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SUPER EDGE BIMAGIC LABELING FOR SOME CLASSES OF CONNECTED GRAPHS DERIVED FROM FUNDAMENTAL GRAPHS

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Abstract: An edge bimagic total labeling on a graph G is a one-to-one map λ from $V(G) \cup E(G)$ onto the integers 1, 2, ..., $|V(G) \cup E(G)|$ with the property that, given any edge (u, v), $\lambda(u) + \lambda(v) + \lambda(uv) = k_1$ or k_2 for some constants k_1 and k_2. Edge bimagic totally labeling was introduced in [1] by J. Baskar Babujee as (1, 1) edge bimagic. In this paper we study super edge-bimagic labeling and exhibit the same for certain classes of connected graphs derived from fundamental graphs.

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1. INTRODUCTION

All graphs in this paper are finite, undirected and simple. A (p, q) graph $G = (V, E)$ with p vertices and q edges is called **total edge magic** if there is a bijection function $f : V \cup E \to \{1, 2, 3, ..., p + q\}$ such that for any edge uv in E we have a constant k with $f(u) + f(v) + f(uv) = k$. A total edgemagic graph is called **super edge-magic** if $f(V(G)) = \{1, 2, ..., p\}$. Wallis [5] called super edge-magic as strongly edge-magic.

J. Baskar Babujee [1] introduced Edge bimagic totally labeling as (1, 1) edge bimagic. In [2] it was proved that the trees $B_{n,n}$ and $\langle < K_{1,n} : 2 > \rangle$ are (1, 1) edge bimagic. A graph $G(p, q)$ with p vertices and q edges is called **total edge bimagic** if there exists a bijection $f : V \cup E \to \{1, 2, 3, ..., p + q\}$ such that for any edge $uv \in E$, we have two constants k_1 and k_2 with $f(u) + f(v) + f(uv) = k_1$ or k_2. A total edge-bimagic graph is called **super edge-bimagic** if $f(V(G)) = \{1, 2, ..., p\}$.

Example 1.1: Super edge bimagic labeling of P_6 is given in Fig. 1.

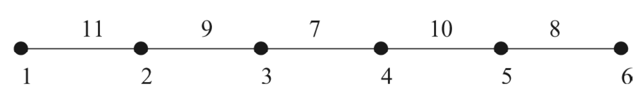

Figure 1: $k_1 = 14, k_2 = 19$

Example 1.2: $P_m \odot K_{1,n}$ is a graph obtained by introducing n new pendent edges at each vertex of path P_m. Super edge bimagic labeling of $P_3 \odot K_{1,4}$ is given in Fig. 2.

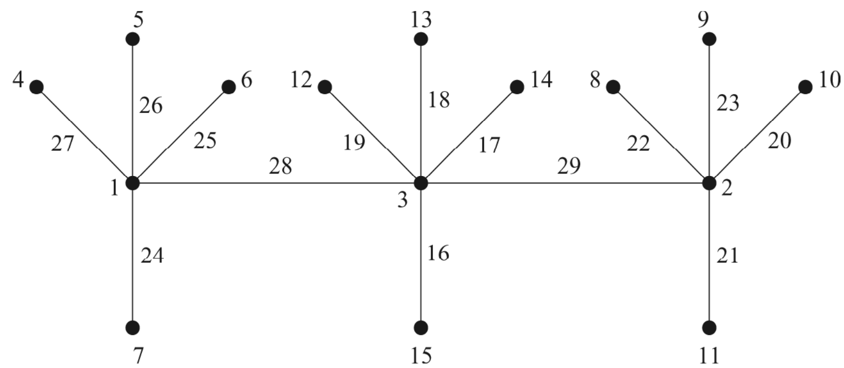

Figure 2: $k_1 = 32, k_2 = 34$

Example 1.3: $P_n + N_2$ is a graph with vertex set $V = \{x_1, x_2, ..., x_n\} \cup \{y_1, y_2\}$ and edge set $E = \{x_i x_{i+1} : 1 \leq i \leq n-1\} \cup \{y_1 x_j, y_2 x_j : 1 \leq j \leq n\}$. Super edge bimagic labeling of $P_5 + N_2$ is given in Fig. 3.

Example 1.4: $P_2 + mK_1$ is a graph with vertex set $V = \{y_1, y_2, x_1, x_2, ..., x_m\}$ and edge set $E = \{y_1 y_2\} \cup \{y_1 x_i, y_2 x_i : 1 \leq i \leq m\}$. Super edge bimagic labeling of $P_2 + 4K_1$ is given in Fig. 4.

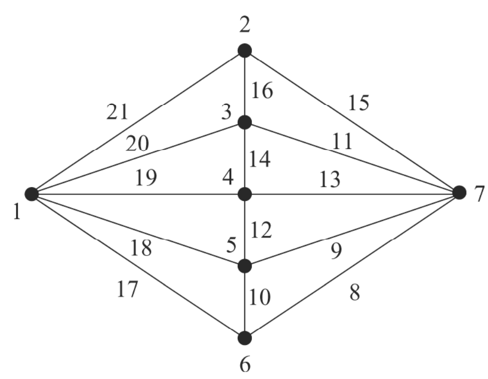

Figure 3: $k_1 = 24, k_2 = 21$

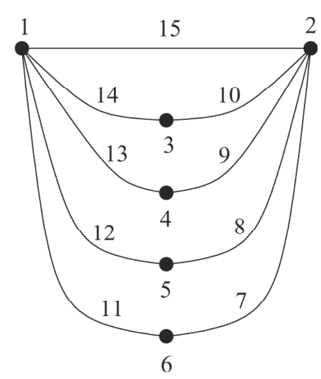

Figure 4: $k_1 = 18, k_2 = 15$

In this paper we prove that $(3, n)$-kite graph $(n \geq 2)$, $P_3 \odot K_{1,n}$ (n is even), $P_n + N_2$ (n is odd and $n \geq 3$), $(P_2 \cup mK_1 + N_2)$ $(m \geq 1)$ and $(P_2 + mK_1)$ $(m \geq 2)$ have super edge bimagic labeling.

2. MAIN RESULTS

Theorem 2.1: The (3, n)-kite graph ($n \geq 2$) has super edge bimagic labeling.

Proof: An (3, n)-kite graph ($n \geq 2$) consisting of cycle of length 3 with n-edge path (the tail) attached to one vertex. Now we design an algorithm for the construction and super edge bimagic labeling for the above graph.

Algorithm 2.1:

Input: The integer n of the (3, n)-kite graph ($n \geq 2$).

Output: Construction and super edge bimagic labeling.

begin

Step 1: The vertex set $V = \{v_1, v_2, v_3, ..., v_{n+2}\}$ and

The edge set $E = \{E_1 \cup E_2\}$

$E_1 = \{v_i v_{i+1}, 3 \leq i \leq n + 1\}$, $E_2 = \{v_1 v_2, v_1 v_3, v_2 v_3\}$

Step 2: $f(v_i) = i;\ 1 \leq i \leq n + 2$

Step 3: If $n \equiv 0 \pmod 2$ go to Step 4 else go to Step 5.

Step 4: $f(v_1 v_2) = 2(n + 2)$

$f(v_1 v_3) = 2n + 3$

$f(v_2 v_3) = 2\,(n + 1)$

$$f(v_j v_{j+1}) = \begin{cases} 2(n - j + 3); & 3 \leq j \leq (n+2)/2 \\ 3n - 2j + 5; & (n+4)/2 \leq j \leq n + 1 \end{cases}$$

Step 5: $f(v_1 v_2) = 2\,(n + 2)$

$f(v_1 v_3) = 2n + 3$

$f(v_2 v_3) = 2\,(n + 1)$

$$f(v_k v_{k+1}) = \begin{cases} 2(n - k + 3); & 3 \leq k \leq (n+3)/2 \\ 3n - 2(k - 3); & (n+5)/2 \leq k \leq n + 1 \end{cases}$$

end.

Next we need to prove that the construction in above algorithm is super edge bimagic. We prove the result in two cases.

Case (i): n is even

Let $f : V \cup E \to \{1, 2, 3, ..., 2n + 4\}$ be the bijective function defined as in the Step 4 of above algorithm.

For $3 \le j \le (n + 2)/2$, $f(v_j) + f(v_{j+1}) + f(v_j v_{j+1}) = j + j + 1 + 2n - 2j + 6 = 2n + 7 = k_1$ (say)

For $(n + 4)/2 \le j \le n + 1$, $f(v_j) + f(v_{j+1}) + f(v_j v_{j+1}) = j + j + 1 + 3n - 2j + 5 = 3n + 6 = k_2$ (say)

Also $f(v_1) + f(v_2) + f(v_1 v_2) = 1 + 2 + 2n + 4 = 2n + 7 = k_1$ (say)

Also $f(v_1) + f(v_3) + f(v_1 v_3) = 1 + 3 + 2n + 3 = 2n + 7 = k_1$ (say)

Also $f(v_2) + f(v_3) + f(v_2 v_3) = 2 + 3 + 2n + 2 = 2n + 7 = k_1$ (say)

Which concludes that there exist two constants k_1 and k_2 such that for any edge uv in the $(3, n)$-kite graph (n is even), $f(u) + f(v) + f(uv) = k_1$ or k_2.

Case (ii): if n is odd

Let $f : V \cup E \to \{1, 2, 3, ..., 2n + 4\}$ be the bijective function defined as in the Step 5 of above algorithm.

For $3 \le k \le (n + 3)/2$, $f(v_k) + f(v_{k+1}) + f(v_k v_{k+1}) = k + k + 1 + 2n - 2k + 6 = 2n + 7 = k_1$ (say)

For $(n + 5)/2 \le k \le n + 1$, $f(v_k) + f(v_{k+1}) + f(v_k v_{k+1}) = k + k + 1 + 3n - 2k + 6 = 3n + 7 = k_2$ (say)

Also $f(v_1) + f(v_2) + f(v_1 v_2) = 1 + 2 + 2n + 4 = 2n + 7 = k_1$ (say)

Also $f(v_1) + f(v_3) + f(v_1 v_3) = 1 + 3 + 2n + 3 = 2n + 7 = k_1$ (say)

Also $f(v_2) + f(v_3) + f(v_2 v_3) = 2 + 3 + 2n + 2 = 2n + 7 = k_1$ (say)

Which concludes that there exist two constants k_1 and k_2 such that for any edge uv in the $(3, n)$-kite graph (n is odd) $f(u) + f(v) + f(uv) = k_1$ or k_2. Hence the result.

Theorem 2.2: The $P_3 \odot K_{1,n}$ (n is even) has super edge bimagic labeling.

Proof: $P_3 \odot K_1$, n is a graph obtained by introducing n new pendent edges at each vertex of path P_3. Now we design an algorithm for the construction and super edge bimagic labeling for the graph $P_3 \odot K_{1,n}$ (n is even).

Algorithm 2.2:

Input: The integer n of the graph $P_3 \odot K_{1,n}$ (n is even).

Output: construction and super edge bimagic labeling.

begin

Step 1: The vertex set $V = \{v_1, v_2, v_3, ..., v_{3n+3}\}$ and

The edge set $E = \{E_1 \cup E_2 \cup E_3 \cup E_4\}$ where $E_1 = \{v_1v_i, 4 \le i \le n+3\}$, $E_2 = \{v_2v_j, n+4 \le j \le 2n+3\}$, $E_3 = \{v_3v_k, 2n+4 \le k \le 3n+3\}$ and $E_4 = \{v_3v_1, v_3v_2\}$.

Step 2: $f(v_i) = i, 1 \le i \le 3n+3$

Step 3: $f(v_1v_i) = 6n + 7 - i, 4 \le i \le n+3$

Step 4: For $n + 4 \le j \le 2n + 3$;

$$f(v_2v_j) = \begin{cases} 6n+6-j & \text{if } j \text{ is even} \\ 6n+8-j & \text{if } j \text{ is odd} \end{cases}$$

Step 5: $f(v_3v_k) = 6n + 7 - k, 2n + 4 \le k \le 3n + 3$

Step 6: $f(v_3v_1) = 6n + 4$

Step 7: $f(v_3v_2) = 6n + 5$

end.

Next we need to prove that the construction in above algorithm is super edge bimagic. Let $f: V \cup E \to \{1, 2, 3, ..., 6n + 5\}$ be the bijective function defined as in above algorithm.

For $4 \le i \le n + 3; f(v_1) + f(v_i) + f(v_1v_i) = 1 + i + 6n + 7 - i = 6n + 8 = k_1$ (say)

For $n + 4 \le j \le 2n + 3$ and if j is even then $f(v_2) + f(v_j) + f(v_2v_j) = 2 + j + 6n + 6j = 6n + 8 = k_1$ (say)

For $n + 4 \le j \le 2n + 3$ and if j is odd then $f(v_2) + f(v_j) + f(v_2v_j) = 2 + j + 6n + 8 - j = 6n + 10 = k_2$ (say)

For $2n + 4 \le k \le 3n + 3, f(v_3) + f(v_k) + f(v_3v_k) = 3 + k + 6n + 7 - k = 6n + 10 = k_2$ (say)

Also $f(v_3) + f(v_1) + f(v_3v_1) = 3 + 1 + 6n + 4 = 6n + 8 = k_1$ (say)

Also $f(v_3) + f(v_2) + f(v_3v_2) = 3 + 2 + 6n + 5 = 6n + 10 = k_2$ (say)

Which concludes that there exist two constants k_1 and k_2 such that for any edge uv in the graph $P_3 \odot K_{1,n}$, $f(u) + f(v) + f(uv) = k_1$ or k_2.

Theorem 2.3: The Graph $P_n + N_2$ $(n \ge 3)$ has super edge bimagic labeling if n is odd.

Proof: For the graph $P_n + N_2$ $(n \ge 3)$, the vertex set $V = \{v_1, v_2, v_3, ..., v_{n+2}\}$ and The edge set $E = \{E_1 \cup E_2 \cup E_3 \cup E_4\}$ where $E_1 = \{v_1v_i, 2 \le i \le n + 1\}$, $E_2 = \{v_{n+2}v_j, 2 \le j \le n\}$, $E_3 = \{v_k v_{k+1}, 2 \le k \le n\}$ and $E_4 = \{v_{n+1}v_{n+2}\}$.

Let $f : V \cup E \to \{1, 2, 3, ..., 4n + 1\}$ be the bijective function defined as

$f(v_i) = i;\ 1 \le i \le n + 2$

$f(v_1 v_i) = 4n + 3 - i;\ 2 \le i \le n + 1$

For $\quad 2 \le j \le n;$

$$f(v_{n+2}v_j) = \begin{cases} 3n + 2 - j & \text{if } j \text{ is even} \\ 2n + 4 - j & \text{if } j \text{ is odd} \end{cases}$$

$f(v_k v_{k+1}) = 3n + 5 - 2k;\ 2 \le k \le n$

$f(v_{n+1} v_{n+2}) = n + 3.$

Next we show the property of super edge bimagic labeling.

For $2 \le i \le n + 1; f(v_1) + f(v_i) + f(v_1 v_i) = 1 + i + 4n + 1 - i + 2 = 4n + 4 = k_1$ (say)

For $2 \le j \le n$ and if j is even then $f(v_{n+2}) + f(v_j) + f(v_{n+2}v_j) = n + 2 + j + 3n + 2 - j = 4n + 4 = k_1$ (say)

For $2 \le j \le n$ and if j is odd then $f(v_{n+2}) + f(v_j) + f(v_{n+2}v_j) = n + 2 + j + 2n + 4 - j = 3n + 6 = k_2$ (say)

For $2 \le k \le n; f(v_k) + f(v_{k+1}) + f(v_k v_{k+1}) = k + k + 1 + 3n + 5 - 2k = 3n + 6 = k_2$ (say)

Also $f(v_{n+1}) + f(v_{n+2}) + f(v_{n+1} v_{n+2}) = n + 1 + n + 2 + n + 3 = 3n + 6 = k_2$ (say)

Which concludes that there exist two constants k_1 and k_2 such that for any edge uv in the graph $P_n + N_2$, $f(u) + f(v) + f(uv) = k_1$ or k_2.

This proves that $P_n + N_2$ $(n \ge 3)$ has super edge bimagic labeling if n is odd.

Theorem 2.4: The graph $(P_2 \cup mK_1 + N_2)$ $(m \ge 1)$ has super edge bimagic labeling.

Proof: For the graph $(P_2 \cup mK_1 + N_2)$ $(m \ge 1)$, the vertex set

$V = \{v_1, v_2, v_3, ..., v_{m+4}\}$ and the edge set

$E = \{E_1 \cup E_2 \cup E_3\}$ where $E_1 = \{v_1 v_i;\ 2 \le i \le m + 3\}$,

$E_2 = \{v_{m+4} v_j;\ 2 \le j \le m + 3\}$ and $E_3 = \{v_2 v_3\}$.

Let $f : V \cup E \to \{1, 2, 3, ..., 3m + 9\}$ be the bijective function defined as

$f(v_i) = i;\ 1 \le i \le m + 4$

$f(v_1 v_i) = 3m + 11 - i;\ 2 \le i \le m + 3$

$$f(v_{m+4}v_j) = 2(m+4) - j;\ 2 \le j \le m+3$$

$$f(v_2v_3) = 2m + 7.$$

Next we show the property of super edge bimagic labeling.

For $2 \le i \le m+3; f(v_1) + f(v_i) + f(v_1v_i) = 1 + i + 3m + 9 - i + 2 = 3m + 12 = k_1$ (say)

For $2 \le j \le m + 3; f(v_{m+4}) + f(v_j) + f(v_{m+4}v_j) = m + 4 + j + 2m + 6 - j + 2 = 3m + 12 = k_1$ (say)

Also $f(v_2) + f(v_3) + f(v_2v_3) = 2 + 3 + 2m + 7 = 2m + 12 = k_2$ (say)

Which concludes that there exist two constants k_1 and k_2 such that for any edge uv in the graph $(P_2 \cup mK_1) + N_2, f(u) + f(v) + f(uv) = k_1$ or k_2.

This proves that $(P_2 \cup mK_1 + N_2)$ $(m \ge 1)$ has super edge bimagic labeling.

Theorem 2.5: The graph $(P_2 + mK_1)$ $(m \ge 2)$ has super edge bimagic labeling.

Proof: For the graph $(P_2 + mK_1)$ $(m \ge 2)$, the vertex set $V = \{v_1, v_2, ..., v_{m+2}\}$ and the edge set $E = \{E_1 \cup E_2\}$ where $E_1 = \{v_1v_i; 2 \le i \le m+2\}$ and $E_2 = \{v_2v_j; 3 \le j \le m+2\}$.

Let $f : V \cup E \to \{1, 2, 3, ..., 3m + 3\}$ be the bijective function defined as

$$f(v_i) = i;\ 1 \le i \le m+2$$

$$f(v_1v_i) = 3m + 5 - i;\ 2 \le i \le m+2$$

$$f(v_2v_j) = 2m + 5 - j;\ 3 \le j \le m+2$$

Next we show the property of super edge bimagic labeling.

For $2 \le i \le m + 2; f(v_1) + f(v_i) + f(v_1v_i) = 1 + i + 3m + 5 - i = 3m + 6 = k_1$ (say)

For $3 \le j \le m + 2; f(v_2) + f(v_j) + f(v_2v_j) = 2 + j + 2m + 5 - j = 2m + 7 = k_2$ (say)

Which concludes that there exist two constants k_1 and k_2 such that for any edge uv in the graph $(P_2 + mK_1)$ $(m \ge 2), f(u) + f(v) + f(uv) = k_1$ or k_2.

This proves that $(P_2 + mK_1)$ $(m \ge 2)$ has super edge bimagic labeling.

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