.

3.1.6. Portfolio of Estimators

Following the work of Jagannathan and Ma (2003), Benninga and Disatnik (2007), and Liu and Lin (2010) this study also compute and evaluates the performance of these portfolio of estimators.

- ***Portfolio composed of Sample, diagonal matrix:*** This portfolio of covariance estimator is composed of equal weighs of sample matrix of variance-covariance and the diagonal matrix of variance-covariance.

$$Port - 1 = \frac{1}{2}\left(\Sigma_{\text{Sample.VCM-2}} + \Sigma_{\text{Diagonal.VCM-2}}\right)$$

- ***Portfolio composed of Sample, single index matrix:*** This portfolio of covariance estimator is composed of equal weighs of sample matrix of variance-covariance and single index matrix.

$$Port - 2 = \frac{1}{2}\left(\Sigma_{\text{Sample.VCM-2}} + \Sigma_{\text{Single-index.VCM-4}}\right)$$

- ***Portfolio composed of Sample, constant correlation covariance matrix:*** This portfolio of covariance estimator is composed of equal weighs of sample matrix of variance-covariance and constant correlation based covariance matrix.

$$Port - 3 = \frac{1}{2}\left(\Sigma_{\text{Sample.VCM-2}} + \Sigma_{\text{Overall-mean.VCM-3}}\right)$$

- ***Portfolio composed of Sample, single index & constant correlation matrix:*** This portfolio of covariance estimator is composed of equal weighs of sample matrix of variance-covariance, single index covariance matrix and constant correlation based covariance matrix.

$$Port - 4 = \frac{1}{3}\left(\Sigma_{\text{Sample.VCM-2}} + \Sigma_{\text{Single-index.VCM-4}} + \Sigma_{\text{Overall-mean.VCM-3}}\right)$$

- ***Portfolio composed of Sample, overall mean & single index matrix:*** This portfolio of covariance estimator is composed of equal weighs of sample matrix of variance-covariance, single index covariance matrix, constant correlation based covariance matrix and the diagonal matrix of variance-covariance.

$$Port - 5 = \frac{1}{4}\left(\Sigma_{\text{Sample.VCM-2}} + \Sigma_{\text{Single-index.VCM-4}} + \Sigma_{\text{Overall-mean.VCM-3}} + \Sigma_{\text{Diagonal.VCM-2}}\right)$$

3.2. Assessment parameters

In line with the study of Jagannathan and Ma (2003) and Liu and Lin (2010), this study uses two types of assessment methods for the comparison of performance of covariance matrix. These methods include root mean square error (RMSE) and estimation of minimum variance portfolios (GMVP).

To estimate the root mean square errors (RMSE) study uses:

$$RMSE = \sqrt{\frac{S(S-1)}{2}\sum_{i=1}^{S}\sum_{u=1,j\neq s}^{S}(\widehat{\emptyset}_{iu} - \emptyset_{iu})^2}$$

Here, $\emptyset_{iu}$ and $\widehat{\emptyset}_{iu}$ represent the actual and the expected covariance in a portfolio between i^{th} *and* u^{th} asset class. A lower value of RMSE is favored compared to a higher value. As, minimum variance portfolios (GMVP) is independent of selection of future return vector hence, study uses risk characteristics of GMVP to make comparison among alternative covariance estimating matrices.

4. Results and discussion

Table-2 presents value of root mean square error (RMSE) for alternative variance-covariance estimators. The table shows pair-wise covariance values and respective out of sample values. A low value of RMSE is preferred over a high. A covariance estimator having relatively lower value outperforms competing estimators.

Table-2 RMSE results of alternative covariance matrices

Sr.	Covariance Estimators	Results
	Traditional variance-covariance estimators	
1	Diagonal matrix of variance-covariance (VCM-1)	0.0279
2	Sample matrix of variance-covariance (VCM-2)	0.0251
3	Constant correlation matrix of variance-covariance (VCM-3)	0.0196
	Factor approach of variance-covariance estimators	
4	Single index matrix of variance-covariance (VCM-4)	0.0125
	Portfolio of Estimators	
5	Portfolio composed of Sample, diagonal matrix (VCM-5)	0.0194
6	Portfolio composed of Sample, single index matrix (VCM-6)	0.0148
7	Portfolio composed of Sample, constant correlation covariance matrix (VCM-7)	0.0179
8	Portfolio composed of Sample, single index &constant correlation matrix (VCM-8)	0.0170
9	Portfolio composed of Sample, overall mean & single index matrix (VCM-9)	0.0128
	Shrinkage based variance-covariance estimators	
10	Shrinkage towards diagonal matrix (VCM-10)	0.0250
11	Shrinkage towards single index matrix (VCM-11)	0.0202
12	Shrinkage towards constant correlation matrix (VCM-12)	0.0201

Value of RMSE it is evident single factor model of covariance estimator, VCM-4 has consistently outperformed all other competing methods. The diagonal matrix of variance-covariance, (VCM-1) and sample matrix of variance-covariance, (VCM-2) performed worst among all other covariance estimators. The important comparison is between the so-called sophisticated covariance estimators developed by Ledoit and Wolf (2003a), Ledoit and Wolf (2003b)and estimators introduced by Jagannathan and Ma (2003).On the RMSE, all average of portfolio estimators performed better compared to complex shrinkage variance covariance estimators. Looking specifically at the results of shrinkage covariance estimators show that (VCM-10) performed worse than (VCM-11) and (VCM-12).

Among portfolio of estimators, the performance of Portfolio composed of sample, overall mean & single index matrix, (VCM-9) is on top while the performance of portfolio composed of sample, diagonal matrix (VCM-5)remain poor. Results also show that constant correlation covariance matrix performed relatively better to all shrinkage covariance estimators and other traditional covariance matrices however, its performance remain poor compared to both single factor covariance estimator (VCM-4) and all competing portfolio of estimators. Overall, the findings suggest that employing a more complicated estimator compared to a simpler approach i.e portfolio of estimators provides no incremental value to investors.

Table-3 provides results of risk behavior with respect to standard deviation for MVPs of twelve alternative covariance estimators. There are certain inconsistencies in the results RMSE and MVPs.

Table-3Standard deviation (SD) values of MVP of alternative covariance matrices

Sr.	Covariance Estimators	Results
	Traditional variance-covariance estimators	
1	Diagonal matrix of variance-covariance (VCM-1)	0.0318
2	Sample matrix of variance-covariance (VCM-2)	0.0299
3	Constant correlation matrix of variance-covariance (VCM-3)	0.0285
	Factor approach of variance-covariance estimators	
4	Single index matrix of variance-covariance (VCM-4)	0.0224
	Portfolio of Estimators	
5	Portfolio composed of Sample, diagonal matrix (VCM-5)	0.0291
6	Portfolio composed of Sample, single index matrix (VCM-6)	0.0287
7	Portfolio composed of Sample, constant correlation covariance matrix (VCM-7)	0.0273
8	Portfolio composed of Sample, single index & constant correlation matrix (VCM-8)	0.0267
9	Portfolio composed of Sample, overall mean & single index matrix (VCM-9)	0.0286
	Shrinkage based variance-covariance estimators	
10	Shrinkage towards diagonal matrix (VCM-10)	0.0290
11	Shrinkage towards single index matrix (VCM-11)	0.0292
12	Shrinkage towards constant correlation matrix (VCM-12)	0.0277

The performance of diagonal matrix of variance-covariance (VCM-1) and sample matrix of variance-covariance (VCM-2) once again remain poor estimators. The performance of constant correlation matrix of variance-covariance (VCM-3) is relatively improved, however, the single index matrix of variance-covariance (VCM-4) remain top performer on the scale of standard deviation of MVP.

Results show that portfolio estimators as group outperformed the most complex variance covariance estimators. The shrinkage towards diagonal matrix (VCM-10) and shrinkage towards single index matrix (VCM-11) performed worst but covariance estimator of shrinkage towards constant correlation matrix (VCM-12) performed relatively much improved compared to its the value of RMSE scale as shown in Table-3.

Overall results show that portfolio of estimator as group outperformed the more complex shrinkage based covariance matrices. It proves that investors would not get addition benefit from the use of so-called sophisticated covariance estimators. The MVP's average mean is reported in appendix at Table A-1 to compare degrees of risk.

Table-5 shows the values of Sharpe ratios for MVPs under alternative covariance matrices which are used to make comparison of resultant portfolios with respect to their respective MVPs. The risk-adjusted return for multiple MVP inputs are represented by the Sharpe ratio. A higher Sharpe value is preferred over lower.

Table-4 Sharpe ratio for MVPs of alternative covariance matrices

Sr.	Covariance Estimators	Results
	Traditional variance-covariance estimators	
1	Diagonal matrix of variance-covariance (VCM-1)	0.11452
2	Sample matrix of variance-covariance (VCM-2)	0.10347
3	Constant correlation matrix of variance-covariance (VCM-3)	0.12264
	Factor approach of variance-covariance estimators	
4	Single index matrix of variance-covariance (VCM-4)	0.07911
	Portfolio of Estimators	
5	Portfolio composed of Sample, diagonal matrix (VCM-5)	0.07726
6	Portfolio composed of Sample, single index matrix (VCM-6)	0.10742
7	Portfolio composed of Sample, constant correlation covariance matrix (VCM-7)	0.09187
8	Portfolio composed of Sample, single index & constant correlation matrix (VCM-8)	0.09883
9	Portfolio composed of Sample, overall mean & single index matrix (VCM-9)	0.08758
	Shrinkage based variance-covariance estimators	
10	Shrinkage towards diagonal matrix (VCM-10)	0.10312
11	Shrinkage towards single index matrix (VCM-11)	0.08271
12	Shrinkage towards constant correlation matrix (VCM-12)	0.11043

In Table-5, the constant correlation matrix of variance-covariance (VCM-3) outperformed all other competing method of covariance matrices as it has highest Sharpe ratio of 0.12264. The single index matrix of variance-covariance (VCM-4) and portfolio composed of Sample, diagonal matrix (VCM-5)however remain the worst performers. For other portfolio of estimators Portfolio composed of Sample, single index matrix (VCM-6) and Portfolio composed of Sample, single index & constant correlation matrix (VCM-8) performed comparatively better compared to other portfolio estimators. As a result, the findings are mixed: no single estimator consistently beats the others, showing that adopting complicated estimators provides no additional value to investors

5. Conclusion

This research aims to compute and assess a set of variance-covariance estimators, which is an important component of portfolio construction. In the study, twelve variance covariance matrices are estimated through data of non-financial sector firms registered on the Pakistan Stock Exchange (PSX). These estimators are evaluated for reliability and efficiency employing two different criteria, the RMSE and MVP procedures, respectively. For covariance estimators, both assessment parameters, RMSE and MVP provide inconsistent findings. The single factor model of covariance estimator, VCM-4 has consistently outperformed all other competing methods at both scales of RMSE and standard deviation (SD) of MVP. The performance of constant correlation matrix of variance-covariance VCM-3 remain worst on RMSE while it shows significant improvement at the scale of standard deviation (SD) of MVP. Overall, findings suggest, as group, the equally weighted covariance estimators introduced by Jagannathan and Ma (2003) produced much improved results in comparison to all other covariance estimators. Findings are in line with the studies of Husnain, Hassan, and Lamarque (2016)and Nguyen (2018) enforcing argument that asset managers or investors would not get any incremental gain by using so called sophisticated method of covariance estimator in comparison to simple method proposed by Jagannathan and Ma (2003) of portfolio of estimators for non-financial firms of equity market of Pakistan. Both individual and institutional investors are recommended to use equally weighted portfolio estimators while crafting their investment strategies as more complex covariance estimator fail to deliver incremental gain in Pakistani context.

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Appendix

Table A-1 Mean values for MVPs for covariance estimators

Sr.	Covariance Estimators	Results
	Traditional variance-covariance estimators	
1	Diagonal matrix of variance-covariance (VCM-1)	0.00232
2	Sample matrix of variance-covariance (VCM-2)	0.00209
3	Constant correlation matrix of variance-covariance (VCM-3)	0.00218
	Factor approach of variance-covariance estimators	
4	Single index matrix of variance-covariance (VCM-4)	0.00155
	Portfolio of Estimators	
5	Portfolio composed of Sample, diagonal matrix (VCM-5)	0.00145
6	Portfolio composed of Sample, single index matrix (VCM-6)	0.00217
7	Portfolio composed of Sample, constant correlation covariance matrix (VCM-7)	0.00182
8	Portfolio composed of Sample, single index & constant correlation matrix (VCM-8)	0.00191
9	Portfolio composed of Sample, overall mean & single index matrix (VCM-9)	0.00173
	Shrinkage based variance-covariance estimators	
10	Shrinkage towards diagonal matrix (VCM-10)	0.00263
11	Shrinkage towards single index matrix (VCM-11)	0.00164
12	Shrinkage towards constant correlation matrix (VCM-12)	0.00239