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Evaluating the forecast performance of Spline-GARCH model for stock market volatility: A comparative study of GARCH family models

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Abstract: Enormous research has been made on volatility modeling using different econometrics and time series techniques for the last two decades. Policies and decisions based on accurate forecasts are like golden heaven for investors that minimize losses. This study investigates the best-fitted volatility model and compares its forecasting ability using stock returns data related to nine stock markets from the group of CPEC related nations. Nonlinear generalized autoregressive conditional heteroscedasticity (GARCH) models, i.e., standard GARCH, PARCH, IGARCH, GARCH-M, GJR-GARCH and EGARCH, are compared with the Spline-GARCH (SP-GARCH) model. The forecast performance of all models are evaluated by root mean square, mean absolute percentage, and mean absolute errors i.e. RMSE, MAPE and MSE respectively. According to the findings, the Sp-GARCH model is found to be a better-fitted model for mostly all the stock markets based on Akaike info (AIC), Schwartz info criteria (BIC). Results are verified by using the likelihood ratio test (LRT). The forecast performance of the SP-GARCH model is also found to be the best among all volatility models for five stock returns, including the founders of CPEC, namely KSE-100, SSE-100, TADAWUL, KASE, and MOEX. In contrast, GARCH-M is the best forecast model for developed stock markets, namely BIST, CAC40, and FTSE. Meanwhile, the predictive power of the SP-GARCH model and other GARCH models is found to be significantly identical except for few cases based on the Diebold-Mariano test. This study shows that the Sp-GARCH model could be a reliable and trustworthy alternate model for any other GARCH model.

Keywords: Forecast performance, high and low frequency volatility, Spline-GARCH, CPEC.

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1. Introduction

There has been a growing literature on modeling and predicting volatility, especially the financial indicators from the last 40 years. The attraction of volatility modeling is due to the central role that volatility plays in most macroeconomic and financial applications (Audrino and Bühlmann, 2009). Over the years, different statistical techniques such as Box-Jenkins ARIMA and traditional volatility models such as ARCH and GARCH models have been used in the financial time series data for forecast purpose. In fact, formulation of a statistical model for financial data is a difficult task, as they hypothesized little knowledge of underlying patterns. Engle (1982) introduced a model to estimated conditional variances, namely (ARCH) model. After a few years, Bollerslev (1986) proposed GARCH model to avoid the higher order of ARCH and to acquire more parsimonious results. They explained that volatility occurs in bunches and is categorized as a time-varying conditional variance. Various extensions of GARCH models are EGARCH by Nelson (1991), CHARMA byTsay (1987), Stochastic Volatility SV suggested by Melino and Turnbull (1990), and the Spline GARCH model by Engle and Rangel (2008) and others are the few examples. Unfortunately, from a vast number of books and research papers that have been published in this area, one cannot conclude whether which linear or nonlinear time series modeling technique is suitable to estimate and forecast the volatility (either high or low fluctuations) from any single or a bunch of countries. If any time series modeling technique is appropriate, then do that technique is also best in the meadow of forecasting? According to Becker and Clements (2007), understanding and forecasting volatility has huge importance in research. Exhibiting long memory features in a time series data is due to the heavy-tailed regime-switching process. Furthermore, one of the causes for heterogeneous time series data is the flow of information. There are many time series, econometrics, and panel data techniques already studied for this purpose. However, no specific study is found in the literature that gives the exact conclusion regarding the techniques. The standard GARCH (1,1) model involves constant expected volatility, i.e., the long-run volatility forecast is constant. According to Engle and Rangel (2008), this specific characteristic is not consistent with the time-series behavior of realized volatilities of stock market returns. The standard GARCH (1,1) model is modified by introducing a trend in the volatility process of returns. This non-parametric trend is modeled by an exponential quadratic spline, which generates a smooth curve describing the long-run volatility component based exclusively on data evidence. The proposed SP-GARCH model by Engle and Rangel(2008) has the ability to estimate the high or low-frequency volatility without losing much information. Many studies related to comparing estimates and forecast performance of GARCH family models based on financial and other time-series variables for different countries are present in the literature. However, no empirical evidence is found in the literature dealing with comparing traditional volatility GARCH family models with the SP-GARCH model. Furthermore, no forecast has been made to verify the forecast performance of the SP-GARCH technique with other GARCH models for many stock markets from different countries, i.e., CPEC related counters. China-Pakistan Economic Corridor (CPEC) has become an eye-catcher for many nations. It has an exceptional attraction towards economists, investors, policymakers, and industrialists. It is an open chapter now that linkages among the global economies in the 21st century are due to highly globalization. Trends in business constantly vary globally, due to which the economic activities quickly vary the developed and developing status of nations. Unfortunately, various geographic parts of the world are restricted. Therefore, the beginning of economic corridors is a ray of sunshine for free transport routes globally. Such developments, namely economic corridors, have always positively impacted macroeconomic activities and become a bridge that links different developed or developing nations as a global production network. The concept of a corridor is a clean source in development in many parts of the world. A shred of evidence is the European Economic Community formed in 1957, namely European Union (EU) since 2009. Many economic corridors are functioning and have become a source of benefits for many countries, i.e., the China-Mongolia-Russia Economic Corridor, New Eurasian Land Bridge, China-Central Asia-West, Asia Economic Corridor, China-Indochina Peninsula Economic Corridor. The East-West Economic corridor, Bengaluru Mumbai, Continental 1 Trade Corridor North America, Bangladesh China India Myanmar Forum for Regional Cooperation(BCIM), Sarawak Corridor of Renewable Energy (SCORE), Eastern Economic Corridor" (EEC), Nanning–Singapore economic corridor and Kalahari Corridor Port of Walvis Bay and Trans European Network Corridors. Mostly, it is seen that a country's role in world politics is determined by its geographical location. The geo-strategic importance of Pakistan can be defined as it is bordered by the most rapidly emerging economic markets, i.e., China and India, while also having neighbors with abundant natural resources, namely Afghanistan and Iran. Therefore, nine emerging and developed stock market indices from CPEC are selected to explore the volatility and forecast performance of GARCH family models. It is a crucial missing space that investors, economists, industrialists, and policymakers from different nations do not understand the significance of CPEC. This research paper will be used as an indicator for the investors, economists, and policymakers from different countries looking to invest and want to be a part of CPEC.

2. Literature Review

Sharma et al. (2021) compared the forecast performance of linear and nonlinear GARCH models on the five selected stock markets from E-7, namely India, China, Brazil, Indonesia, and Mexico. They found that the forecasting ability of the standard GARCH model is better than the EGARCH and TGARCH models. Similarly, Ampountolas (2021) compared the forecast performance of standard GARCH and EGARCH with multiple layer ANN, Holt-winter exponential smoothing, ARIMA, and seasonal ARMA with exogenous indicators. He used daily demand data from US metropolitan hotels from 2015 to 2019 with exogenous indicators, i.e., holidays and temperature. He revealed that the standard GARCH and GJR-GARCH models have superior forecasting power compared to other models included in the study based on MAPE criteria. He further verified the results using the DM and Harvey-Leybourne-Newbold tests. Nguyen and Walther (2018) studied time-varying volatility patterns. They used GARCH-MIDAS and Spline GARCH approach for economic and financial variables of different frequencies. Using commodity futures for Crude Oil, Gold, Silver, and Platinum, and commodity index of daily prices from the period of 1st January 1996 to 31st December 2015. Their findings show that the long-term and short-term volatility of commodity futures leads to a better in-sample fit utilizing the Spline GARCH and the GARCH MIDAS models. They also found that the forecasting performance of Spline GARCH and GARCH MIDAS models is poor compared to single model fits for all the commodities. Their findings depend on RMSE, MSE, and QLIKE forecast criteria. On same year, Lin (2018) also studied the volatility forecasting of daily closed prices of the SSE composite index from July 2013 to July 2017.

He compared symmetric GARCH, TARCH, and EGARCH models. He found that the EGARCH model outperforms other GARCH models. A year before, Alsheikhmubarak and Giouvris (2017) evaluated the ability of symmetric and asymmetric GARCH models. They used the FTSE 100 Implied Volatility Index (IVI) volatility. They estimated the GARCH, EGARCH, GJR-GARCH, and GARCH-MIDAS models. They also used FTSE 100 index returns and several macroeconomic variables, namely UK industrial production, three months LIBOR, GBP effective exchange rate, and unemployment rate from a period from 1/4/2000 to 12/31/2015. Their results show that the GARCH (1,1) model best estimates volatility for market returns and macroeconomic factors, i.e., IVI indices. Furthermore, the GARCH-MIDAS approach also presented evidence of the effect of these explanatory factors on estimating IVI. Aliyu (2014) studied generalized autoregressive conditional heteroscedasticity (GARCH) models. He worked on the relation of inflation with stock market returns and volatility. He used monthly data covers from 1998 to 2010 for the two countries, namely Nigeria and Ghana. He found that any bad news has much impact on the stock market volatility compared to any good news having equal magnitude for Nigeria but Ghana; the results are opposite. Moreover, he also found that the inflation rate and its three-month average have a strong and significant impact on the volatility of the stock market. Cho and Elshahat (2014) also developed a volatility forecast model, namely Modified component, MC-GARCH that accounts for the effect of fundamental macroeconomic variables that reflect the state of the economy. They used S&P 500 index as the proxy for market portfolio and other macroeconomic variables, namely GDP, CPI, Inflation, and growth rate of M2 from 1960 to 2008. These variables have been documented in the empirical literature or in the economic theory to affect equity volatility. According to their findings, the estimates from purposed MC GARCH models are consistent with literature that shows a significant negative relationship between the business cycle and the equity market volatility. They also stated that the relation with macroeconomic variables was not statistically significant using the MC GARCH model. Lim and Sek (2013) studied the volatility of the Malaysian stock market by using different GARCH models. They distributed the monthly data from January 1990 to December 2010 into three parts, i.e., pre, during, and post-crisis 1997. They found that the different GARCH models are suitable for different crises periods. Their results are based on MSE, RMSE, and MAPE. The evolution of SP-GARCH was started when Engle and Rangel (2008) studied modeling equity volatilities as a combination of macroeconomic effects and time-series dynamics. They proposed a modified volatility model, namely the spline GARCH model. This component is estimated for nearly 50 countries over various sample periods of daily stock data till 25th June 2004 for many developed countries. They worked on both, i.e., low and high-frequency components of volatility for macroeconomic factors, namely GDP, inflation, and short-term interest rates. According to their findings, volatility is higher for emerging markets and large economies. The proposed spline GARCH model allows long-horizon forecasts of volatility. Depending on the macroeconomic developments, they only compare the estimates and forecast performance of the purposed GARCH model with the classical GARCH (1,1) time series model. According to the proposed Spline GARCH model by Engle and Rangel (2008), the model has the ability to capture short-term dynamic behavior and long-term dynamic behavior of volatility. Without violating the classical properties of GARCH modeling, this model fits and forecasts high and medium frequencies for volatility. Additionally, the proposed model also handles low-frequency volatility changes and captures cyclical patterns. Furthermore, the Spline GARCH model is evaluated in two parts. The first empirical analysis depends on stock market returns. Daily data of 23 developed countries and 18 emerging markets as mentioned by Bekaert and Harvey (2000). After comparing the results of the proposed model with standard GARCH(1,1) model based on BIC and the likelihood ratio test, the SP-GARCH model is found to be better than the GARCH(1,1) model for all the countries. The ARCH effect was present in all-time series data sets. Also, the mean value suggests slightly less persistence in the Spline GARCH model as compared to GARCH (1,1). Lastly, the second empirical analysis depends on the question regarding the economic determinants of low-frequency volatility. By using cross-sectional and time-series data, they focus on macroeconomic fundamental variables and variables related to the market structure of each exchange. The proposed model estimates are better than the GARCH (1,1) model for developed countries and emerging markets.

Moreover, these GARCH family models are used for the volatility studies of macroeconomic and financial variables of many countries. According to the literature, no empirical study has been conducted to compare the model and forecast performance of the SP-GARCH model with other GARCH family models using the low and high-frequency structural breaks in volatility. This study will fill the gap regarding the forecast performance of the SP-GARCH model against other traditional GARCH family models.

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3. Methodology

To explore the volatility and evaluate the forecast performance of SP-GARCH and other GARCH models, stock market indices from nine countries that are related to CPEC are used in this study. Daily data starts from 1st week of December 2014 to last week of June 2021. KSE 100 (Pakistan), SSE 100 (China), KASE (Kazakhstan), TADAWUL (Kingdom of Saudi Arabia), KLSE (Malaysia), BIST (Turkey), MOEX (Russia), FTSE (United Kingdom) and CAC40 (France) are taken. All stock markets shows nonlinear behavior. The data set of all indices are filtered. Difference in Logarithm is applied on all market indices for each closing price to analyze the index returns

$$St = 100 \times [\ln(Rt) - \ln(Rt\text{-}1)] \qquad 1$$

here St stands for closing price at the period of time t, so that St is the percent return for the daily closing price from period t-1 to period t.

To identify the nonstationarity in the stock returns time series, break point unit root test is used based on Akaike information criteria (AIC) and Schwarz information criteria (BIC). After that, the ARCH LM test is used to identify any presence of arch effect in stock returns volatility. Moreover, the data is split into two parts for estimation and forecast comparison. For the estimation purpose of the model, data is selected from December 2, 2014, to June 8, 2020. To evaluate the forecast performance of models, data is used from June 9, 2020, to June 8, 2021. All the GARCH models of a different order of lag variances, i.e., p and residual errors, i.e., q, included in this study are selected based on AIC and BIC information criteria. All GARCH family models with order (1,1) are found to be the best among all (p,q) orders. Finally, the forecast performance is evaluated based on three forecast accuracy criteria i.e. RMSE, MAE, and MAPE. Lastly, the predictive power of the SP-GARCH and other GARCH models is evaluated by using the Diebold-Mariano test.

3.1 Standard GARCH model

Engle (1982) introduced autoregressive conditional heteroscedasticity (ARCH) technique which is used to estimate the variation in volatility. Generalized ARCH was firstly studied by Bollerslev (1986). According to Bollerslev (1986), conditional variances in GARCH model depend on its previous lags. The variance of ARCH i.e. $\delta_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2$ is known as ARCH model of order 1. Here α and β are ARCH, GARCH terms respectively.

$$y_t = \beta_1 + \beta_2 x_{2t} + \cdots + \beta_k x_{kt} + \mu_t \text{where} \alpha_0 \sim N(0, \delta_t^2) \qquad 2$$

Therefore, the GARCH (1,1) model can be written as:

$$\delta_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \beta \delta_{t-1}^2 \qquad 3$$

3.2 Exponential GARCH (EGARCH) model

GARCH model, sometimes less effective to explain the leverage effects, which can be seen in the case of financial time series. The Concept of leverage effects, which were first observed by Black (1976) A commonly used exponential GARCH model purposed by Nelson (1991) is the EGARCH (1, 1) given by:

$$\log(\sigma_t^2) = \alpha_0 + \alpha_1 \left|\frac{\mu_{t-1}}{\sigma_{t-1}}\right| + \beta_1 \log(\sigma_{t-1}^2) + \gamma \left|\frac{\mu_{t-1}}{\sigma_{t-1}}\right| \qquad 4$$

3.3 GJR-GARCH model

Glostenet. Al. (1993) another modification in GARCH modeling namely GJR GARCH model which can be use in presence of asymmetric effect of volatility i.e. leverage effect. The model is:

$$h_t = \omega_1 + [\alpha_1 + \in I(\varepsilon_{t-1} < 0)]\varepsilon_{t-1}^2 + \beta_1 h_{t-1} \qquad 5$$

Here indicator function is $I(\varepsilon_{t-1} < 0)$. Also, $\omega_1 > 0, \alpha_1 \geq 0$ and the $\beta_1 \geq 0$

3.4 GARCH-M model

For the phenomenon i.e. return of security while it influenced by its own volatility. Usually this phenomena works in Finance. For such scenarios, the GARCH-M model is suitable. Here M denotes GARCH in the mean. GARCH-M (1,1) can be written as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \alpha_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad 6$$

3.5 IGARCH

The IGARCH model is known as the unit-root GARCH model. Its features are quite similar as compare to Box-Jenkins ARIMA models. It is well-known for its feature i.e. the impact of past squared shocks is persistent in IGARCH model. The IGARCH model can be written as:

$$\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1 - \beta_1) \alpha_{t-1}^2 \qquad 7$$

3.6 Power ARCH model

The asymmetric power ARCH model was introduced by the Ding et al. (1993). The PARCH model equation is:

$$\sigma_t^d = \alpha_0 + \sum_{i=1}^{p} \alpha_i (|e_{t-i}| + r_i e_{t-i})^d + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^d \quad 8$$

Where d is parameter power term.

3.7 Spline-GARCH model

Engle and Rangel (2008) purposed a GARCH model i.e. modification of GARCH model namely Spline GARCH. This model allows time-variation in the unconditional level in volatility. If a return time series such that: $r_t=\mu+\varepsilon_t$, where μ is to be the expected return while ε_t is white noise.Here ε_t not necessary to be serially independent. Specific parametric form is assumed for conditional heteroscedasticity i.e. the Spline GARCH model can be written as :

$$\varepsilon_t = \sqrt{\sigma_t^2 + \tau_t + z_t} \quad 9$$

Where z_t is a Gaussian standardized series.

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \,. \quad 10$$

$$\tau_t = exp\left(\sum_{i=1}^{k} \emptyset_i \,(t-t_i)^2\right) \quad 11$$

is the exponential of a quadratic Spline with "k"knots i.e. $t_1,t_2,\ldots,t_k$

3.8 Forecast evaluation Criteria

For the purpose of evaluating out-of-sample forecasting performance, the examination of forecast accuracy will depend on three evaluation statistics criteria given below:

$$MAE = \frac{1}{N}\sum_{t=1}^{N}(y_{t+s} - f_{t,s}) \quad 12$$

$$MAPE = 100 \text{ x} \frac{1}{N}\sum_{t=1}^{N}\left(\frac{y_{t+s}- f_{t,s}}{y_{t+s}}\right) \quad 13$$

$$RMSE = \sqrt{\sum\left(\frac{Y-\hat{Y}}{h}\right)\sum_{t=1}^{N}(y_{t+s} - f_{t,s})} \quad 14$$

Where, $f_{t,s}$ as the forecast made at time t for s steps ahead (i.e. the forecast made for time $t+s$, and y_{t+s} as the realised value of y at time $t+s$.

3.9 Diebold-Mariano test

The forecast criteria are not enough to illustrate the forecast performance of two different models. To check the significant predictive power between the forecasts of two estimated models, Diebold-Mariano (DM) test is used. Mark (1995), Swanson and White (1997) and Fraz et al. (2020) used DM test to compare the significant forecasting power of different time series models. It depends on two loss differential functions i.e. MAE and MSE error criteria.

4. Findings and Discussion

All the stock market returns show high volatility (Figure. It is evidence to use the most suitable time series models, i.e., GARCH models, to estimate and forecast these volatilities. High and low frequency in volatility can be seen in all stock market indices (Figure 1). All the stock market returns are not normally distributed based on the Jarque-Bera test findings. It shows evidence of fat tails in stock returns. According to the ARCH-LM test, all the stock returns are also serially correlated on level (Table 1).

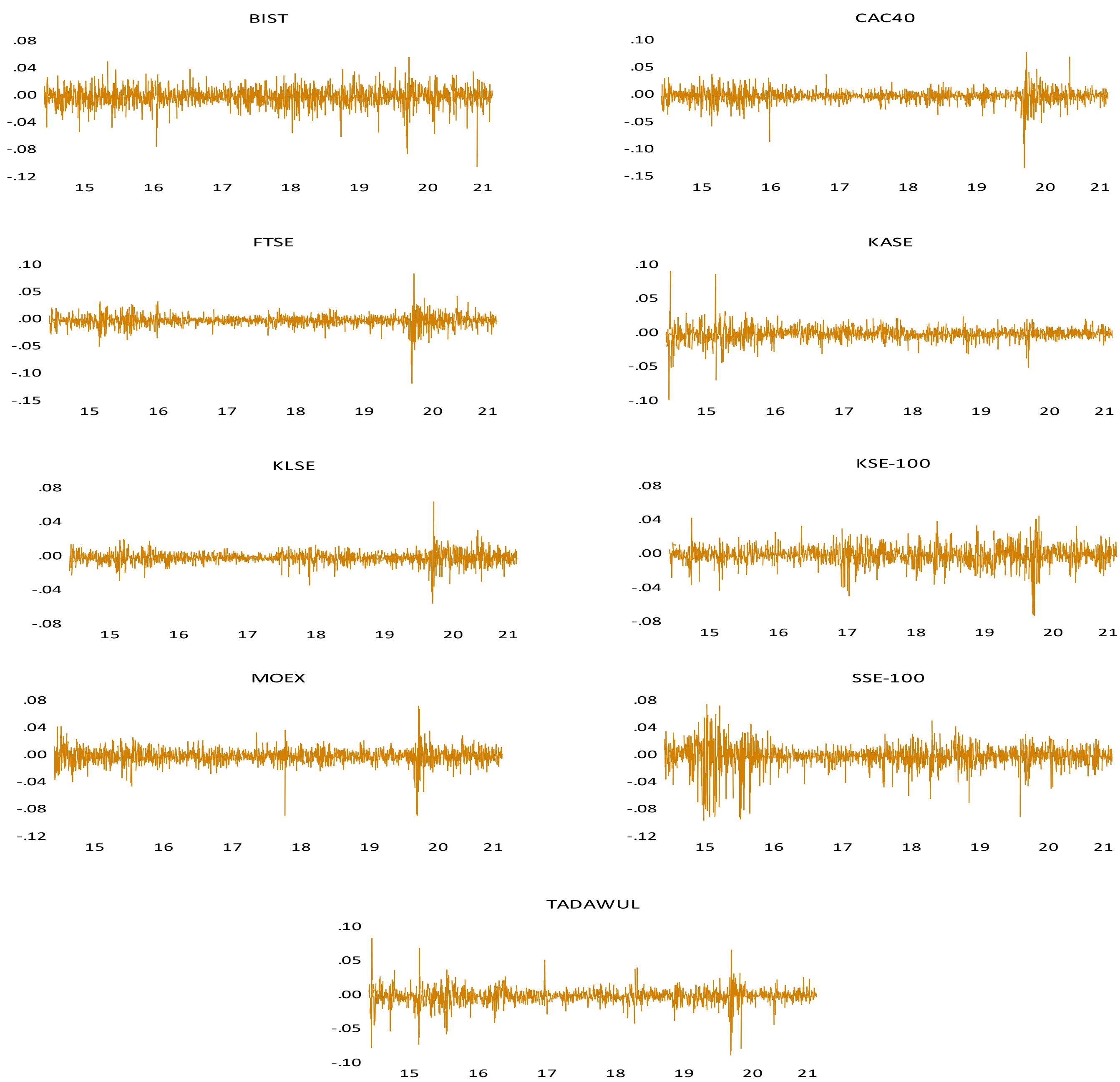

Figure 1: Stock market returns

Table 1. Descriptive statistics and ARCH-LM test

Stock returns	*Mean*	*Std. Dev.*	*Skew.*	*Kurt.*	*Jarque-Bera*	*ARCH-LM*
BIST	0.00031	0.014	-0.858	8.069	2034.912	0.494*
CAC40	0.00024	0.012	-1.071	16.080	12480.710	0.279*
FTSE	0.00003	0.011	-0.929	16.951	14072.110	0.032*
KASE	0.00072	0.010	0.014	19.345	18979.480	0.127*
KLSE	0.00007	0.007	-0.288	13.170	7371.126	1.763*
KSE-100	0.00025	0.011	-0.622	7.801	1747.501	0.050*
MOEX	0.00052	0.011	-0.703	12.303	6288.193	0.213*
SSE-100	0.00020	0.017	-1.179	9.550	3442.704	1.186*
TADAWUL	0.00012	0.012	-0.808	13.841	8535.024	0.004*

Source: Author's findings * significant at 1%, ** significant at 5%

The unit root on level is present in all the stock market indices given in Table 2. All the stock market indices become stationary after taking 1st difference. Breakpoint unit root ADF test is used. The results are based on AIC (Akaike info criteria) and BIC (Schwarz info criteria).

Table 2. Break point unit root evidence

	AIC-Criterion		**Schwarz- Criterion**	
Stock returns	At level	1st difference	At level	1st difference
	t-Statistic	t-Statistic	t-Statistic	t-Statistic

BIST	-3.898	-42.583*	-4.004	-40.921*
CAC40	-3.325	-41.052*	-3.654	-39.872*
FTSE	-4.374	-41.949*	-4.436	-40.718*
KASE	-2.924	-41.281*	-2.872	-40.958*
KLSE	-3.669	-41.125*	-3.876	-40.440*
KSE100	-2.893	-36.383*	-3.043	-36.177*
MOEX	-4.686	-42.482*	-5.156	-41.586*
SSE100	-4.099	-38.288*	-4.252	-37.063*
TADAWUL	-3.718	-37.383*	-4.001	-37.383*

Source: Author's findings * significant at 1%, ** significant at 5%

Furthermore, According to Bartlett's test, the volatilities of stock returns are statistically significant among equity markets. (Table 3).

Table 3. Bartlet test

Indices	*BIST*	*CAC40*	*FTSE*	*KASE*	*KLSE*	*KSE-100*	*MOEX*	*SSE-100*	*TADAWUL*	*Bartlet*	*DF*
Variance	0.00018	0.00016	0.00011	0.00010	0.00005	0.00012	0.00012	0.00029	0.00014	1496.0*	8

Source: Author's findings * significant at 1%, ** significant at 5%

Table 4. Estimation results GARCH family models

Model	*Returns*	*BIST*	*CAC40*	*FTSE*	*KASE*	*KLSE*	*KSE-100*	*MOEX*	*SSE-100*	*TADAWUL*	*Mean*
GARCH		0.066*	0.153*	0.139*	0.071*	0.077*	0.130*	0.072*	0.186*	0.178*	0.12
GARCH-M		0.070*	0.150*	0.139*	0.071*	0.077*	0.131*	0.073*	0.061*	0.177*	0.11
EGARCH		0.078*	0.158*	0.478*	0.163*	0.155*	0.192*	0.123*	0.129*	0.213*	0.19
IGARCH	Alpha	0.017*	0.079*	0.063*	0.046*	0.062*	0.092*	0.067*	0.049*	0.098*	0.06
PARCH		0.071*	0.096*	0.078*	0.078*	0.067*	0.101*	0.057*	0.044*	0.115*	0.08
GJR-GARCH		0.037*	0.009*	0.003*	0.063*	0.044*	0.014*	0.030*	0.056*	0.052*	0.03
SP-GARCH		0.080*	0.186*	0.184*	0.130*	0.180*	0.324*	0.090*	0.240*	0.228*	0.18
GARCH		0.854*	0.829*	0.822*	0.909*	0.917*	0.827*	0.900*	0.793*	0.782*	0.85
GARCH-M		0.808*	0.600*	0.820*	0.909*	0.917*	0.824*	0.899*	0.930*	0.782*	0.83
EGARCH		0.902*	0.970*	0.903*	0.982*	0.987*	0.958*	0.975*	0.986*	0.961*	0.96
IGARCH	Beta	0.983*	0.921*	0.937*	0.954*	0.938*	0.908*	0.933*	0.951*	0.902*	0.94
PARCH		0.824*	0.899*	0.913*	0.901*	0.924*	0.872*	0.916*	0.917*	0.868*	0.89
GJR-GARCH		0.776*	0.868*	0.874*	0.900*	0.924*	0.847*	0.913*	0.927*	0.828*	0.87
SP-GARCH		0.760*	0.719*	0.816*	0.870*	0.820*	0.676*	0.910*	0.760*	0.772*	0.79
Knots		5	5	5	5	3	2	3	5	4	5
Obs/Knot		341	341	341	341	569	853	569	341	427	341

Source: Author's findings * significant at 1%, ** significant at 5%

In Table 4 the estimation results summarize for all the stock returns present in this study. The optimal number of knots in the SP-GARCH model is 5 for all stock returns except KLSE, KSE-100, MOEX, and TADAWUL which are 3, 2, 3, and 4 respectively. Variation in knots is related to volatility patterns and the data frequency which indicate the cyclical pattern in the low-frequency volatility. It indicates that BIST, CAC40, FTSE, KASE, and SSE-100 stock market returns have average number of observation per knots one time that number in KLSE, KSE-100, MOEX and TADAWUL. Therefore, KLSE, KSE-100, MOEX and TADAWUL stock markets show an average at least one time more cycles as compare to remaining stock markets. Since the SP-GARCH model has the ability to explore changes in the dependence structure therefore as according to Engle and Rangel (2008), the estimated spline GARCH models coefficients associated with temporal dependence are compared with a standard GARCH family models. According to the findings, the variation between ARCH effects in the SP-GARCH and other GARCH models are not close to each other i.e. the mean values are found to be 0.18 for spline GARCH model while 0.11 for traditional GARCH (1,1) model (Table 4). It shows an effect of knots for spline GARCH. Also, the mean value is found to be marginally low in the persistence of the SP-GARCH model i.e. 0.78 as compared to the all other standard GARCH model (Table 4). According to likelihood ratio test, Moreover, the estimated SP-GARCH model is also compared with other GARCH family models included GARCH (1,1) model based on AIC, BIC criteria (Table 6) and Likelihood ratio test (Table 5). According to LR test, SP-GARCH model is preferred over all other GARCH family models for all stock markets related to CPEC. The findings suggested that there is a mixed conclusion concerning the best-estimated model for all the emerging and developed countries. For the BIST stock market, Standard GARCH (1,1) model is found to be the best-fitted model based on AIC and BIC criteria. While power ARCH (PARCH) is found to be best fitted for two developed stock markets i.e. CAC40 and FTSE. Both these countries belong to European Union. It is a shred of empirical evidence that can be used by investors and policymakers. For KLSE and MOEX, GJR-GARCH is the best-fitted model as the two countries have a good correlation in many diplomatic and traveling purposes. Also, for two emerging stock markets namely KASE and SSE-100 which are also neighboring countries and have mostly similar cultures, SP-GARCH is the best-fitted model among all other GARCH families. Lastly, EGARCH is found to be the best-fitted model for two emerging markets i.e. KSE-100 and TADAWUL. Both countries Pakistan and Saudi Arabia are religiously linked to each other. Also, the economic relations are outstanding between these two countries.

Table 5. Likelihood ratio test for Spline GARCH with other GARCH models

Stock Returns	*Likelihood*							*LRT*					
	GARCH	EGARCH	IGARCH	GARCH-M	GJR-GARCH	PARCH	SP-GARCH	GARCH	EGARCH	IGARCH	GARCH-M	GJR-GARCH	PARCH
BIST	4958.0	4989.9	4931.0	4987.2	4986.9	4990.4	4805.7	304.5	368.3	250.6	362.9	362.3	369.3
CAC40	5401.7	5443.9	5360.4	5433.5	5436.3	5451.0	5326.9	149.7	234.1	67.0	213.3	218.8	248.3
FTSE	5646.7	5624.2	5612.0	5648.8	5673.1	5685.7	5532.5	228.3	183.2	158.9	232.6	281.1	306.4
KASE	5682.5	5676.8	5663.7	5682.6	5683.8	5684.0	5625.2	114.6	103.3	77.0	114.9	117.2	117.6
KLSE	6313.5	6317.0	6302.7	6314.8	6318.3	6318.3	6108.7	409.6	416.6	388.0	412.2	419.2	419.2
KSE100	5483.7	5526.5	5448.2	5484.9	5523.3	5528.5	5417.5	132.2	218.0	61.4	134.8	211.5	221.9
MOEX	5448.5	5449.1	5448.5	5448.9	5455.0	5455.3	5444.0	9.2	10.2	9.2	10.0	22.1	22.7
SSE100	4874.8	4857.6	4849.4	4873.9	4874.4	4877.0	4791.9	165.7	131.3	114.9	164.0	164.9	170.3
TADAWUL	5488.5	5512.5	5434.8	5489.4	5510.0	5514.0	5431.9	113.1	161.2	5.7	115.0	156.2	164.2

Source: Author's findings

Since the study aims to compare the forecast performance of SP-GARCH, therefore, in Table 7, all forecast performances are given. Firstly, for all the stock market indices, either developed or emerging markets, Spline GARCH outer performs all other GARCH models. It is found to be the best forecast model that captures the high and low frequency of volatility compared to the other GARCH family models. These findings are evidence of how the SP-GARCH model can be used instead of the other traditional GARCH models. The findings are based on the forecast evaluation criteria, namely MSE, RMSE, MAPE, SMAPE, Theil U1, and Theil U2. Similarly, the forecast performances of the remaining GARCH family models are also compared with the SP-GARCH model and each other. The results indicate that the conclusion is not supporting only a single GARCH model. The results are a little mixed. Unexpectedly, the GARCH-M model is the best and most precise forecast technique for three developed stock market indices, namely BIST, CAC40, and FTSE, compared to all GARCH models. These three developed stock markets belong to Turkey, France, and the UK, respectively. Even though the forecasting ability of the SP-GARCH model is found to be very close to the GARCH-M model. Still, the forecast performance of the GARCH-M model is slightly superior to the SP-GARCH model and all other GARCH models included in this study. Additionally, based on six forecast error info criteria, the GARCH-M model is found to be the best forecast technique for developed stock markets especially belongs to CPEC. Standard GARCH (1,1) is found to be the best forecast model for the emerging stock market, namely KLSE belongs to Malaysia. GARCH (1,1) is also found to be better in forecast ability than the SP-GARCH model. Nevertheless, GARCH (1,1) and SP-GARCH forecast performance are very close. Additionally, the traditional standard GARCH (1,1) model outer performs the SP-GARCH model only based on RMSE and MAE. Finally, the results support the results for the remaining five stock markets, namely KASE, KSE-100, MOEX, SSE-100, and TADAWUL from Kazakhstan, Pakistan, Russia, China, and Saudi Arabia, respectively SP-GARCH model. The SP-GARCH model is the best and most precise forecast model for all five remaining emerging stock markets. Even though the forecasting ability of the GARCH (1,1), EGARCH, and GARCH-M models are also appreciable, the forecast performance of the SP-GARCH model is undoubtedly superior to GARCH (1,1) model as well as all other GARCH models included in this study. Lastly, based on six forecast error info criteria, the SP-GARCH model is found to be the best forecast technique for emerging stock markets especially belongs to CPEC.

Table 6. Estimated GARCH models evaluation based on AIC and BIC criteria

INDICES	*Info Criteria*	*EGARCH*	*GARCH*	*GARCHM*	*GJR-GARCH*	*PARCH*	*IGARCH*	*SP-GARCH*
BIST	AIC	-5.847	-5.922	-5.843	-5.844	-5.847	-5.782	-5.628
	BIC	-5.831	-5.906	-5.824	-5.828	-5.828	-5.775	-5.602
CAC40	AIC	-6.380	-6.332	-5.852	-6.371	-6.387	-6.285	-6.238
	BIC	-6.364	-6.319	-5.836	-6.355	-6.368	-6.279	-6.209
FTSE	AIC	-6.008	-6.619	-6.620	-6.649	-6.662	-6.581	-6.478
	BIC	-5.992	-6.606	-6.604	-6.633	-6.643	-6.574	-6.446
KASE	AIC	-6.653	-6.661	-6.660	-6.661	-6.660	-6.641	-6.693
	BIC	-6.637	-6.648	-6.644	-6.645	-6.641	-6.635	-6.658
KLSE	AIC	-7.404	-7.401	-7.402	-7.406	-7.404	-7.391	-7.157
	BIC	-7.388	-7.388	-7.386	-7.390	-7.385	-7.384	-7.135
KSE100	AIC	-6.477	-6.428	-6.428	-6.473	-6.478	-6.389	-6.347
	BIC	-6.461	-6.415	-6.412	-6.457	-6.459	-6.382	-6.324
MOEX	AIC	-6.386	-6.387	-6.386	-6.393	-6.392	-6.358	-6.376
	BIC	-6.370	-6.374	-6.370	-6.377	-6.373	-6.352	-6.351
SSE100	AIC	-5.146	-5.112	-5.111	-5.146	-5.145	-5.082	-5.610
	BIC	-5.130	-5.099	-5.095	-5.130	-5.126	-5.076	-5.582
TADAWUL	AIC	-6.460	-6.433	-6.433	-6.458	-6.461	-6.373	-6.433
	BIC	-6.444	-6.421	-6.417	-6.442	-6.442	-6.366	-6.404

Source: Author's findings

Table 7. Forecast performance evolution of GARCH models

Stock Returns	*Models*	*EGARCH*	*GARCH*	*GARCHM*	*GJR-GARCH*	*IGARCH*	*PARCH*	*SP-GARCH*
BIST	RMSE	19.283	19.268	19.256	19.285	19.271	19.281	19.265
	MAE	12.834	12.796	12.791	12.84	12.806	12.83	12.793
	MAPE	0.986	0.983	0.983	0.987	0.984	0.986	0.983
CAC40	RMSE	59.483	59.347	59.247	59.438	59.376	59.494	59.344
	MAE	41.92	41.822	41.741	41.88	41.837	41.931	41.82
	MAPE	0.788	0.787	0.786	0.788	0.787	0.788	0.787
FTSE	RMSE	68.094	68.02	67.851	68.071	68.027	68.094	68.01
	MAE	49.166	49.054	48.865	49.119	49.07	49.166	49.053
	MAPE	0.777	0.776	0.773	0.776	0.776	0.777	0.775
KASE	RMSE	16.32	16.279	16.272	16.303	16.275	16.309	16.239
	MAE	12.271	12.271	12.27	12.271	12.272	12.271	12.269
	MAPE	0.449	0.449	0.449	0.449	0.449	0.449	0.449
KLSE	RMSE	13.316	13.312	13.275	13.314	13.312	13.314	13.267
	MAE	9.975	9.961	9.962	9.97	9.962	9.971	9.958
	MAPE	0.634	0.633	0.634	0.634	0.633	0.634	0.633
KSE100	RMSE	392.675	391.838	391.676	392.753	391.579	392.703	391.564
	MAE	296.2	295.559	295.286	296.251	295.325	296.218	295.266
	MAPE	0.704	0.703	0.702	0.704	0.702	0.704	0.701
MOEX	RMSE	30.117	30.102	30.079	30.133	30.084	30.134	30.023
	MAE	23.505	23.495	23.478	23.521	23.483	23.522	23.412

	MAPE	0.741	0.74	0.74	0.741	0.74	0.741	0.74
SSE-100	RMSE	80.135	80.166	80.168	80.181	80.23	80.182	80.131
	MAE	59.445	59.423	59.427	59.425	59.437	59.425	59.421
	MAPE	0.861	0.861	0.861	0.861	0.861	0.861	0.861
TADAWUL	RMSE	64.282	63.442	63.543	63.948	63.374	64.017	63.371
	MAE	45.058	44.217	44.355	44.665	44.172	44.745	44.171
	MAPE	0.516	0.507	0.508	0.512	0.506	0.513	0.506

Source: Author's findings * significant at 1%, ** significant at 5%

Table 8. Diebold-Mariano Test

Models	***Loss fn***	***Stock market indices***								
		BIST	***CAC40***	***FTSE***	***KLSE***	***KASE***	***KSE-100***	***MOEX***	***SSE-100***	***TADAWUL***
GARCH vs SP-GARCH	Abs Error	-1.420	-0.792	-0.072	-6.285*	-0.143	0.103	-0.942	-0.061	1.977*
	Sq Error	-0.525	-0.943	0.020	-5.611*	-2.068*	0.029	-1.144	0.199	2.968*
EGARCH vs SP-GARCH	Abs Error	-1.725	-1.439	0.547	-6.156*	-0.308	0.266	-1.187	0.331	3.587*
	Sq Error	-0.699	-1.351	0.369	-5.500*	-2.229*	0.258	-1.198	0.139	3.469*
GARCH-M vs SP-GARCH	Abs Error	-1.406	-0.947	-0.925	-6.654*	-0.164	0.007	-0.907	0.045	1.549
	Sq Error	-0.853	-1.056	-0.753	-6.067*	-2.119*	-0.029	-1.228	0.294	1.292
GJR-GARCH vs SP-GARCH	Abs Error	1.617*	-1.244	0.366	-6.199*	-0.240	0.277	1.245	-0.045	2.963*
	Sq Error	0.724	-1.254	0.306	-5.537*	-2.167*	0.275	1.258	0.246	3.288*
IGARCH vs SP-GARCH	Abs Error	-1.450	-0.954	0.254	-6.275*	-0.128	0.023	0.008	0.012	-1.986*
	Sq Error	-0.575	-1.074	0.145	-5.602*	-2.051*	-0.067	0.003	0.361	-2.979*
PARCH vs SP-GARCH	Abs Error	-1.685	-1.483	0.545	-6.195*	-0.268	0.270	1.245	-0.044	3.117*
	Sq Error	-0.687	-1.372	0.368	-5.533*	-2.190*	0.264	1.259	0.248	3.327*

Source: Author's findings * significant at 1%, ** significant at 5%

Since the SP-GARCH model is revealed as better than the other GARCH models based on MAE, RMSE, MAPE, SMAPE, Theil-U1, and Theil-U2 forecast criteria for significant results, DM-test is used to check the predictive power between these two GARCH models (Table 8). According to the Diebold-Mariano test given in Table 8, the SP-GARCH and all other GARCH models have identical forecast abilities for six stock market returns. Therefore, all GARCH models are equal in predictive power at a 5% significance level except for KLSE, KASE, BIST, and TADAWUL. The predictive ability of the SP-GARCH model is significantly higher compared to other GARCH models. It is highly recommended to use the SP-GARCH model to forecast the stock returns volatility, especially for KLSE, KSAE, and TADWULK stock markets.

5. Conclusion

To model and forecast the stock market volatility, especially with the high and low frequency, is quite a grueling task in economics and finance. It is considered a productive space for research due to the enormous use of the volatility forecasts of stock markets in risk management, option pricing, and portfolio management. Often the study of volatility in notional stock markets by investors, economists, actuaries, etc., is an unpretentious burden for the government policy-makers. Especially the stock markets related to CPEC has been a big fish since 2014. This economic corridor has similar advantages to neighbor-friendly nations, i.e., China and Pakistan. Mostly, it is known as the game-changer for Pakistan. The effects are not only around in Asia but also in Europe and Russia. Most nations from Europe or Asia wanted to be a part of this corridor to enhance their strategic network and economic activities. According to Hussain and Hussain (2017), the CPEC will be expected to connect more than 50 nations to this well-established economic hub, turning this region into an economic block. This study uses daily data of only nine stock market indices, including Pakistan and China. Firstly, volatility is studied and explored in all stock markets. All the stock markets showed slow and low-frequency volatility. Nonlinear models are used after testing the presence of ARCH effects. Standard GARCH (1,1), GARCH-M (1,1), IGARCH (1,1), PARCH (1,1), EGARCH (1,1), GJR-GARCH (1,1), and SP-GARCH models are used for estimation. Data was split into two parts. The daily stock market closed prices from December 2014 to July 2020 are used for estimation, while the remaining is used for the one-step-ahead forecast comparison. In the presence of volatility features, i.e., slow and low-frequency volatility, SP-GARCH and other GARCH models are estimated. The first task was to determine whether the SP-GARCH model is better fitted than the traditional GARCH family models. It is revealed that for all the cases, SP-GARCH is found to be the most suitable model among all GARCH models based on AIC, BIC, and LRT. Furthermore, all GARCH models are estimated and compared to each other based on AIC and BIC. The findings suggested a mixed conclusion concerning the best-estimated model for all the emerging and developed countries. For the BIST stock market, Standard GARCH (1,1) model is found to be the best-fitted model, While PARCH is found to be best fitted for two developed stock markets, i.e., CAC40 and FTSE. For KLSE and MOEX, GJR-GARCH is found to be the best-fitted model. Also, for two emerging stock markets, namely KASE and SSE-100, SP-GARCH is the best-fitted model among all other GARCH family models. Lastly, EGARCH is found to be the best-fitted model for two emerging markets, i.e., KSE-100 and TADAWUL. Moreover, the forecast performance for all the stock market indices is evaluated. Engle and Rangel (2008) proposed the SP-GARCH model and showed that it is the best-fitted model. The findings of this study are pretty similar to Engle and Rangel (2008) but in forecast performance. It is revealed that the Spline GARCH outer performs to the traditional standard GARCH (1,1) model for all stock markets. It is found to be the best forecast model that captures the slow and low frequency of volatility compared to all other GARCH models. These findings are evidence of how the SP-GARCH model can be used instead of the GARCH (1,1) model by investors and economists. The findings are based on the forecast evaluation criteria, namely MSE, RMSE, MAPE, SMAPE, Theil U1, and Theil U2. Moreover, the DM-tests concluded that there is no significant difference between the forecasts of Spline-GARACH and GARCH (1,1) models. Therefore, both GARCH models have equal predictive power. Similarly, the forecast performances of the remaining GARCH family models are also compared with the SP-GARCH model and each other. The results indicate that the conclusion is not supporting only a single GARCH model. Unpredictably, the GARCH-M model is found to be the best and most precise forecast technique for three developed stock market indices, namely BIST, CAC40, and FTS. The forecast performance of the SP-GARCH model is close to the GARCH-M model. Likewise, GARCH (1,1) is found to be the best forecast model for the emerging stock market, namely KLSE. Lastly, for the remaining five stock markets, namely KASE, KSE-100, MOEX, SSE-100, and TADAWUL, the results effusively support the SP-GARCH model. The SP-GARCH model is found to be the best and most precise forecast model for all five remaining emerging stock markets. The SP-GARCH model is found to be the best forecast technique for emerging stock markets especially belongs to CPEC. According to these results, the SP-GARCH model can be used as an alternative volatility model for stock market forecast purposes, especially for the CPEC related nations.

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