

A Cusp Catastrophe Model to analyze the impact of COVID-19 on stock market

Amina Amirat¹, Makram ZAIDI²

¹Department of Finance and Economics, University of Jeddah, Jeddah 23436, Saudi Arabia.

²Department of General Courses, College of Applied Studies and Community Service, Imam Abdulrahman Bin Faisal University, P.O. Box 1982, Dammam, Saudi Arabia.

Received: 21st January 2021

Revised: 13th March 2021

Accepted: 18th April 2021

Abstract: The COVID-19 outbreak not only alters the health and lives of humans but also causes world economy to stagnate. Stock market are sensitive to any event that affect the economy as whole such as infectious diseases. This study investigates whether the catastrophe model can analyze and describe the volatility of stock market under the effect of COVID-19 virus. We employed a stochastic cusp catastrophe model to data from the Saudi stock market from November 2019 to June 2020. We find that cusp catastrophe model outperforms linear regression and that the market is in dominated by chartist where the chosen variables: investor sentiment, Covid-19 cases and oil prices are in the bifurcation set.

Keywords: Cups catastrophe model, volatility, sentiment, stock market, bifurcation

JEL classifications:C01, C53

1- Introduction

Unexpected virus called coronavirus disease 2019 (COVID-19) has been a nightmare for the worldwide since the ending of 2019. The novel (COVID-19) outbreak appeared in December 2019, at Wuhan, a central city in China and fast spread throughout China and the world (Figure1.1). Until now, no drug has been developed for treating the virus and the outbreak causes a negative effect on economic growth. Coronaviruses, named for their crown-like shape, are a large family of pathogens responsible for diseases that range in severity from the common cold to the frequently fatal Middle East Respiratory Syndrome (MERS). New ones pop up now and then, often moving from an animal host to humans. Unlike the

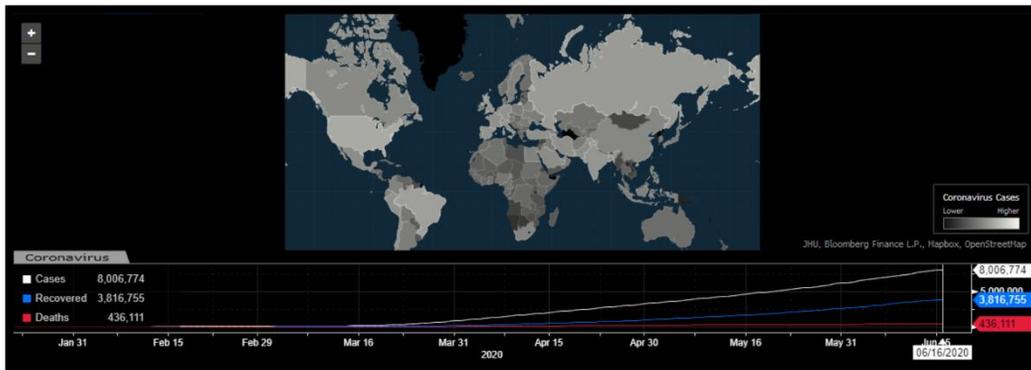
¹Assistant Professor in Finance, Department of finance and economics, College of Business, University of Jeddah, aamirat@uj.edu.sa (corresponding author)

² Assistant professor in quantitative methods, College of Applied Studies and Community Service, Imam Abdulrahman Bin Faisal University, mmzaidi@iau.edu.sa

coronavirus responsible for the 2002-2003 outbreak in Asia of severe acute respiratory syndrome or SARS, this new one can spread via people who are infected but haven't yet developed or won't develop symptoms.

The global outbreak brought a lot of economic and socio-economic uncertainties and forced governments around the world to respond. Many opted for movement restrictions with various degrees to curb the spread of the disease, with often severe direct economic implications. Saudi Arabia is facing the double shock of the coronavirus (Figure 1.2) and the collapse in oil prices (Figure 1.3). Either of these is painful enough by itself. Their combined effect will be devastating. The timid pandemic response means the economy will not be able to restart at full speed once the containment measures have been lifted. Without support, firms will go bankrupt and workers will be laid off. The result will be an economy contracting at 2.9% ~ the slowest rate since the 1980s.

Figure 1.1: Coronavirus spread in the world



Saudi Arabia will contract in 2020 as Covid-19 ripple effects flow through various channels, leaving energy markets bearish (Figure 1.3), trade flows interrupted, international travel constrained, and major events cancelled. An easing of lockdown situations in the country, which together with supportive fiscal and monetary policy adjustments will help to return to growth in late 2020 and 2021. Though, the recovery will be restrained by ongoing social distancing and public health protocols, as well as cautious expenditure by more financially stressed households, businesses and governments.

Figure 1.2: Cases trend in Saudi Arabia

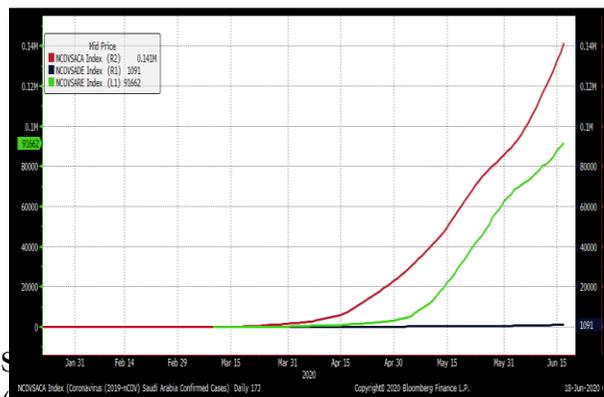


Figure 1.3: Oil Prices



(NCOVSAC: Confirmed cases, NCOVSARE: Recovered case)

Economic turmoil coupled with the COVID-19 pandemic has had wide-ranging and serious impacts upon stock markets. Key events included a described Russia–Saudi Arabia oil price war after failing to reach an OPEC+ accord that resulted in a collapse of crude oil prices and a stock market collapse in March 2020. The effects upon markets in Saudi Arabia are part of the coronavirus recession and among the impacts of the pandemic.

The objective of this paper is to study the impact of Covid-19 on market volatility in Saudi Arabia. This paper deal with an empirical analysis of catastrophe theory and may contribute to this research as few papers in literature used this theory. We used the model of Zeeman and main aim of our paper is to check if catastrophe models can explain the stock market volatility during crashes.

The catastrophe model tries to disclose information that will help us to better understand the formation of crash. It explains how little adjustments in control variables can have unexpected impacts on dependent parameters. In our article, we employ the theory to quick stock market variations that are known as crashes by using real world data during the actual crisis of COVID-19.

The research question addressed in this article is of vital importance to policy makers and investors alike. This is the first paper that estimates the impact of daily reported Covid-19 cases, oil prices and investor sentiment on volatility of the returns of TASI index. We fit the catastrophe model to the data from 24th November 2019 to 11th June 2020. Zeeman (1974) was the pioneer to use qualitative methodology of catastrophe theory to explain how stock markets can behave unstably. We follow his work by integrating quantitative methodology related to the actual pandemic.

Our paper is organized as follow. The first section introduces the paper. The second section summarizes the related literature review. The third section describes the methodology. The fourth section presents the results and discussion and the last section will conclude.

2- Literature review

There is a little but fast-growing study that analyses the effect of Covid-19 on the stock market (e.g., Gormsen & Koijen (2020), Yilmaz kuday (2020), and Baker et al. (2020)). Yilmaz kuday (2020) studies the impact of the number of deaths related to Covid-19 on the S&P500. Baker et al. (2020) quantifies the impact of news related to Covid-19 on stock market volatility using textual analysis and report that Covid-19 has had a much greater impact on volatility than other similar diseases, such as Ebola.

The impact of the COVID-19 is of critical importance, especially since its first outbreak happened in China and spread to other countries. Researchers believe that COVID-19 and SARS belong to the similar family, but these two epidemics diverge significantly. Many previous researches related to the economic impact of the infectious virus epidemic could be referred to as we examine the impact of COVID-19.

Looking at the impact on stock markets, Nippani and Washer (2004) examined the impact of SARS on Canada, China, the particular administrative region of Hong Kong, Indonesia, China, Singapore, the Philippines, Vietnam and Thailand and concluded that SARS only affected the stock markets of China and Vietnam. Del and Paltrinieri (2017) evaluated the mutual equity funds in African countries and suggested that Ebola and the Arab Spring seriously affect the funds flows, spending, and returns of the market. Macciocchi et al., (2016) considered the short-term economic impact of the Zika virus outbreak on Brazil,

Argentina and Mexico, and their results showed that markets did not show large negative returns the day after each shock. Ming-Hsiang Chen, Shawn, and Gon (2007) checked the SARS outbreak influences on the efficiency of Taiwanese hotel stocks using an event study approach and found declines in income and stock price. According to Bai (2014) and Baker, Wurgler, and Yuan (2012) investors may feel pessimistic about investment prospects in a given market during disease outbreak which will impacts stock returns and volatility.

Because in our paper we are using the cusp catastrophe theory to assess the impact of COVID-19 on stock markets, it would be relevant to go through the application of this theory in the social sciences. Unfortunately, the use of catastrophe theory has not been as extensive as in the natural sciences, although it was applied early in its existence. Zeeman's (1974) cooperation with Thom and his own popularization of the theory through the use of nontechnical examples (Zeeman 1975, Zeeman 1976) led to the development of many applications in the fields of economics, psychology, sociology, political studies and others (Barunik and Kukacka, 2015). Zeeman (1974) also proposed the application of the cusp catastrophe model to stock markets.

The main difficulty in the use of catastrophe theory occurs from the fact that it stems from deterministic systems. Hence, it is hard to apply it precisely to systems that are subject to random influences, which are common in the behavioral sciences. Cobb and Watson (1980), Cobb (1981) and Cobb and Zacks (1985) provided the necessary bridge and took catastrophe theory from determinism to stochastic systems. The main constraint of Cobb's method of estimation was the need of a constant variance, which forces researchers to suppose that the volatility of the stock markets is steady. Quantitative verification of Zeeman's (1974) hypotheses about the use of the theory to stock market crashes was pioneered by Barunik and Vosvrda (2009), where they applied the cusp model to two separate, large stock market crashes. But, the successful application of Barunik and Vosvrda (2009) brought only initial results in a limited environment.

In the current research, we propose an upgraded method of application that we believe brings us closer to an answer regarding whether cusp catastrophe theory is capable of explaining stock market volatility during crisis.

3- Data and Methodology

3-1- Cusp Catastrophe theory

Initiated by the French mathematician Rene Thom (1975), catastrophe theory is a special area of dynamical systems theory. It analyzes and classifies phenomena characterized by sudden and unexpected shifts in behavior arising from small variations in circumstances.

To detect the existence of catastrophe, we need five elements that represent the flags of catastrophe (Glimor, 1993). These elements are (1) Bimodality, where the distribution shows two clearly different modes; (2) sudden jump, where the outcome variate brusquely between the modes even with minor changes in the predictors; (3) inaccessibility, where an outcome value in the zone between the modes is improbable; (4) hysteresis, where change from one mode to the other cannot be determined by the same values for control factors; and (5) divergence, where a small modification in the control variable cause a large shift in the outcome and deviation from the linear model.

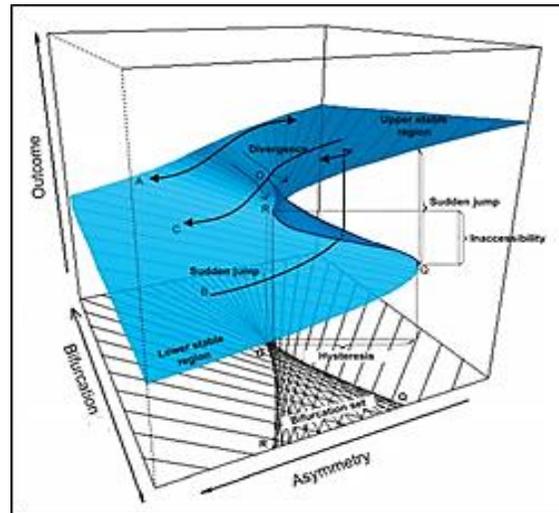
The deterministic cusp catastrophe model is specified using three components: two control factors (i.e., α and β) and one outcome variable (i.e., z_t). This model is described by a differential equations-based dynamic system:

$$\frac{dz_t}{dt} = \frac{dV(z_t; \alpha; \beta)}{dz_t} \tag{1}$$

(t is time from 0 to T)

For the potential function V , the argument α is called asymmetry or normal control factor where the outcome z_t changes asymmetrically from one mode to the other eventually as α increases. The argument β is called bifurcation or splitting control factor, which produces the outcome side to divide and bifurcate from soft variations to sudden jumps as β increases. Both α and β are linked to determine the outcome variable z_t in a three-dimensional outcome response surface. When the right side of Equation 1 moves toward 0, the outcome z_t will not change with time. Such status is called equilibrium; this hypothesis is necessary to explain cusp models based on cross-sectional data. In general, the behavior of the outcome z_t , that is, how it changes with time t, is in general complicated, but each subject will move toward an equilibrium status. Figure (3.1) clearly describes the stability plane, which mirrors the reaction side of the outcome measure (z_t) at several permutations of asymmetry control factor (α) and bifurcation control factor (β).

Figure 2.1: Cusp catastrophe model



The cusp model is built on the nonlinear deterministic dynamical system described in equation (1), where the behavior of z_t (the volatility of stock market index in our study) will change over time t according to the derivative of the cusp potential function defined by:

$$V(z_t; \alpha; \beta) = -\frac{1}{4}z_t^4 + \frac{1}{2}\beta z_t^2 + \alpha z_t \tag{2}$$

Which will be in equilibrium at:

$$\frac{dV(z_t; \alpha; \beta)}{dz_t} = -z_t^3 + \beta z_t + \alpha = 0 \quad (3)$$

Where α and β are the control variables which specify the behavior of the system. These variables are factors which will depend upon the actual measured independent variables. So, lets assume a set of n independent variables $\{X_1, X_2, \dots, X_n\}$ and x_i are the realizations of X_i , and the control variables α_x and β_x .

$$\alpha_x = \alpha_0 + \sum_{i=1}^n \alpha_i x_i \quad (4)$$

$$\beta_x = \beta_0 + \sum_{i=1}^n \beta_i x_i \quad (5)$$

So, for each value x_i we will get three forecasted values of the state variable. The predicted values will be roots of the following equation:

$$\frac{dV(z_t; \alpha_x; \beta_x)}{dz_t} = -z_t^3 + \beta_x z_t + \alpha_x = 0 \quad (6)$$

Which describes the cusp catastrophe response surface containing a smooth pleat as shown in figure (3.1).

Additionally, Cobb (1981) used λ and σ which are strictly positive and represent the location and scale parameters, respectively; and the equilibrium will be:

$$\frac{dV(z_t; \alpha_x; \beta_x)}{dz_t} = -\left(\frac{z_t - \lambda}{\sigma}\right)^3 + \beta_x \left(\frac{z_t - \lambda}{\sigma}\right) + \alpha_x = 0 \quad (7)$$

Now, the statistical estimation problem it to estimate the $2n+4$ parameters: $\lambda, \sigma, \alpha_0, \dots, \alpha_n, \beta_0, \dots, \beta_n$.

According to dynamic system, the equation (7) can be seen as the surface of equilibrium points for the dynamic system of the variable z_t which is following the equation (1)

$$dz_t = \frac{dV(z_t; \alpha_x; \beta_x)}{dz_t} dt \quad (8)$$

When using data from real-world, it is important to add non-deterministic behavior to this system because the actual system does not determine its next states completely.

$$dz_t = -\frac{dV(z_t; \alpha_x; \beta_x)}{dz_t} dt + \sigma_{z_t} dW_t \quad (9)$$

Where:

$-\frac{dV(z_t; \alpha_x; \beta_x)}{dz_t}$ is a drift or deterministic part that represent the equilibrium state of the cusp model

σ_{z_t} is the instantaneous variance of the process z_t

W_t is the standard Wiener process and $dW_t \sim N(0, dt)$

Researchers put a link between PDF corresponding to the solution of the equation (9) and the one corresponding to a limiting stationary stochastic system (Cobb (1981), Cobb and Watson (1980), Hartelman (1997), Hartelman et al. (1998) and Wagenmakers et al. (2005)). They demonstrate that PDF $f(z_t)$ converges in time to the PDF $f_s(z|x)$ as the dynamics of z_t are supposed to be much faster than changes in x_i . This has led to an exact definition of stochastic equilibrium state that is conform with its deterministic counterpart. According to Hartelman (1997) and Wagenmakers et al. (2005), the limiting PDF of z is:

$$f_s(z|x) = \xi \exp \left(-\frac{1}{4} \left(\frac{z-\lambda}{\sigma_z} \right)^4 + \frac{\beta_x}{2} \left(\frac{z-\lambda}{\sigma_z} \right)^2 + \alpha_x \left(\frac{z-\lambda}{\sigma_z} \right) \right) \quad (10)$$

The constant ξ normalizes the PDF, which mean that the integral of the normalized PDF over its entire range will be equal to 1. The implicit cusp surface can be found by solving the equation $\frac{df_s(z|x)}{dz} = 0$ and obtaining the modes and anti-modes of the cusp catastrophe PDF. The parameters will be estimated following the method of Hartelman (1997) and Wagenmakers et al. (2005).

By elimination z from the canonical equation (7) and its equation of double roots, we can obtain the statistic that discriminates between the unimodal and bimodal cases. It is called Cardan's discriminant:

$$\delta_x = \left(\frac{\alpha_x}{2} \right)^2 - \left(\frac{\beta_x}{3} \right)^3 \quad (11)$$

If the Cardan's discriminant is negative resp. nonnegative, the PFD is bimodal or unimodal. The equation (11) also determines the shape of the cusp model. It shows the locus of fold bifurcations which separate the region with two stable solutions from the one with unique stable solution.

3-2- Application of Cusp theory to stock markets

In our empirical test, we follow Hartelman (1997), who designed an application for estimating a cusp model. A comparison between the cusp and the linear regression models is done using of a likelihood ratio test, which is asymptotically chi-squared distributed with degrees of freedom equal to the difference in degrees of freedom for two compared models. Hartelman (1997) also compares catastrophe model to a nonlinear logistic model:

$$z = \lambda + \frac{\sigma}{1 + e^{-(\alpha_x/\beta_x^2)}} \quad (12)$$

Where $\sigma > 0$

Cusp catastrophe model and the logistic model will be compared using Akaike information criterion (AIC) and Bayesian information criterion (BIC) statistics. So, we need two conditions to approve that the cusp model outperform the linear and logistic models. The first is that the chi-square test shows that the likelihood of cusp is significantly greater than the one of linear regression. The second is that AIC and BIC will be lower for the cusp model.

In this paper, we will follow the work of Zeeman (1974) who was the first who stated that the cusp catastrophe model can explain uncertain behavior of the stock market. One of the major hypotheses of Zeeman is that in stock market we have two types of investors: fundamentalists and chartists.

Fundamentalists act based on estimates of economic factors and fundamental analysis of company stocks. Chartists act according to the behavior of stocks and usually use charts and historical prices for predictions. So, the two variables are C which represent the proportion of speculative money in the capital market and F who represent excess demand for the stock.

In our research the static variable z_t is the volatility of Saudi stock index TASI. Fundamentalist that are external driving forces represent an asymmetry control variable α_x and chartists which are part of external process of stock markets constitute the bifurcation variable β_x .

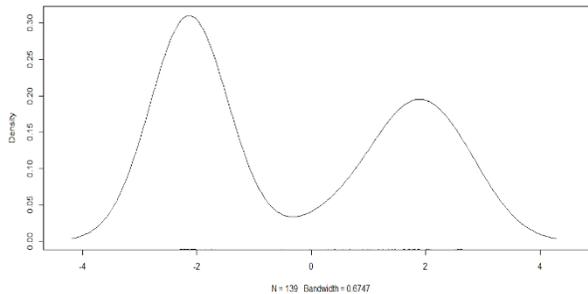
3-3- Data

We will test the cusp catastrophe model on the set of daily volatility of the Saudi stock market index TASI during the most recent crisis of COVID-19. The data lies between 24th November 2019 to 11th June 2020 with total of 140 observation. Although the first Covid-19 confirmed case in Saudi Arabia was in 3rd March 2020 but we extended our database to the end of November 2019 because this pandemic started in China in early December 2020. All data are extracted from bloomberg terminal.

The state variable in our research is the volatility of TASI index defined as the relative rate at which the price of a security moves up and down, found by calculating the annualized standard deviation of daily change in price. We use the Augmented Dickey-Fuller test (ADF test) to check the stationarity of the volatility of returns. The test statistic was -1.5537 with a p-value 0.7622 which means that we accept the alternative hypothesis of stationarity. To check the bimodality, we apply kernel density estimation using gaussian kernel with smoother bandwidth (0.6747). As shown in figure (3.1), kernel density indicates bimodality in stock index volatility which will allow us to test for bifurcations.

The percentage change in oil prices and confirmed covid-19 cases correlate with volatility of prices and represent a good measure for fundamental investors. For the ratio of advance/decline stocks (A/D) is a breadth indicator used to show how many stocks are participating in a stock market rally or decline. We put 0 for days where A/D is going down to show a bearish divergence and 1 for days where A/D is turning upward which is a sign of bullish. We expect that this variable will be on the asymmetry side of the model.

Figure 3.1: Kernel density estimate of TASI volatility



For control variables, we have chosen the daily change in oil prices, the daily change in confirmed cases of COVID-19 and the ratio of advancing to declining stocks as a measure of investor sentiment.

The descriptive statistics of our data are shown in table 1.

Table 1: Descriptive statistics of data.

Statistics	Volatility	Confirmed Cases COVID-19	Oil prices
Minimum	14.01	0	-0.24
Maximum	43.72	4	0.21
Mean	25.9	0.11	-0.002
Standard deviation	12.09	0.39	0.055
Median	16.17	0	0.001
Mode	Multi	0	Multi
Kurtosis	1.24	72.12	8.85
Skewness	1.26	7.6	-0.59

The result in the table confirm the bimodality of the state variable which also platykurtic.

4 Results and discussion

At first, we consider Hartigans' dip test for multimodality (Hartigan (1985) and Hartigan and Hartigan (1985)), which can be used to test the presence of bifurcation points in the data. The dip test measures multimodality in a sample by the maximum difference, over all sample points, between the empirical distribution function, and the unimodal distribution function that minimizes that maximum difference. We find that the statistic of the test is 0.1167 with p-value 0.000, so we accept the alternative hypothesis of non- unimodal data which means at least bimodal. This result confirms the existence of one bifurcation point at least.

We move now to the estimation of the cusp catastrophe model using real-world data. The results are shown in table (2). The estimation was done using the software R.

The control variables are expressed as follow

$$\alpha_x = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 \quad (13)$$

$$\beta_x = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \quad (14)$$

Where:

x_1 is sentiment indicator extracted from the A/D ratio,

x_2 is the percentage change in the daily number of COVID-19 confirmed cases,

x_3 is the percentage change in oil prices.

α_i and β_i are parameters (ifrom 0 to 1)

Table 2: Cusp catastrophe results

	α_0	α_1	α_2	α_3	β_0	β_1	β_2	β_3	λ	σ	R^2	LL	AIC	BIC
Unrestricted cusp model	-0.217 (0.076)	0.093 (0.457)	0.829 (0.018) **	1.593 (0.082)	3.740 (0.000) ***	1.010 (0.000) ***	-1.651 (0.002) ***	4.629 (0.032) **	-4.624 (0.000) ***	0.165 (0.000) ***	0.934	-55.10	130.21	159.634
Restricted cusp model	-0.131 (0.006) ***	-	0.784 (0.023) **	-	3.758 (0.000) ***	0.996 (0.000) ***	-1.649 (0.002) ***	4.514 (0.036) **	-4.624 (0.000) ***	0.165 (0.000) ***	0.934	-56.92	129.84	153.377
Linear	-	-	-	-	-	-	-	-	-	-	0.037	- 544.49	1098.9 7	1113.68

(**) significant at 5%, (***) significant at 1%

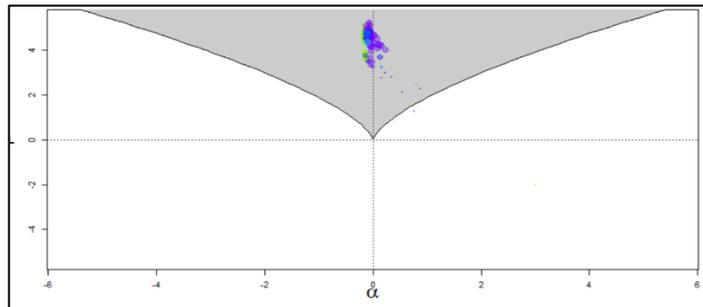
On the left side of table (2), we fit the cusp catastrophe model to the stock index volatility without any restrictions and we use all control variables as appeared in equations (13) and (14). In terms of the log likelihood, the cusp model describes the data better than the linear regression model. The coefficient σ is far away from zero. In term of r-squared, the cusp model strongly outperforms the linear model. For the unrestricted model, all parameters are significant at 95% and 99% in the bifurcation side. But in the asymmetry side only the percentage change in the number of COVID-19 confirmed cases is significant at 95% level. Other coefficients in the asymmetry side are non-significant suggesting that they are not needed in the model. So, we fit again the model leaving these regressors out of the model. We set $\alpha_1 = \alpha_3 = 0$:

$$\alpha_x = \alpha_0 + \alpha_2 x_2 \tag{15}$$

Now all the regression coefficients 'are significant'. If we turn attention to the information criteria, we see that the AIC and BIC for the second model are smaller than the first. We conclude that all variables contribute to the bifurcation side but only the Covid-19 variable contributes to the asymmetry side.

The results suggest strong evidence that over the of study, the stock markets are better described by the cusp catastrophe model. Using our two-step modelling approach, we have shown that the cusp model fits the data well and the fundamental side is controlled by only the Covid-19 cases while the bifurcation side is controlled by investor sentiment, Covid-19 cases and oil prices.

Figure 4.1: Cusp region



A control plane scatter plot of our data is presented in Figure (4.1). A three-dimensional display of the model fit as generated with cusp3d is presented in Figure (4.2).

A couple of things may be noted from the scatter plot. First of all, the sizes of the dots differ. In fact, the size of the dots, each of which corresponds to a single case, varies according to the observed bivariate density of the control factor values at the location of the point. A second observation is all cases lie inside the bifurcation. Almost points are colored in more intense purple, so they are associated with high values of the state variable. This means that our variables lead to high volatility and create bifurcation points.

Figure 4.2: Cusp 3D pre

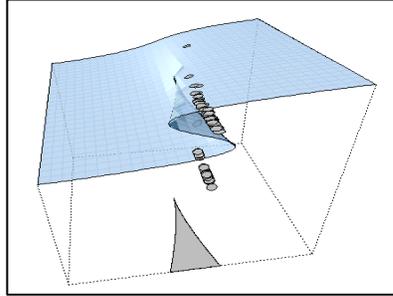


Figure (4.2) confirm that all points are in the bifurcation set and that that the volatility in our period of study lies in catastrophe area. This means that small changes in our parameters : Investor sentiment, Covid19- cases and oil prices can cause equilibria to appear or disappear, or to change from attracting to repelling and vice versa, leading to large and sudden changes of the volatility of returns. The results show that the market is dominated by chartists during this period which are part of external process of stock markets.

5- Conclusion

The novel coronavirus that emerged late last year in central China has infected millions of people around the world. It's taken health officials time to understand that it behaves in ways atypical of similar viruses, producing missteps in the response. The list of symptoms of Covid-19, the disease it causes, keeps growing, complicating efforts to diagnose and treat it. And the virus has yet to give up all its mysteries, creating uncertainties about when the pandemic might end.

In this paper, we analyzed the volatility of Saudi stock market index using cusp catastrophe. This theory has a strong theoretical base and the promising results from the application of this theory in research make the cusp catastrophe model an extremely useful tool for researchers interested in broadening the horizons of their data analysis. We used data from November 2019 to June 2020 as a total of 140 data point. After applying the Hartigans' dip test we found evidence of multimodality which encourage us to fit the data to the cusp catastrophe model taking the volatility of TASI index as a state variable and investors sentiment, daily change in Covid-19 case and daily change in oil prices as control variables. The results show that bifurcation has higher priority than asymmetry which means that the market is dominated by chartists and external forces. The control variables lie in the bifurcation set which mean that these variables are responsible of the sudden jump in stock market volatility during our period of study.

This study should be extended in future research by adding new variables in order to understand more the behavior of stock market during crisis and use our results for prediction purposes.

Declaration of Conflicting Interests

The authors declare that there is no conflict of interest.

Data availability statement

Data is available on request

References

- [1] Bai (2014) Bai, Y. Cross-border sentiment: An empirical analysis on EU stock markets. *Appl. Financ. Econ.* 2014, 24, 259–290.
- [2] Baker, S. R., Bloom, N., Davis, S. J., Kost, K., Sammon, M. & Viratyosi, T. (2020), ‘The unprecedented stock market reaction to covid-19’, White paper - Becker Friedman Institute for economics at UChicago .
- [3] Baker, M.; Wurgler, J.; Yuan, Y. Global, local, and contagious investor sentiment. *J. Financ. Econ.* 2012, 104, 272–287.
- [4] Barunik J and Kukacka J, 2015, *quantitative finance*, vol15, n6, 959–973, <http://dx.doi.org/10.1080/14697688.2014.950319>
- [5] Barunik, J. and Vosvrda, M., Can a stochastic cusp catastrophe model explain stock market crashes? *J. Econ. Dyn. Control*, 2009, 33(10), 1824–1836.
- [6] Cobb, L. and Watson, B., Statistical catastrophe theory: An overview. *Math. Model.*, 1980, 1, 311–317.
- [7] Cobb, L. and Zacks, S., Applications of catastrophe theory for statistical modeling in the biosciences. *J. Am. Stat. Assoc.*, 1985, 392, 793–802.
- [8] Cobb, L., Parameter estimation for the cusp catastrophe model. *Behav. Sci.*, 1981, 26(1), 75–78.
- [9] Del Giudice, A. and Paltrinieri, An., 2017. "The impact of the Arab Spring and the Ebola outbreak on African equity mutual fund investor decisions," *Research in International Business and Finance*, Elsevier, vol. 41(C), pages 600-612.
- [10] Gilmore, R. (1993). *Catastrophe theory for scientists and engineers*. New York, NY: Dover Publications.
- [11] Gormsen, N. J. & Kojien, R. S. J. (2020), ‘Coronavirus: Impact on stock prices and growth expectations’, *SSRN Electronic Journal* . Yilmazkuday (2020)
- [12] Hartelman, P.A.I., 1997. *Stochastic catastrophe theory*. Dissertatiereeks 1997-2, Faculteit Psychologie, Universiteit van Amsterdam. Hartelman, P.A.I., van der Maas, H.L.J., Molenaar, P.C.M., 1998. Detecting and modeling developmental transitions. *British Journal of Developmental Psychology* 16, 97–122.
- [13] Hartigan, J.A. and Hartigan, P., The dip test of unimodality. *Ann. Stat.*, 1985, 13(1), 70–84.
- [14] Hartigan, P., Algorithm as 217: Computation of the dip statistic to test for unimodality. *J. R. Stat. Soc. Ser. C (Appl. Stat.)*, 1985, 34(3), 320–325.
- [15] Macciocchi, D.; Lanini, S.; Vairo, F.; Zumla, A.; Figueiredo, L.T.M.; Lauria, F.N.; Strada, G.; Brouqui, P.; Puro, V.; Krishna, S.; et al. Short-term economic impact of the Zika virus outbreak. *New Microbiol.* 2016, 39, 287–289.
- [16] Ming-Hsiang, C., SooCheong, J. and Woo, G.K, 2007. “The impact of the SARS outbreak on Taiwanese hotel stock performance: An event-study approach” *Int J HospManag.* 2007 Mar; 26(1): 200–212. 10.1016/j.ijhm.2005.11.004

- [17] Nippani, Srinivas, and Kenneth M. Washer. 2004. "SARS: A Non-event for Affected Countries' Stock Markets?" *Applied Financial Economics* 14 (15): 1105-1110.
- [18] Wagenmakers, E.J., Molenaar, P.C.M., Hartelman, P.A.I., van der Maas, H.L.J., 2005. Transformation invariant stochastic catastrophe theory. *Physica D* 211, 263.
- [19] Zeeman E (1974). "On the Unstable Behavior of the Stock Exchanges." *Journal of Mathematical Economics*, 1, 39-44.