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Monitoring Multiple Linear Profile Based on EWMA Control Charts by Using Ridge Regression Estimators: An Application to Wind Tunnel Data of NASA Langley Research Centre

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Abstract: In many quality control studies the performance of a product or process is usually characterized by a single response variable However, in some applications of quality control, the performance of a product or a process can be best characterized by a linear relationship between a response variable and one or more explanatory variables (Noorossana et al., 2011). But, when more than one explanatory variables are involved in the profile it may indicate the presence of high collinearity among explanatory variables which is called multicollinearity (Gujarati et al., 2012). It should be noted that if the multicollinearity is neglected during the profile monitoring, then the designed control charts applied in phase II lack the sufficient effectiveness in detecting shifts or out of control signals. In this paper, the effect of the multicollinearity on the monitoring of multiple linear profiles has been studied and some new type of Exponentially Weighted Moving Average (EWMA) control charts for Intercept, Slopes and Mean Squared Error (MSE) by using Ridge Regression Estimators (RRE) have been proposed in order to provide the solution for multicollinearity. An application of wind tunnel data by NASA Langley Research Centre was used for monitoring profiles based on proposed EWMA control charts for Intercept, Slopes and MSE. The performance of the proposed EWMA control charts have been evaluated on the Average Run Length (ARL) criterion, the results indicated that the proposed EWMA control charts obtained from RRE for Intercept, Slopes and MSE outperform the existing control charts obtained from Ordinary Least Squared (OLS) estimator.

Keywords: Quality Control, Multicollinearity, RRE, EWMA, MSE, OLS

Introduction

In many service and manufacturing organizations the quality of the process and product can play an important role in the prosperity. In the traditional control charts, monitoring the performance of a product or process measurements by using a single quality characteristic at a given time span. However, in new developments, the process and product's performance can be measure by using a linear functional relationship. This linear functional relationship is known as a profile. Mahmoud et al., (2007) defined the profile as "the quality of a process is best characterized and summarized by a linear functional relationship between a response variable and one or more explanatory variables over time". The purpose of the profile monitoring is used to check the stability of the profile parameters (Intercept, Slopes and MSE) in different time span.

In multiple linear profile monitoring the regression analysis was used to estimate the profile's parameters. In multiple linear regression analysis, usually we consider that the profile's predictors are not linearly related to each other. In practice, there may be some type of relationships between the profile's predictors can exist. In this case, the assumption of independence of the profile's predictors is violated, so that the violation of this assumption causes the problem of multicollinearity. Chatterjee et al., (2006) highlighted that the existence of multicollinearity in a data set of two or more explanatory variables give same or approximately same information. Gujarati et al., (2012) highlighted that if there is perfect multicollinearity exist among the independent variables the regression coefficients of the X-variables are indeterminate and their standard errors are infinite. They also commented that if there is high or near to perfect multicollinearity, it effect on the OLS estimators with large variances and covariances and due to large variances the confidence intervals also wider and it is the cause of null hypothesis will be accepted. Also, due to large variances the t ratio of one or more regression coefficients may be tends to statistically insignificant andR^2 will be high and misleading the model interpretation. Smith H. (1981) stated that as a result of multicollinearity, the X^tX, information matrix, is near singular that leads to large standard errors for OLS estimates. Mahmoud et al., (2007) commented in reducing the multicollinearity issue which might lead to an ill-conditioned matrix and provided inaccurate estimates of the profile parameters by using polynomial regression.

The control charts developed for the monitoring of multiple linear profiles may also be effect due to ignore the multicollinearity problem. Kang & Albin (2000), Kim et al., (2003),Kim et al., (2003) and Mahmoud, (2008)have been proposed monitoring the multiple linear profile's coefficients individually using by Shewhart Control Charts or EWMA Control Charts or Multivariate Exponentially Weighted Moving Average (MEWMA) Control charts. They said the coefficients of the polynomial profiles can be made independently by using orthogonal polynomials. Kazemzadeh et al., (2009) extended the approach to polynomial profile in MEWMA control chart and they suggested that centering the X-values to reduce the multicollinearity problem. Bradley & Srivastava (1979) illustrate that centering the X-values does not completely alleviate the problem of ill conditioning. This would result inaccurate or underestimated error coefficient and eventually lead to a poor estimation of the likelihood ratio statistic. Williams et al., (2006) demonstrated the superiority of the change point approach as compared to the Hotteling T^2 statistic and Mahmoud & Woodall (2004) F-approach. They believed the multicollinearity among regression variables will result in an ill conditioning of the HAT matrix and will lead to unstable coefficients. Gupta, (2010) propose to tackle the problem by either normalizing the X-values or by using orthogonal polynomials.

In order to overcome the problem of multicollinearity among explanatory variables, Hoerl & Kennard (1970a) introduced Ridge Regression Estimator (RRE) method instead of Ordinary Least Square (OLS) estimates method for Multiple Linear Regression analysis. This study used different types of ridge regression estimators to handle the multicollinearity problem.

This article is organized as follows: Section 2 contains the methodology related to the detection of multicollinearity with the help of Variance Inflation Factor (VIF), Tolerance, Condition Index and Coefficient of Determination (R^2). Section 3presents an alternative approach that is Ridge Regression Estimators for the analysis of multiple linear profiles in the presence of multicollinearity. The Section 4 represents an application of the wind tunnel experiment data by NASA Langley Research Centre and based on this data develop the multiple linear profile and estimated its parameters by using Ridge Regression Estimators. In section 5 proposed new improved EWMA control chart for Intercept, Slopes and MSE for multiple linear profile monitoring in the presence of multicollinearity for phase II monitoring. The comparison of the proposed RRE based EWMA control charts and existing OLS based EWMA control charts also represent in section 5. The last section contains the final comments and conclusions of the study.

Methodology

Multiple Linear Profile

Eyvazian et al., (2011) described, multiple linear profile is a linear relationship between a response variable and two or more independent variables that should be monitored over time. This relationship can be represents as:

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{1ij} + \beta_{2j}X_{2ij} + \dots + \beta_{kj}X_{kij} + \varepsilon_{ij} \tag{1}$$

Where $i = 1, 2, \dots, n_j$ denotes sample size, $k = 1, 2, \dots, p$ denotes number of independent variables and $j = 1, 2, \dots, m$ denotes number of samples.

The above model (1) can be defined as:

$$Y_j = X_j\beta_j + \varepsilon_j \tag{2}$$

Where, $j = 1, 2, \dots, m$, $Y_j = (y_{1j}, y_{2j}, \dots, y_{n_j j})^T$, $\varepsilon_j = (\varepsilon_{1j}, \varepsilon_{2j}, \dots, \varepsilon_{n_j j})^T$, $\beta_j = (\beta_{1j}, \beta_{2j}, \dots, \beta_{pj})^T$.

The matrix X_j, *say Information Matrix,* is as follows:

$$X_j = \begin{bmatrix} 1 & x_{11j} & x_{21j} & \cdots & x_{p1j} \\ 1 & x_{12j} & x_{22j} & \cdots & x_{p2j} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{1n_jj} & x_{2n_jj} & \cdots & x_{pn_jj} \end{bmatrix}$$

The OLS estimators for estimating the parameters of the multiple linear profiles as follows:

$$\hat{\beta}_{j(OLS)} = (X^TX)^{-1}X^TY_j \tag{3}$$

With var-cov matrix of β is

$$\Sigma = Var(\hat{\beta}_{j(OLS)}) = \sigma^2(X^TX)^{-1} \tag{4}$$

The $\hat{\beta}_{OLS}$ and $Var(\hat{\beta}_{j(OLS)})$ depend on X^TX matrix. The variances computed from the OLS estimate becomes large if the X^TX matrix is near to singular. The OLS estimators are efficient under the assumptions of the Classical Linear Regression Model (CLRM). The multicollinearity is also one of the hottest problem in CLRM. The next section describe the method of detection multicollinearity from multiple linear profiles.

Detection of Multicollinearity

There are several methods to use to detection of Multicollinearity.

1. **Variance Inflation Factor *(VIF)***
2. **Tolerance *(TOL)***
3. **Coefficient of Determination *(R^2)***
4. **Condition Index *(CI)***

Variance Inflation Factor (VIF), Tolerance (TOL) and Coefficient of Determination (R^2)

By the definition of *VIF* described by Gujarati et al., (2012) "How the variance of an estimator is inflated by the presence of multicollinearity". In two variable regression model if coefficient of determination (r^2_{23}) approaches to 1 the *VIF* approaches to infinity. In other words, as multicollinearity among independent variables is increasing the variances of OLS estimator will be also increasing. The formula of *VIF* is as:

$$VIF_j = \frac{1}{(1-R_j^2)}$$

If the value of *VIF* will be exceeds to 10 it indicate that the R^2will be exceeds 0.90, that variable is said be highly collinear.

The inverse of the *VIF* is called tolerance. The formula of *TOL* is as:

$$TOL_j = \frac{1}{VIF_j} = (1 - R_j^2)$$

The value of TOL_j is closer to 0 or R^2_j also close to 1 indicated that that *independent variables* are highly collinear.

Condition Index (*CI*)

The *CI* is also used to check the multicollinearity among the explanatory variables. The condition index is computed from the condition number "k" and condition number is computed from the maximum and minimum eigenvalues. The formula of the condition number is as follows:

$$k = \frac{Maximum\ Eigenvalue}{Minimum\ Eigenvalue}$$

and the formula of *CI* is defined as

$$CI = \sqrt{\frac{Maximum\ Eigenvalue}{Minimum\ Eigenvalue}} = \sqrt{k}$$

If *k* is between 100 and 1000 or the *CI* is between 10 and 30, there is moderate to strong multicollinearity and if it exceeds 1000 or 30, CI indicated high multicollinearity among the independent variables.

Ridge Regression Estimators

Gujarati et al., (2012) concluded that under the CLRM if all the assumption are fulfilled, the OLS estimators provided Best Linear Unbiased Estimators (BLUE). The common procedure in regression analysis that is OLS is not robust to multicollinearity problem and will result in inaccurate model. By solving this problem, a number of methods were developed in the literatures and the most common method was used of Ridge Regression Estimator (RRE)

Hoerl & Kennard, (1970) firstly introduced RRE to overcome the problem of multicollinearity among explanatory variables in Multiple Linear Regression Model. They used a small positive number, say, "k" (≥0) to be added to diagonal of X^TX matrix to reduce the effects of multicollinearity. So that the Ridge Regression Estimator is describe as follow:

$$\hat{\alpha} = (X'X + KI)^{-1} X'Y \qquad (5)$$

The "k" is known as *Ridge Parameter* (constant) and will be finding out from the data. For any positive value of "k", the RRE provides a Minimum Mean Square Error (MMSE) as compared OLS. When k=0, $\hat{\alpha}$ equal to $\hat{\beta}_{OLS}$ and when "k" increases to zero it introduced more bias, but the variance of the regression estimator stabilizes. So that it is quite helpful to allowing a small bias in order to achieve the MMSE.

Hoerl & Kennard (1970b) also described the formula for computing MSE of Ridge Regression Estimator, which is defined as,

$$MSE(\hat{\alpha}(k)) = \sigma^2 \sum_{j=1}^{p} \frac{\lambda_j}{(\lambda_j + k)^2} + \sum_{j=1}^{p} \frac{k^2 \alpha_j^2}{(\lambda_j + k)^2} \qquad (6)$$

The 1st term on right hand side of above MSE is a variance and the second term is an amount of bias, where, k is the ridge parameter, $\sigma^2$is the variance the model computed from OLS, α_i is the ith element of the α and λ_j is the jth eigen value of the X matrix.

In the existing literature, many researchers described many methods for estimation of Ridge parameter "k" based on the canonical form of regression model. The following table showed some existing methods for estimation of Ridge Parameter "k".

Table: Existing Ridge Parameter "k"

Sr. No	Ref	Notation	Formula
1	(Hoerl & Kennard, 1970a)	K_{HK}	$\hat{K}_{HK} = \frac{\hat{\alpha}^2}{\hat{\alpha}_{max}^2}$
2	Dwivedi & Srivastava, (1978)	K_{DS}	$\hat{K}_{DS} = \frac{\hat{\sigma}^2}{\hat{\beta}'\hat{\beta}}$
3	Kibria, (2003)	K_{GM}	$\hat{K}_{GM} = \frac{\hat{\sigma}^2}{(\prod_{j=1}^{p} \hat{\alpha}_j^2)^{\frac{1}{p}}}$
4	Kibria, (2003)	K_{MED}	$\hat{K}_{MED} = median\left\{\frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}\right\}$
5	Muniz & Kibria, (2009)	K_{KM2}	$\hat{K}_{KM2} = max\left\{\frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}}}\right\}$
6	Muniz & Kibria, (2009)	K_{KM3}	$\hat{K}_{KM3} = max\left\{\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}}\right\}$

7	Muniz & Kibria, (2009)	K_{KM4}	$\hat{K}_{KM4} = \left(\prod_{j=1}^{p} \frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}}}\right)^{\frac{1}{p}}$
8	(Muniz & Kibria, 2009)	K_{KM5}	$\hat{K}_{KM5} = \left(\prod_{j=1}^{p} \sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}}\right)^{\frac{1}{p}}$
9	(Muniz & Kibria, 2009)	K_{KM6}	$\hat{K}_{KM6} = median\left(\frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}}}\right)$
10	(Muniz et al., 2012)	K_{KM8}	$\hat{K}_{KM8} = max\left(\frac{1}{\sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}_j^2}}}\right)$
11	(Muniz et al., 2012)	K_{KM9}	$\hat{K}_{KM9} = max\left(\sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}_j^2}}\right)$
12	(Muniz et al., 2012)	K_{KM10}	$\hat{K}_{KM10} = \left(\prod_{j=1}^{p} \frac{1}{\sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}_j^2}}}\right)^{\frac{1}{p}}$
13	Muniz et al., (2012)	K_{KM11}	$\hat{K}_{KM11} = \left(\prod_{j=1}^{p} \sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}_j^2}}\right)^{\frac{1}{p}}$
14	(Muniz et al., 2012)	K_{KM12}	$\hat{K}_{KM12} = median\left(\frac{1}{\sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}_j^2}}}\right)$
15	Dorugade, (2014)	$K_{4(AD)}$	$\hat{K}_{AD} = \frac{2p}{\lambda_{max}} \sum_{j=1}^{p} \frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}$

Where $\hat{\sigma}^2$ is the variance of residuals obtained from OLS and unbiased estimator of σ^2 and it is computed by $\frac{(y-\hat{y})^t(y-\hat{y})}{(n-p-1)}$, $\hat{\alpha}_{max}^2$ is the maximum value of $\hat{\alpha}^2$and λ_{max} is the maximum eigen value of the X matrix.

Alheety et al., (2021), Ozbay, Nimet (2019), Ali et al., (2019) and Ertas, Hassan (2018) also proposed some new ridge regression estimators under the problem of multicollinearity. Guler (2017) developed robust Lui-Type estimator based on M-estimator for multicollinerity.

EWMA Control Charts based on Ridge Regression Estimator

EWMA Control Chart for Y-Intercept

Kim et al., (2003) proposed the EWMA control charts for Intercept, Slope and MSE which were also use for multiple linear profile for Phase II monitoring. The new proposed EWMA chart statistic for monitoring the Y-intercept (A_o) by using Ridge Regression Estimators is as follow:

$$EWMA_I(j) = \vartheta . A_{oj} + (1 - \vartheta). EWMA_I(j-1) \tag{7}$$

Where "A_o" is the Y-intercept of multiple linear profile computed from ridge regression estimators and j = 1, 2, ..., m represents the number of samples and "ϑ" is a smoothing constant of EWMA statistic and its value lies between 0 to 1 and $EWMA_I(0) = A_o$.
The control limits for the $EWMA_I(j)$= control chart for Y-intercept as follow:

$$LCL = A_o - L_1 \sigma \sqrt{\frac{\vartheta}{(2-\vartheta)n}}$$

$$CL = A_o$$

$$UCL = A_o + L_1 \sigma \sqrt{\frac{\vartheta}{(2-\vartheta)n}}$$

L_1 (> 0) is chosen to give a specified in-control ARL = 200. An alarming situation will be arisen when $EWMA_I(j)$ < LCL or $EWMA_I(j)$ > UCL.

EWMA Control Chart for Slopes

The new proposed $EWMA_{Sk}(j)$ chart statistic for monitoring the slopes (A_{kj}) by using Ridge Regression Estimators is as follow:

$$EWMA_{Sk}(j) = \vartheta . A_{kj} + (1 - \vartheta). EWMA_{Sk}(j-1) \tag{8}$$

Where "A_{kj}" are the slopes of the multiple linear profiles computed from the ridge regression estimators and k= 1,2, ...,p denotes the number of independent variables and j = 1, 2, . . . ,m denotes number of samples and $EWMA_{Sk}(0) = A_k$.

The control limits for the $EWMA_{Sk}(j)$ control chart for Slop "A_k" as follow:

$$LCL = A_k - L_{Ak}\sigma\sqrt{\frac{\vartheta}{(2-\vartheta)S_{x_k x_k}}}$$

$$CL = A_k$$

$$UCL = A_k + L_{Ak}\sigma\sqrt{\frac{\vartheta}{(2-\vartheta)S_{x_k x_k}}}$$

EWMA Control Chart for Mean Square Error (MSE)

The EWMA chart for monitoring the MSE based on RRE can be calculated by using the following $EWMA_E(j)$ statistic.

$$EWMA_E(j) = \max\{\vartheta \ln\left(MSE_{(R)j}\right) + (1-\vartheta).EWMA_E(j-1), \ln(\sigma_0^2)\}$$

Where $MSE_{(R)j}$ is the MSE of multiple linear profiles computed from the ridge regression estimators and $j = 1, 2, \ldots, m$ represents the number of samples with ϑ ($0 < \vartheta < 1$) again a smoothing constant and $EWMA_E(0) = ln(\sigma^2)$. This EWMA chart for monitoring MSE by using RRE is a one-sided chart.

In our proposed method, the assumption that σ^2, the in-control value of σ^2=1 is made without loss of generality, so we have $EWMA_E(0) = 0$. Lawless JF (1976) provided an exact expression for $Var[ln(MSE_{(R)j})]$ using the log-gamma distribution, but for convenience we use the following approximation that is very similar to a result derived by Crowder & Hamilton, (1992):

$$Var\left[ln\left(MSE_{(R)j}\right)\right] \sim \frac{2}{n-2} + \frac{2}{(n-2)^2} + \frac{4}{3(n-2)^3} - \frac{16}{15(n-2)^5}$$

The procedure signals when $EWMA_{E(j)}$ is greater than an upper control limit given by

$$UCL = L_E\sqrt{\frac{\vartheta}{(2-\vartheta)} Var[\ln(MSE_{(R)j})]}$$

and the multiplier L_E(> 0) is again chosen to give a specified in-control ARL.

An Application of Wind tunnel experiments data at NASA Langley Research Centre

Mahmoud (2008) used the application of wind tunnel experiments data at the NASA Langley Research Center for monitoring multiple linear profiles. The data was consist of one response variable "Y" is represented an adjusted axial response and six axial forces. This study only used two axial forces and represented as X_{1ij} and X_{2ij}. These two axial forces X_{1ij} and X_{2ij} are highly collinear with correlation coefficient is "r" 0.99. The sample size n= 64, 73 or 74 with m=11 different samples. The multiple linear profile under the study is given below:

$$Y_{ij} = A_o + A_1 X_{1ij} + A_2 X_{2ij} + e_{ij}$$

$$Y_{ij} = 12 + 21X_{1ij} + 0.1X_{2ij} + e_{ij} \qquad (9)$$

where the e_{ij}'s I.I.D. N(0, σ^2) normal random variables. The all simulations regarding ARL based on error term e_{ij} with μ =0 and $\sigma^2 = 1$.

Kim et al., (2003) set the ARL values for all EWMA charts is at 200. For the EWMA chart for monitoring the Y-intercept, L_I is chosen as 3.0156 to give the in-control ARL of 200. Also the EWMA chart for monitoring the slope, L_S is equal to 3.0109 to have an in-control ARL of 200. Finally, L_E is chosen as 1.3723 to achieve the in-control ARL of 200 for the EWMA chart for monitoring the error variance.

Results and discussions

In this study there were fifteen different ridge regression estimators were used and the table 1.2 shows the values of those fifteen ridge regression estimators with respect to the m=11 different samples of computed for the each multiple linear profiles. The results showed that for each sample the ridge estimators produce different values for each multiple linear profiles.

Table 1.2 Different Ridge Parameters "k" Values, m = 11

	1	2	3	4	5	6	7	8	9	10	11
K_{DS}	0.3402	0.2567	0.2131	0.2143	0.2141	0.2388	0.2075	0.1965	0.2173	0.2262	0.2125
K_{HK}	0.6805	0.5133	0.4263	0.4287	0.4281	0.4776	0.4149	0.3929	0.4346	0.4525	0.4250
K_{GM}	1.1762	0.8652	0.7114	0.7683	0.7084	0.8025	0.7254	0.6871	0.7518	0.7588	0.7477
K_{MED}	2.0331	1.4583	1.1874	1.3770	1.1721	1.3484	1.2681	1.2016	1.3005	1.2725	1.3156
K_{KM2}	1.6335	1.8752	2.0553	2.0661	2.0486	1.9434	2.0945	2.1524	2.0442	1.9961	2.0711
K_{KM3}	1.9213	1.6224	1.4622	1.5874	1.4512	1.5595	1.5193	1.4790	1.5368	1.5146	1.5486
K_{KM4}	0.9221	1.0751	1.1856	1.1409	1.1881	1.1163	1.1741	1.2064	1.1533	1.1480	1.1565
K_{KM5}	1.0845	0.9301	0.8435	0.8765	0.8417	0.8958	0.8517	0.8289	0.8671	0.8711	0.8647
K_{KM6}	1.0770	1.2458	1.3696	1.3481	1.3688	1.2923	1.3764	1.4143	1.3475	1.3282	1.3584
K_{KM8}	1.6336	1.8753	2.0554	2.0662	2.0487	1.9435	2.0946	2.1525	2.0443	1.9961	2.0712
K_{KM9}	1.9201	1.6216	1.4617	1.5867	1.4506	1.5589	1.5187	1.4785	1.5363	1.5141	1.5480
K_{KM10}	0.9224	1.0754	1.1858	1.1411	1.1884	1.1166	1.1744	1.2066	1.1536	1.1482	1.1567
K_{KM11}	1.0842	0.9299	0.8433	0.8763	0.8415	0.8956	0.8515	0.8288	0.8669	0.8709	0.8645
K_{KM12}	1.0772	1.2460	1.3698	1.3482	1.3690	1.2925	1.3765	1.4145	1.3476	1.3283	1.3586
$K_4(AD)$	0.6804	0.5133	0.4262	0.4286	0.4281	0.4775	0.4149	0.3929	0.4345	0.4525	0.4249

Table 1.3, Table 1.4 and Table 1.5 shows the estimated profile's coefficients (A_o, A_1, A_2) computed for multiple linear profiles by using fifteen different ridge estimators with respect to m=11 different samples. The result indicated that due to a small change in ridge estimators the estimated profile's coefficients (A_o, A_1, A_2) approximately same with in the samples with a slight change. Also there is a slight change in multiple linear profile's coefficients by using all the ridge estimators with respect to OLS estimators, when k=0.

Table 1.3 Intercept B_0, m =11											
	1	2	3	4	5	6	7	8	9	10	11
k=0	4.51	11.43	12.69	18.49	17.15	11.78	13.04	20.43	7.58	6.94	14.27
K_{DS}	4.42	11.36	12.62	18.41	17.08	11.70	12.99	20.37	7.50	6.89	14.21
K_{HK}	4.33	11.30	12.54	18.34	17.01	11.62	12.94	20.31	7.42	6.84	14.15
K_{GM}	4.19	11.21	12.44	18.22	16.91	11.50	12.86	20.22	7.30	6.77	14.05
K_{MED}	3.96	11.06	12.28	18.00	16.76	11.32	12.72	20.07	7.10	6.65	13.89
K_{KM2}	4.07	10.96	11.98	17.75	16.47	11.11	12.51	19.78	6.83	6.48	13.68
K_{KM3}	3.99	11.02	12.18	17.92	16.67	11.24	12.66	19.98	7.02	6.59	13.82
K_{KM4}	4.26	11.16	12.28	18.08	16.76	11.40	12.74	20.06	7.16	6.68	13.94
K_{KM5}	4.22	11.20	12.40	18.18	16.87	11.47	12.83	20.18	7.26	6.74	14.02
K_{KM6}	4.22	11.12	12.21	18.01	16.70	11.34	12.69	20.00	7.09	6.63	13.88
K_{KM8}	4.07	10.96	11.98	17.75	16.47	11.11	12.51	19.78	6.83	6.48	13.68
K_{KM9}	3.99	11.02	12.18	17.92	16.67	11.24	12.66	19.98	7.02	6.59	13.82
K_{KM10}	4.26	11.16	12.28	18.08	16.76	11.40	12.74	20.06	7.16	6.68	13.94
K_{KM11}	4.22	11.20	12.40	18.18	16.87	11.47	12.83	20.18	7.26	6.74	14.02
K_{KM12}	4.22	11.12	12.21	18.01	16.70	11.34	12.69	20.00	7.09	6.63	13.88
$K_4(AD)$	4.33	11.30	12.54	18.34	17.01	11.62	12.94	20.31	7.42	6.84	14.15

Table 1.4 Slope B_1, m =11											
	1	2	3	4	5	6	7	8	9	10	11
k=0	20.46	20.89	20.72	21.63	21.18	20.97	21.45	22.06	21.05	20.76	21.70
K_{DS}	20.45	20.89	20.71	21.62	21.17	20.97	21.44	22.05	21.04	20.75	21.70
K_{HK}	20.43	20.88	20.70	21.61	21.16	20.96	21.43	22.05	21.03	20.74	21.69
K_{GM}	20.42	20.87	20.69	21.60	21.15	20.94	21.42	22.04	21.01	20.73	21.68
K_{MED}	20.39	20.85	20.67	21.57	21.13	20.92	21.41	22.02	20.99	20.72	21.66
K_{KM2}	20.40	20.84	20.64	21.54	21.10	20.90	21.38	21.98	20.95	20.70	21.63
K_{KM3}	20.39	20.85	20.66	21.56	21.12	20.91	21.40	22.01	20.98	20.71	21.65
K_{KM4}	20.43	20.86	20.67	21.58	21.13	20.93	21.41	22.02	20.99	20.72	21.66
K_{KM5}	20.42	20.87	20.68	21.59	21.15	20.94	21.42	22.03	21.01	20.73	21.67
K_{KM6}	20.42	20.86	20.66	21.57	21.13	20.92	21.40	22.01	20.99	20.72	21.66
K_{KM8}	20.40	20.84	20.64	21.54	21.10	20.90	21.38	21.98	20.95	20.70	21.63
K_{KM9}	20.39	20.85	20.66	21.56	21.12	20.91	21.40	22.01	20.98	20.71	21.65
K_{KM10}	20.43	20.86	20.67	21.58	21.13	20.93	21.41	22.02	20.99	20.72	21.66
K_{KM11}	20.42	20.87	20.68	21.59	21.15	20.94	21.42	22.03	21.01	20.73	21.67
K_{KM12}	20.42	20.86	20.66	21.57	21.13	20.92	21.40	22.01	20.99	20.72	21.66
$K_4(AD)$	20.43	20.88	20.70	21.61	21.16	20.96	21.43	22.05	21.03	20.74	21.69

Table 1.5 Slope B_2, m =11											
	1	2	3	4	5	6	7	8	9	10	11
k=0	0.177	0.041	0.105	-0.197	-0.057	0.056	-0.101	-0.347	0.000	0.098	-0.217
K_{DS}	0.180	0.044	0.108	-0.194	-0.054	0.059	-0.099	-0.344	0.003	0.100	-0.215
K_{HK}	0.184	0.046	0.111	-0.191	-0.051	0.062	-0.097	-0.342	0.007	0.103	-0.213
K_{GM}	0.189	0.049	0.114	-0.186	-0.048	0.066	-0.094	-0.338	0.011	0.106	-0.209
K_{MED}	0.198	0.055	0.121	-0.178	-0.042	0.073	-0.089	-0.332	0.019	0.111	-0.203
K_{KM2}	0.194	0.059	0.132	-0.169	-0.031	0.081	-0.080	-0.320	0.030	0.118	-0.195
K_{KM3}	0.197	0.057	0.124	-0.175	-0.039	0.076	-0.086	-0.328	0.022	0.113	-0.200
K_{KM4}	0.186	0.051	0.121	-0.181	-0.042	0.070	-0.090	-0.332	0.017	0.109	-0.205
K_{KM5}	0.188	0.050	0.116	-0.185	-0.046	0.068	-0.093	-0.336	0.013	0.107	-0.208
K_{KM6}	0.188	0.053	0.123	-0.178	-0.040	0.073	-0.087	-0.329	0.020	0.111	-0.202
K_{KM8}	0.194	0.059	0.132	-0.169	-0.031	0.081	-0.080	-0.320	0.030	0.118	-0.195
K_{KM9}	0.197	0.057	0.124	-0.175	-0.039	0.076	-0.086	-0.328	0.022	0.113	-0.200
K_{KM10}	0.186	0.051	0.121	-0.181	-0.042	0.070	-0.090	-0.332	0.017	0.109	-0.205
K_{KM11}	0.188	0.050	0.116	-0.185	-0.046	0.068	-0.093	-0.336	0.013	0.107	-0.208
K_{KM12}	0.188	0.053	0.123	-0.178	-0.040	0.073	-0.087	-0.329	0.020	0.111	-0.202
$K_4(AD)$	0.184	0.046	0.111	-0.191	-0.051	0.062	-0.097	-0.342	0.007	0.103	-0.213

Table 1.6 & 1.7 compared the VIF and TOL values for detection of multicollinearity among independent variables from multiple linear profiles and compare the results by using the fifteen different ridge estimators with OLS estimators (k=0) from m=11 different samples. The thumb rule of the VIF values if the value to be exceeds 10 and TOL values near to 0 indicated the high multicollinearity. The result indicated that when use OLS estimators (k=0) the VIF values exceeded to 10 and TOL values near to 0. It implies that there is a high correlation between X_1 and X_2 which may be the cause of multicollinearity problem in the monitoring multiple linear profiles. After using the ridge estimators the results indicated that there is no VIF values exceeds to 10 and not all TOL near to 0, it implies that there is slightly correlation between X_1 and X_2 and multicollinearity is not creating problem during the profile monitoring.

Table 1.6 VIF, m =11											
	1	2	3	4	5	6	7	8	9	10	11
k=0	27.02	24.72	35.34	35.69	30.14	34.3	26.86	31.91	45.16	30.24	33.04
K_{DS}	0.256	0.331	0.343	0.34	0.362	0.315	0.39	0.384	0.311	0.344	0.353
K_{HK}	0.159	0.195	0.207	0.206	0.213	0.194	0.223	0.223	0.197	0.205	0.21
K_{GM}	0.106	0.135	0.15	0.142	0.153	0.139	0.152	0.155	0.142	0.146	0.146
K_{MED}	0.064	0.088	0.104	0.092	0.106	0.093	0.100	0.103	0.095	0.099	0.095
K_{KM2}	0.079	0.07	0.063	0.062	0.063	0.066	0.062	0.06	0.063	0.065	0.062
K_{KM3}	0.068	0.08	0.087	0.081	0.088	0.082	0.085	0.086	0.082	0.085	0.083
K_{KM4}	0.128	0.115	0.104	0.107	0.104	0.109	0.106	0.103	0.105	0.107	0.106
K_{KM5}	0.113	0.128	0.134	0.13	0.136	0.128	0.136	0.136	0.129	0.132	0.132
K_{KM6}	0.114	0.102	0.092	0.093	0.093	0.097	0.093	0.09	0.092	0.095	0.093
K_{KM8}	0.079	0.07	0.063	0.062	0.063	0.066	0.062	0.06	0.063	0.065	0.062
K_{KM9}	0.068	0.08	0.087	0.081	0.088	0.082	0.085	0.086	0.082	0.085	0.083
K_{KM10}	0.128	0.115	0.104	0.107	0.104	0.109	0.106	0.103	0.105	0.107	0.106
K_{KM11}	0.113	0.128	0.134	0.13	0.136	0.129	0.136	0.137	0.129	0.132	0.132
K_{KM12}	0.114	0.102	0.092	0.093	0.093	0.097	0.093	0.09	0.092	0.095	0.093
K_4(AD)	0.159	0.195	0.207	0.206	0.213	0.194	0.223	0.223	0.197	0.205	0.21

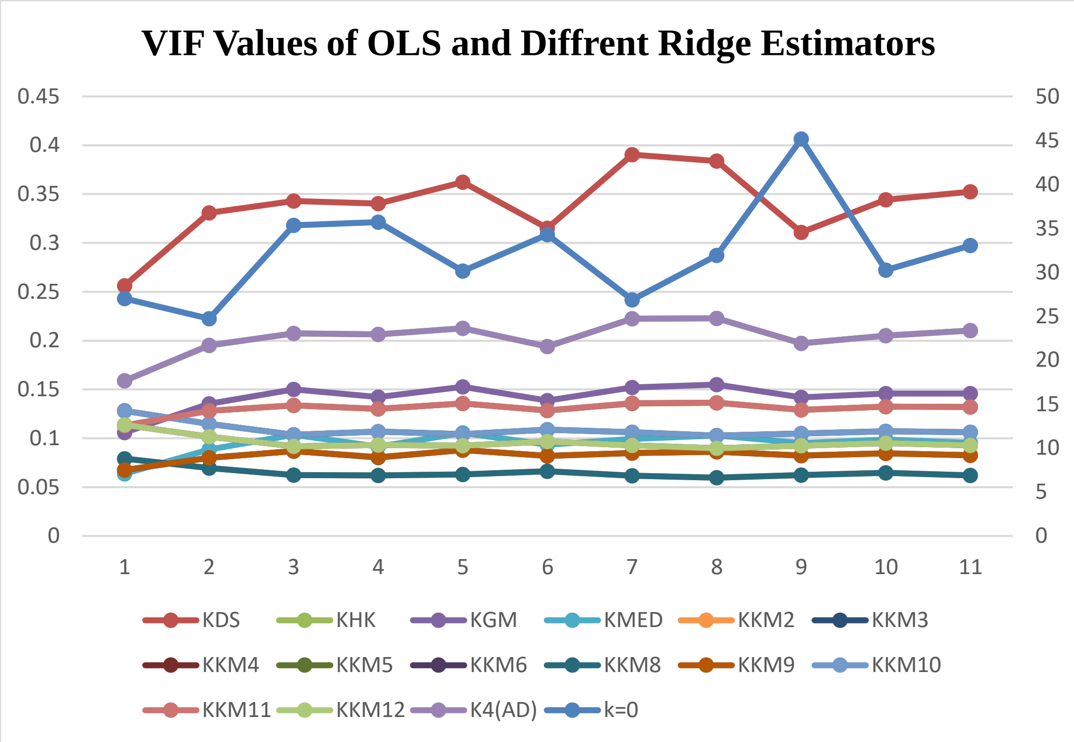


Table 1.7 TOL, m =11											
	1	2	3	4	5	6	7	8	9	10	11
k=0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
K_{DS}	0.279	0.225	0.191	0.192	0.193	0.210	0.189	0.181	0.192	0.201	0.192
K_{HK}	0.451	0.376	0.327	0.329	0.330	0.355	0.324	0.310	0.331	0.343	0.328
K_{GM}	0.610	0.521	0.462	0.485	0.462	0.497	0.470	0.454	0.477	0.481	0.477
K_{MED}	0.759	0.672	0.611	0.654	0.609	0.648	0.632	0.616	0.637	0.632	0.642
K_{KM2}	0.703	0.739	0.760	0.762	0.760	0.747	0.766	0.772	0.758	0.754	0.763
K_{KM3}	0.745	0.701	0.671	0.694	0.670	0.689	0.683	0.675	0.684	0.681	0.687
K_{KM4}	0.539	0.585	0.611	0.600	0.613	0.594	0.610	0.617	0.602	0.602	0.605
K_{KM5}	0.586	0.542	0.511	0.523	0.512	0.529	0.516	0.508	0.518	0.521	0.519
K_{KM6}	0.584	0.627	0.652	0.648	0.653	0.636	0.656	0.663	0.647	0.644	0.651
K_{KM8}	0.703	0.739	0.760	0.762	0.760	0.747	0.766	0.772	0.758	0.754	0.763
K_{KM9}	0.745	0.701	0.671	0.694	0.670	0.689	0.683	0.675	0.684	0.681	0.687
K_{KM10}	0.539	0.585	0.611	0.600	0.613	0.594	0.610	0.617	0.602	0.602	0.605
K_{KM11}	0.586	0.542	0.511	0.523	0.512	0.529	0.516	0.508	0.518	0.521	0.519
K_{KM12}	0.584	0.627	0.652	0.648	0.653	0.636	0.656	0.663	0.647	0.644	0.651
$K_4(AD)$	0.451	0.376	0.327	0.329	0.330	0.355	0.324	0.310	0.331	0.343	0.328

Table 1.8 compared the Condition Index values among fifteen different ridge estimators with OLS estimators (k=0) from m=11 different samples. The thumb rule of the condition index if the value to be exceeds to 30 it indicate multicollinearity is problematic. The result indicated that when we use OLS estimators (k=0) the condition index values exceeded until 30, showed high multicollinearity among independent variables. It implies that there is a strong correlation between X_1 and X_2 which is the cause of multicollinearity. After using the ridge estimators the results indicated that there are no condition index values exceeds to 30, it implies that multicollinearity is not problematic in the monitoring multiple linear profiles.

Table 1.8 Condition Index, m =11

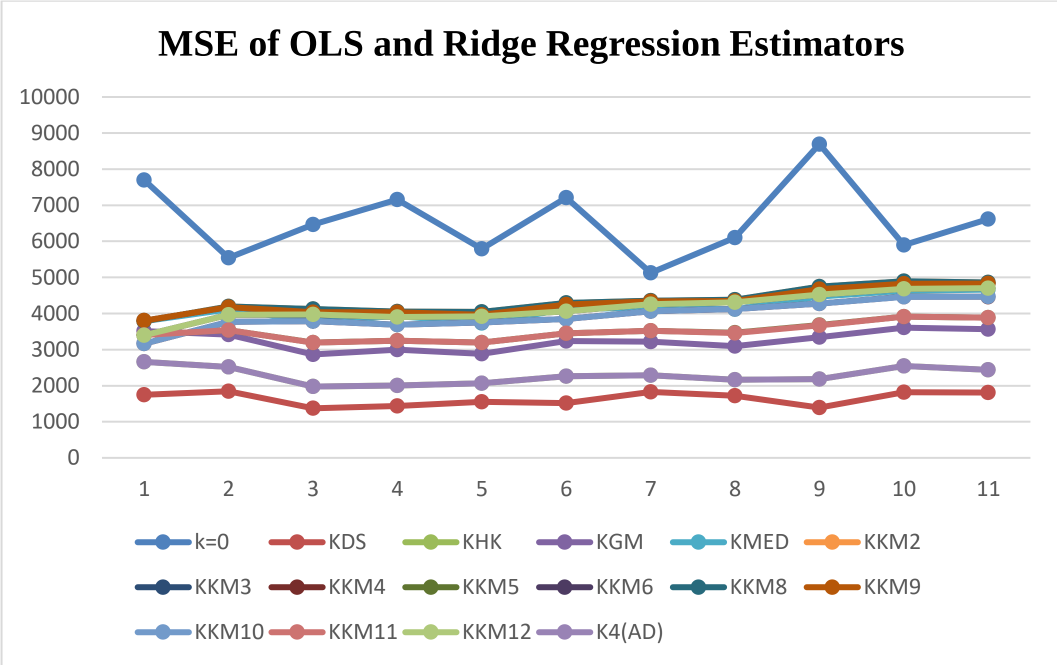
	1	2	3	4	5	6	7	8	9	10	11
k=0	106.1	96.88	139.4	140.8	118.6	135.2	105.4	125.7	178.6	118.9	130.2
K_{DS}	6.47	8.07	9.67	9.63	9.52	8.77	9.67	10.27	9.66	9.1	9.65
K_{HK}	3.81	4.67	5.48	5.45	5.42	5	5.52	5.82	5.44	5.19	5.47
K_{GM}	2.64	3.21	3.72	3.52	3.71	3.41	3.64	3.8	3.59	3.54	3.58
K_{MED}	1.96	2.32	2.64	2.42	2.65	2.45	2.52	2.62	2.51	2.53	2.48
K_{KM2}	2.19	2.03	1.95	1.95	1.95	2.01	1.93	1.91	1.96	1.98	1.94
K_{KM3}	2.01	2.19	2.34	2.23	2.34	2.25	2.28	2.32	2.28	2.28	2.26
K_{KM4}	3.09	2.79	2.64	2.71	2.63	2.74	2.65	2.61	2.7	2.69	2.68
K_{KM5}	2.78	3.06	3.3	3.21	3.29	3.16	3.25	3.33	3.25	3.22	3.24
K_{KM6}	2.79	2.55	2.42	2.45	2.42	2.51	2.41	2.38	2.46	2.46	2.43
K_{KM8}	2.19	2.03	1.95	1.95	1.95	2.01	1.93	1.91	1.96	1.98	1.94
K_{KM9}	2.01	2.19	2.34	2.23	2.34	2.25	2.28	2.32	2.28	2.28	2.26
K_{KM10}	3.09	2.79	2.64	2.71	2.63	2.74	2.64	2.61	2.7	2.69	2.68
K_{KM11}	2.78	3.06	3.3	3.21	3.29	3.16	3.25	3.33	3.25	3.22	3.24
K_{KM12}	2.79	2.55	2.42	2.45	2.42	2.51	2.41	2.38	2.46	2.46	2.43
$K_4(AD)$	3.81	4.67	5.48	5.45	5.42	5	5.52	5.82	5.44	5.19	5.47

Table 1.9 compared the coefficient of determination (R^2) values of fifteen different ridge estimators with OLS estimators (k=0) from m=11 different samples. The result indicated that when used OLS estimators (k=0) the values of coefficient of determination (R^2) implies that approximately 99% variation explained by the independent variables into the dependant variables which is due to a strong correlation between X_1 and X_2 and it is also the indication of multicollinearity problem in monitoring multiple linear profiles. After using the ridge estimators the coefficient of determination (R^2) values showed a different percentage of variation but not indicated approximately 99% variation. The ridge estimators remove the multicollinearity effect and then computed the coefficient of determination.

Table 1.9 Coefficient of determination (R^2), m =11											
	1	2	3	4	5	6	7	8	9	10	11
k=0	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
K_{DS}	0.721	0.775	0.810	0.808	0.807	0.791	0.811	0.820	0.808	0.799	0.808
K_{HK}	0.549	0.624	0.673	0.671	0.670	0.645	0.676	0.690	0.669	0.657	0.672
K_{GM}	0.390	0.479	0.538	0.515	0.538	0.503	0.530	0.546	0.523	0.519	0.523
K_{MED}	0.241	0.328	0.389	0.346	0.391	0.352	0.368	0.384	0.363	0.368	0.358
K_{KM2}	0.298	0.261	0.240	0.238	0.240	0.253	0.234	0.228	0.242	0.246	0.237
K_{KM3}	0.255	0.299	0.329	0.306	0.330	0.311	0.317	0.325	0.316	0.319	0.313
K_{KM4}	0.461	0.415	0.389	0.400	0.387	0.407	0.390	0.383	0.398	0.398	0.396
K_{KM5}	0.414	0.458	0.489	0.477	0.488	0.471	0.484	0.492	0.482	0.479	0.481
K_{KM6}	0.416	0.373	0.348	0.352	0.347	0.364	0.345	0.337	0.353	0.356	0.349
K_{KM8}	0.298	0.261	0.240	0.238	0.240	0.253	0.234	0.228	0.242	0.246	0.237
K_{KM9}	0.255	0.299	0.329	0.306	0.330	0.311	0.317	0.325	0.316	0.319	0.313
K_{KM10}	0.461	0.415	0.389	0.400	0.387	0.406	0.390	0.383	0.398	0.398	0.396
K_{KM11}	0.414	0.458	0.489	0.477	0.488	0.471	0.484	0.493	0.482	0.479	0.481
K_{KM12}	0.416	0.373	0.348	0.352	0.347	0.364	0.344	0.337	0.353	0.356	0.349
$K_4(AD)$	0.549	0.624	0.673	0.671	0.670	0.645	0.676	0.690	0.669	0.657	0.672

Table 1.10 compared the values of Mean Square Error (MSE) of fifteen different ridge estimators with OLS estimators (k=0) from m=11 different samples. The result showed that OLS estimators (k=0) obtained the higher MSE values as compare to the all different ridge parameters in all samples. It is due to high multicollinearity among the independent variables. But when using ridge estimators and computed MSE, it provides minimum MSE values in the presence of multicollinearity. Among all the ridge estimators the K_{DS} obtained the minimum MSE with a maximum coefficient of determination (R^2) values with respect to all samples.

Table 1.10 Mean Square Error (MSE), m =11											
	1	2	3	4	5	6	7	8	9	10	11
k=0	7696	5539	6467	7159	5787	7206	5126	6105	8693	5894	6616
K_{DS}	1745	1845	1373	1440	1554	1520	1831	1723	1393	1816	1810
K_{HK}	2659	2516	1976	2004	2070	2258	2290	2161	2183	2548	2443
K_{GM}	3514	3411	2865	2998	2882	3237	3218	3094	3346	3608	3567
K_{MED}	3786	4118	3790	3925	3735	4113	4165	4115	4473	4628	4662
K_{KM2}	3787	4194	4120	4056	4041	4297	4352	4382	4746	4893	4864
K_{KM3}	3802	4177	4032	4036	3969	4244	4335	4348	4670	4827	4831
K_{KM4}	3163	3768	3787	3689	3753	3850	4064	4120	4274	4465	4465
K_{KM5}	3407	3536	3196	3244	3196	3454	3522	3464	3677	3914	3888
K_{KM6}	3397	3966	3969	3903	3917	4062	4254	4309	4523	4687	4702
K_{KM8}	3787	4194	4120	4056	4041	4297	4352	4382	4746	4893	4864
K_{KM9}	3802	4177	4032	4036	3969	4244	4334	4347	4670	4826	4831
K_{KM10}	3164	3769	3788	3689	3754	3851	4064	4120	4274	4465	4466
K_{KM11}	3407	3536	3196	3244	3195	3454	3522	3463	3676	3914	3888
K_{KM12}	3398	3966	3970	3903	3917	4062	4254	4309	4523	4687	4703
$K_4(AD)$	2659	2516	1976	2004	2070	2258	2290	2161	2183	2548	2443



EWMA Control Charts Statistics for Intercept, Slop and MSE

Table 1.11, Table 1.12 and Table 1.13 compare the EWMA control chart values for profile's coefficients (A_o, A_1, A_2) including Control limits of fifteen different ridge estimators and OLS estimators with respect to m=11 different samples. The EWMA control limits for profile's coefficients (A_o, A_1, A_2) by using the OLS estimator wider than the all ridge estimators. It indicated that the control limits of EWMA chart for monitoring multiple linear profiles under the profile's coefficients (A_o, A_1, A_2) by using ridge estimators have quickly detect shift or out of control signal as compare to the EWMA control chart for monitoring multiple linear profiles under the profile's coefficients (A_o, A_1, A_2) by using OLS estimator. Among all ridge estimators the K_{DS} ridge estimator provides more narrow control limits of EWMA control chart for monitoring multiple linear profile's coefficients (A_o, A_1, A_2).

Table 1.11 EWMA Control Chart Statistics for Intercept (A_o)

	1	2	3	4	5	6	7	8	9	10	11	LCL	CL	UCL
k=0	10.50	10.69	11.09	12.57	13.49	13.14	13.12	14.58	13.18	11.94	12.40	4.81	12	20.19
K_{DS}	10.48	10.66	11.05	12.52	13.44	13.09	13.07	14.53	13.12	11.88	12.34	9.91	12	15.09
K_{HK}	10.50	10.69	11.09	12.57	13.49	13.14	13.12	14.58	13.18	11.94	12.40	8.80	12	17.17
K_{GM}	10.47	10.63	11.01	12.48	13.38	13.03	13.01	14.47	13.06	11.82	12.28	9.00	12	17.00
K_{MED}	10.44	10.59	10.96	12.41	13.31	12.95	12.93	14.39	12.97	11.73	12.20	8.84	12	17.16
K_{KM2}	10.39	10.53	10.88	12.30	13.19	12.82	12.80	14.25	12.82	11.59	12.05	8.92	12	17.08
K_{KM3}	10.41	10.52	10.81	12.20	13.06	12.67	12.64	14.06	12.62	11.39	11.85	7.72	12	16.28
K_{KM4}	10.40	10.52	10.85	12.27	13.15	12.77	12.75	14.19	12.76	11.52	11.98	8.76	12	16.24
K_{KM5}	10.45	10.59	10.93	12.36	13.24	12.87	12.85	14.29	12.86	11.63	12.09	8.11	12	16.89
K_{KM6}	10.44	10.59	10.95	12.40	13.29	12.93	12.91	14.36	12.94	11.70	12.17	8.57	12	16.43
K_{KM8}	10.44	10.58	10.91	12.33	13.20	12.83	12.80	14.24	12.81	11.57	12.04	8.90	12	17.10
K_{KM9}	10.41	10.52	10.81	12.20	13.06	12.67	12.64	14.06	12.62	11.39	11.85	9.72	12	17.28
K_{KM10}	10.40	10.52	10.85	12.27	13.15	12.77	12.75	14.19	12.76	11.52	11.98	8.76	12	17.24
K_{KM11}	10.45	10.59	10.93	12.36	13.24	12.87	12.85	14.29	12.86	11.63	12.09	8.11	12	17.89
K_{KM12}	10.44	10.59	10.95	12.40	13.29	12.93	12.91	14.36	12.94	11.70	12.17	8.57	12	18.43
K_4(AD)	10.44	10.58	10.91	12.33	13.20	12.83	12.80	14.24	12.81	11.57	12.04	7.90	12	17.10

Table 1.12 EWMA Control Chart Statistics for Slop (A_1)													
	1	2	3	4	5	6	7	8	9	10	11	LCL	CL	UCL
k=0	20.89	20.89	20.86	21.01	21.04	21.03	21.11	21.30	21.25	21.15	21.26	16.26	21	25.74
K_{DS}	20.89	20.89	20.85	21.01	21.04	21.02	21.11	21.30	21.25	21.15	21.26	19.63	21	22.37
K_{HK}	20.89	20.89	20.86	21.01	21.04	21.03	21.11	21.30	21.25	21.15	21.26	18.27	21	23.73
K_{GM}	20.89	20.89	20.85	21.00	21.03	21.02	21.10	21.29	21.24	21.14	21.25	18.21	21	23.79
K_{MED}	20.88	20.88	20.84	20.99	21.02	21.01	21.09	21.28	21.23	21.13	21.24	18.67	21	24.33
K_{KM2}	20.88	20.87	20.83	20.98	21.01	20.99	21.08	21.26	21.21	21.11	21.22	17.24	21	24.76
K_{KM3}	20.88	20.87	20.82	20.97	20.99	20.98	21.06	21.24	21.18	21.09	21.20	17.15	21	24.85
K_{KM4}	20.88	20.87	20.83	20.98	21.01	20.99	21.07	21.26	21.20	21.10	21.21	17.17	21	23.83
K_{KM5}	20.89	20.88	20.84	20.99	21.02	21.00	21.08	21.27	21.21	21.12	21.23	17.33	21	24.67
K_{KM6}	20.88	20.88	20.84	20.99	21.02	21.01	21.09	21.28	21.22	21.12	21.23	17.54	21	24.46
K_{KM8}	20.88	20.88	20.84	20.98	21.01	20.99	21.08	21.26	21.21	21.11	21.22	17.23	21	24.77
K_{KM9}	20.88	20.87	20.82	20.97	20.99	20.98	21.06	21.24	21.18	21.09	21.20	17.15	21	24.85
K_{KM10}	20.88	20.87	20.83	20.98	21.01	20.99	21.07	21.26	21.20	21.10	21.21	17.17	21	24.83
K_{KM11}	20.89	20.88	20.84	20.99	21.02	21.00	21.08	21.27	21.21	21.12	21.23	17.33	21	24.67
K_{KM12}	20.88	20.88	20.84	20.99	21.02	21.01	21.09	21.28	21.22	21.12	21.23	17.54	21	24.46
K_4(AD)	20.88	20.88	20.84	20.98	21.01	20.99	21.08	21.26	21.21	21.11	21.22	17.23	21	24.77

Table 1.13 EWMA Control Chart Statistics for Slop (A_2)													
	1	2	3	4	5	6	7	8	9	10	11	LCL	CL	UCL
k=0	0.12	0.10	0.10	0.04	0.02	0.03	0.00	-0.07	-0.05	-0.02	-0.06	-0.45	0.1	0.65
K_{DS}	0.12	0.10	0.10	0.04	0.02	0.03	0.01	-0.06	-0.05	-0.02	-0.06	-0.11	0.1	0.16
K_{HK}	0.12	0.10	0.10	0.04	0.02	0.03	0.00	-0.07	-0.05	-0.02	-0.06	-0.25	0.1	0.25
K_{GM}	0.12	0.10	0.10	0.05	0.03	0.03	0.01	-0.06	-0.05	-0.02	-0.06	-0.15	0.1	0.22
K_{MED}	0.12	0.10	0.11	0.05	0.03	0.04	0.01	-0.06	-0.05	-0.02	-0.05	-0.19	0.1	0.19
K_{KM2}	0.12	0.11	0.11	0.05	0.03	0.04	0.02	-0.05	-0.04	-0.01	-0.05	-0.13	0.1	0.23
K_{KM3}	0.12	0.11	0.11	0.06	0.04	0.05	0.02	-0.05	-0.03	0.00	-0.04	-0.16	0.1	0.26
K_{KM4}	0.12	0.11	0.11	0.05	0.03	0.04	0.02	-0.05	-0.04	-0.01	-0.05	-0.16	0.1	0.26
K_{KM5}	0.12	0.10	0.11	0.05	0.03	0.04	0.01	-0.06	-0.04	-0.01	-0.05	-0.10	0.1	0.30
K_{KM6}	0.12	0.10	0.11	0.05	0.03	0.04	0.01	-0.06	-0.04	-0.01	-0.05	-0.03	0.1	0.23
K_{KM8}	0.12	0.10	0.11	0.05	0.03	0.04	0.02	-0.05	-0.04	-0.01	-0.05	-0.14	0.1	0.24
K_{KM9}	0.12	0.11	0.11	0.06	0.04	0.05	0.02	-0.05	-0.03	0.00	-0.04	-0.16	0.1	0.26
K_{KM10}	0.12	0.11	0.11	0.05	0.03	0.04	0.02	-0.05	-0.04	-0.01	-0.05	-0.16	0.1	0.36
K_{KM11}	0.12	0.10	0.11	0.05	0.03	0.04	0.01	-0.06	-0.04	-0.01	-0.05	-0.10	0.1	0.20
K_{KM12}	0.12	0.10	0.11	0.05	0.03	0.04	0.01	-0.06	-0.04	-0.01	-0.05	-0.13	0.1	0.23
K_4(AD)	0.12	0.10	0.11	0.05	0.03	0.04	0.02	-0.05	-0.04	-0.01	-0.05	-0.14	0.1	0.24

Table 1.14 showed the $EWMA_{(e)j}$ values under the EWMA control chart for MSE, including one sided upper control limit. The table also compared the $EWMA_{(e)j}$ values based on fifteen different ridge estimators and $EWMA_{(e)j}$ values based on OLS estimators with m=11 different samples. The upper control limit of EWMA control chart for MSE by using the OLS estimator showed that the $EWMA_{(e)j}$ values are in-control although the all other fifteen upper control limit of EWMA control chart for MSE by using ridge estimators indicated that out of control signal. So we concluded that when monitoring multiple linear profiles based on EWMA control charts for MSE by using ridge estimators, they have more ability to detect shift as compare to the monitoring multiple linear profiles based on EWMA control charts for MSE by using OLS estimator. Among all ridge estimators the K_{DS} provides very narrow upper control limit for EWMA control charts under MSE.

Table 1.14 EWMA Control Chart for MSE												
	1	2	3	4	5	6	7	8	9	10	11	UCL
k=0	0.777	1.370	1.859	2.258	2.559	2.819	2.997	3.155	3.311	3.403	3.487	7.86
K_{DS}	0.648	1.172	1.565	1.884	2.145	2.353	2.535	2.675	2.769	2.867	2.945	1.41
K_{HK}	0.777	1.370	1.858	2.257	2.558	2.817	2.996	3.153	3.310	3.402	3.486	3.81
K_{GM}	0.685	1.228	1.642	1.974	2.242	2.464	2.644	2.782	2.893	2.996	3.074	1.73
K_{MED}	0.709	1.274	1.711	2.064	2.343	2.576	2.763	2.908	3.031	3.137	3.220	1.99
K_{KM2}	0.716	1.295	1.752	2.120	2.411	2.651	2.845	2.999	3.129	3.236	3.323	2.47
K_{KM3}	0.716	1.297	1.761	2.130	2.425	2.667	2.861	3.017	3.149	3.257	3.343	2.70
K_{KM4}	0.716	1.297	1.759	2.128	2.422	2.663	2.858	3.014	3.145	3.253	3.339	2.62
K_{KM5}	0.700	1.275	1.736	2.102	2.397	2.634	2.829	2.986	3.115	3.222	3.308	2.88
K_{KM6}	0.706	1.275	1.721	2.079	2.364	2.599	2.788	2.939	3.064	3.170	3.254	1.86
K_{KM8}	0.706	1.285	1.747	2.116	2.412	2.651	2.847	3.004	3.134	3.242	3.328	2.93
K_{KM9}	0.716	1.297	1.761	2.130	2.425	2.667	2.861	3.017	3.149	3.257	3.343	2.70
K_{KM10}	0.716	1.297	1.759	2.128	2.422	2.663	2.858	3.014	3.145	3.253	3.339	2.62
K_{KM11}	0.700	1.275	1.736	2.102	2.397	2.634	2.829	2.986	3.115	3.222	3.308	2.88
K_{KM12}	0.706	1.275	1.721	2.079	2.364	2.599	2.788	2.939	3.064	3.170	3.254	1.86
K_4(AD)	0.706	1.285	1.747	2.116	2.412	2.651	2.847	3.004	3.134	3.242	3.328	2.93

ARL Comparison

In this section, we compare the ARL performance of newly proposed EWMA-based control charts for profile's coefficients (A_0, A_1, A_2) and MSE in Phase II by using the OLS estimator and all fifteen ridge estimators. All chart combinations studied are designed to have the same overall in-control ARL of 200. The smoothing constant θ = 0.2 is fixed for all EWMA control charts for Intercept, Slopes and MSE.

Table 1.15: ARL Comparisons Under Intercept Shifts From A_0 To $A_0 + \lambda\sigma$ (In-control ARL = 200)										
	Shift = λ									
Ridge Parameter	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
k=0	168	141	95	68	43	29	18	11	7	5
K_{DS}	136	106	68	43	21	15	8	3	1	1
K_{HK}	145	108	71	48	25	22	9	6	3	2
K_{GM}	149	125	79	51	28	26	10	6	4	1
K_{MED}	154	131	83	54	29	26	12	8	6	2
K_{KM2}	162	138	91	62	37	31	15	11	8	3
K_{KM3}	157	131	85	58	36	30	17	9	7	3
K_{KM4}	152	129	81	51	28	25	11	7	5	1
K_{KM5}	151	127	81	50	27	25	11	7	5	1
K_{KM6}	155	132	84	55	30	27	13	8	6	2
K_{KM8}	162	137	90	62	36	31	16	11	9	4
K_{KM9}	156	130	86	57	33	28	15	7	5	3
K_{KM10}	152	129	82	51	28	24	11	7	5	1
K_{KM11}	151	127	80	51	28	26	10	6	4	1
K_{KM12}	155	131	83	55	31	27	14	8	5	2
K_4(AD)	143	108	71	48	25	20	9	6	3	2

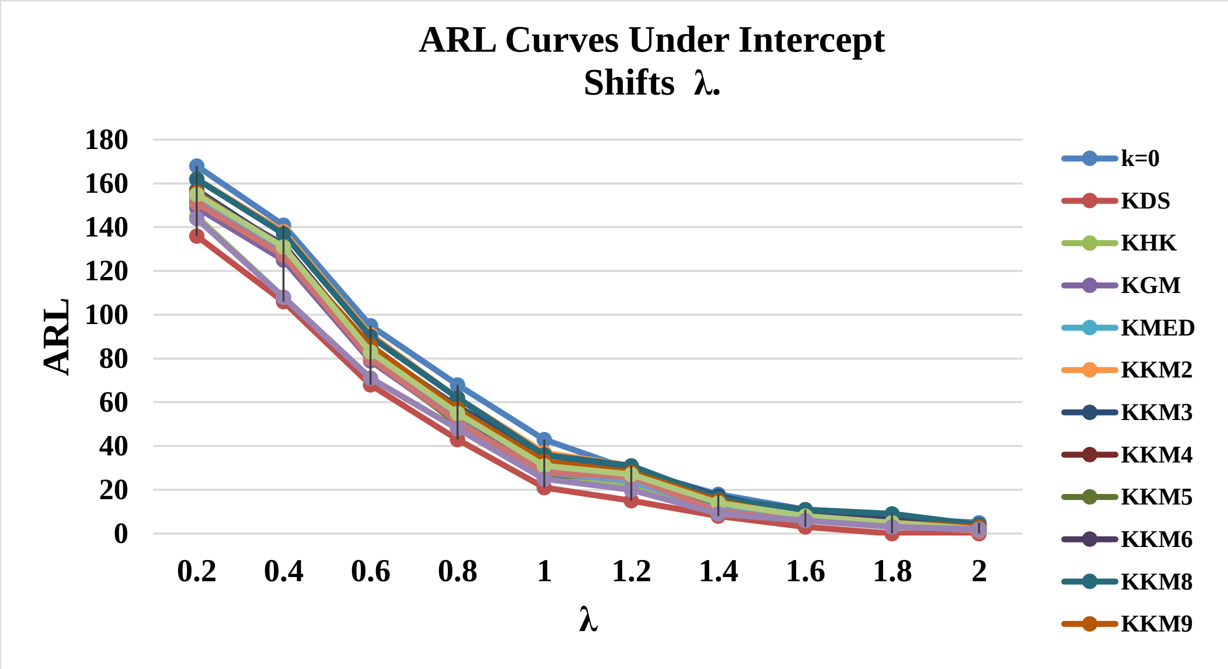


Table 1.16: ARL Comparisons Under Slope Shifts in Model From A_1 To A_1 + Aσ (In-control ARL = 200)										
Ridge Parameter	Shift = σ									
	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
k=0	127	51	37	25	18	14	11	9	5	3
K_{DS}	54	23	9	4	2	1	1	1	1	1
K_{HK}	63	28	15	8	5	3	2	1	1	1
K_{GM}	81	30	17	11	7	5	3	1	1	1
K_{MED}	98	41	28	17	13	6	4	3	2	1
K_{KM2}	112	47	32	21	14	9	7	6	3	1
K_{KM3}	109	46	31	19	13	8	6	5	3	1
K_{KM4}	93	39	25	15	11	5	4	3	1	1
K_{KM5}	89	31	19	11	8	6	3	1	1	1
K_{KM6}	101	43	30	18	13	7	5	4	2	1
K_{KM8}	111	48	32	21	15	9	7	6	3	1
K_{KM9}	108	45	30	19	14	8	7	6	3	1
K_{KM10}	93	39	25	15	11	5	4	3	1	1
K_{KM11}	88	32	19	12	8	6	3	1	1	1
K_{KM12}	102	43	29	17	12	7	5	3	1	1
K_4(AD)	61	27	14	7	5	3	2	1	1	1

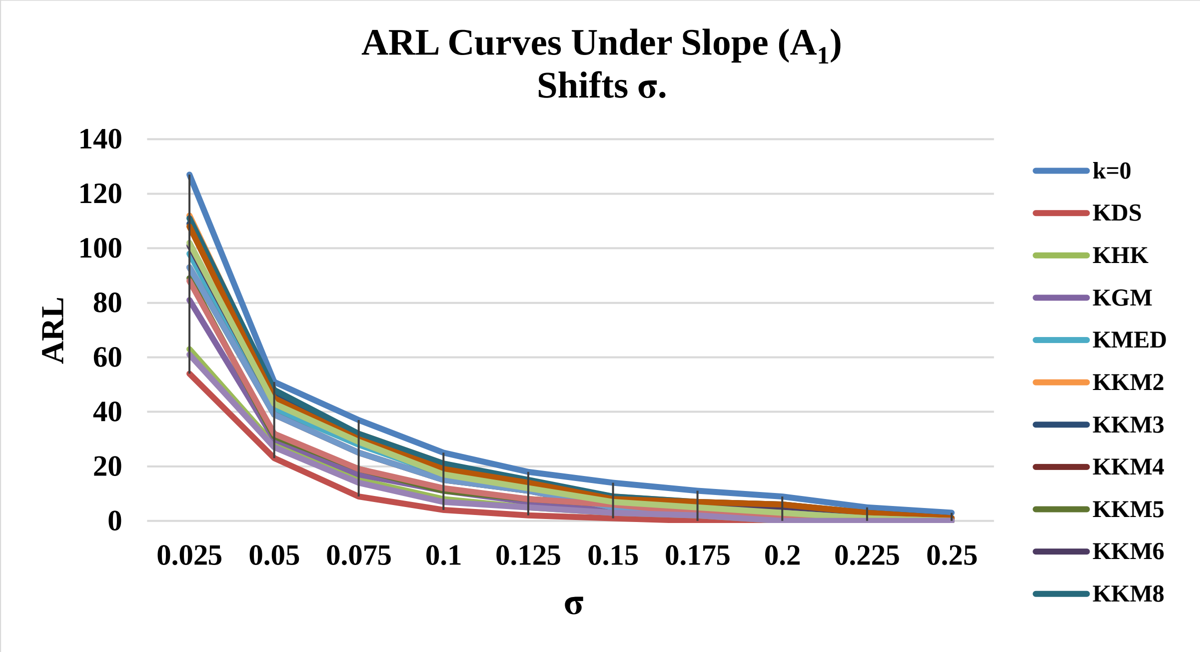
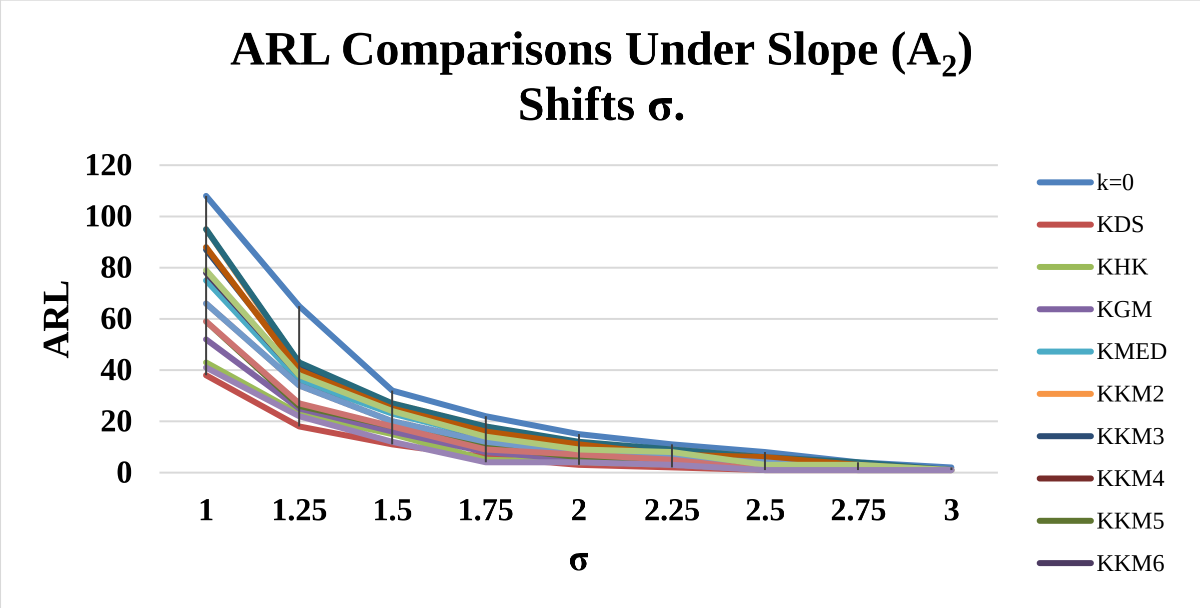


Table 1.17: ARL Comparisons Under Slope Shifts in Model From A_2 To A_2 + Aσ (In-control ARL = 200)

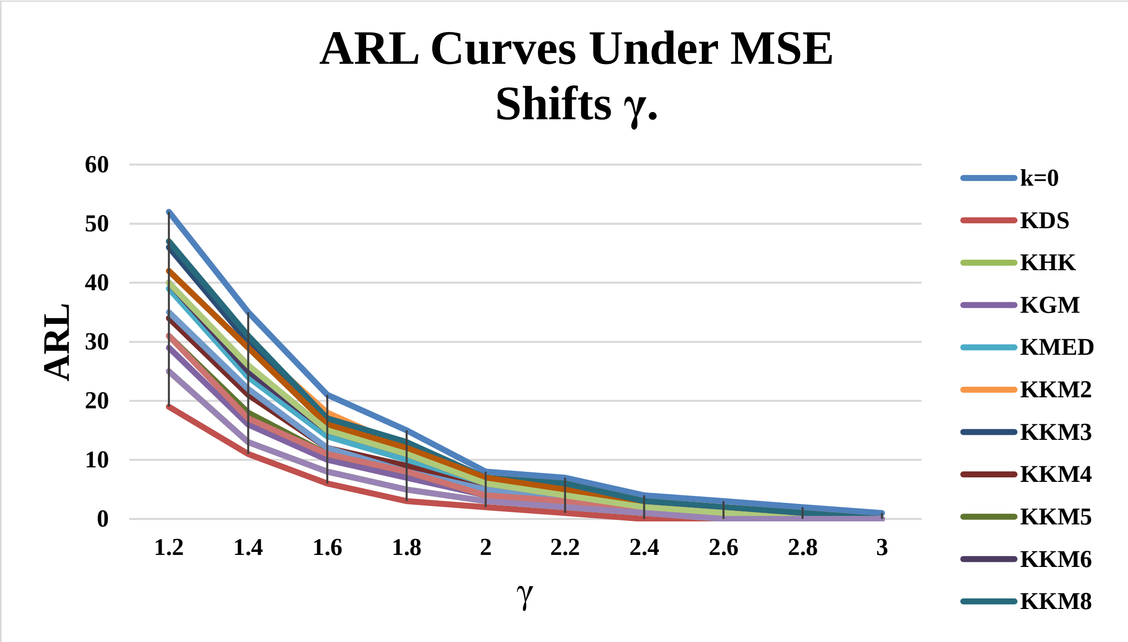
Ridge Parameter	Shift = σ								
	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
k=0	108	65	32	22	15	11	8	4	2
K_{DS}	38	18	11	6	3	2	1	1	1
K_{HK}	43	23	15	5	4	3	1	1	1
K_{GM}	52	25	16	8	5	3	1	1	1
K_{MED}	75	36	23	14	10	7	3	2	1
K_{KM2}	95	42	27	18	11	9	6	3	1
K_{KM3}	87	41	26	16	10	8	5	3	1
K_{KM4}	66	34	20	12	8	6	3	1	1
K_{KM5}	59	26	18	10	6	4	2	1	1
K_{KM6}	78	38	25	15	10	6	4	2	1
K_{KM8}	95	43	27	18	12	9	6	4	1
K_{KM9}	88	40	25	16	11	7	6	3	1
K_{KM10}	66	34	20	12	8	6	4	1	1
K_{KM11}	59	27	18	9	7	5	1	1	1
K_{KM12}	79	38	24	14	9	8	3	3	1
K_4(AD)	41	22	12	4	4	3	1	1	1



The table 1.15, table 1.16, table 1.17 and table 1.18 compare the ARL values for the performance evaluation of EWMA control chart for profile's coefficients (A_o, A_1, A_2) and MSE in Phase II based on the OLS estimator and all fifteen ridge regression estimators with respect to m=11 different samples. The ARL values under OLS estimators for profile's coefficients (A_o, A_1, A_2) and MSE were larger than the all fifteen ridge estimators. It indicated that OLS estimates have a low performance in the presence of multicollinearity. The all other ARL values indicated that the profile monitoring in the presence of multicollinearity by using fifteen ridge regression estimators provides a quick indication of small shift or out of control signals. Among all the ridge estimators K_{DS} provides the smallest ARL, which is also the indication that the EWMA control charts under K_{DS} out performs among all other EWMA control charts based on other ridge estimators.

Table 1.18: ARL Comparisons Under MSE Shifts From σ To γσ (In-control ARL = 200)

Ridge Parameter	Shift = γ								
	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8
k=0	52	35	21	15	8	7	4	3	2
K_{DS}	19	11	6	3	2	1	1	1	1
K_{HK}	25	13	8	5	3	2	1	1	1
K_{GM}	29	16	10	7	4	3	2	1	1
K_{MED}	39	24	14	10	6	4	2	1	1
K_{KM2}	47	30	18	12	7	6	3	2	1
K_{KM3}	46	30	17	13	6	5	2	1	1
K_{KM4}	34	21	12	9	5	4	2	1	1
K_{KM5}	31	18	11	8	4	3	2	1	1
K_{KM6}	40	25	15	11	6	4	2	1	1
K_{KM8}	47	31	17	13	7	6	3	2	1
K_{KM9}	42	29	16	12	7	5	2	1	1
K_{KM10}	35	22	12	8	5	3	2	1	1
K_{KM11}	31	17	11	8	4	3	2	1	1
K_{KM12}	40	26	15	11	6	4	2	1	1
K_4(AD)	25	13	8	5	3	2	1	1	1



Conclusion

In the presence of multicollinearity among explanatory variables the OLS estimators provided large MSE. The EWMA control charts for Intercept, Slopes and MSE by using OLS estimates provide low efficiency of detecting shift or out of control signal when multicollinearity among independent variables are high. The VIF, TOL, CI and R^2 have been used for detection of multicollinearity. Ridge regression estimators provided MMSE in the presence of multicollinearity. This study provided new proposed EWMA control charts for Intercept, Slopes and MSE based on ridge regression estimators. An application of Wind Tunnel data taken from NASA Legacy Research Centre has been used for built up multiple linear profiles. The data was consist of one dependent variable which is represented an adjusted axial response and two axial forces. These two axial forces are highly collinear with correlation coefficient "r" 0.99. This data based on m=11 different samples with sample size 64, 73 or 74. The results indicated that EWMA control charts outperform under ridge regression estimators on the basis of ARL values. Among all ridge regression estimators, the K_{DS} estimator provided minimum values of ARL which showed the good performance among all ridge regression estimators.

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