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DIRECTED VERTEX DOMINATING FUNCTION

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ABSTRACT: In this paper, we prove the necessary and sufficient condition for a directed vertex dominating function to be a minimal directed vertex dominating function. Also we find the directed fractional vertex domination number $a_\alpha(D)$ for digraphs CT (n), Unidirected caterpillar, $\overrightarrow{C_m^n}$.

KEYWORDS: Digraph, Directed vertex dominating function, Minimal directed vertex dominating function, Directed fractional vertex dominating number $a_\alpha(D)$.

1. INTRODUCTION

Consult [1] for notation and terminology which are not defined here. The concept of dominating function and fractional domination number in graphs were introduced in [6]. A dominating function (DF) of a graph G = (V, E) is a function $f : V \to [0, 1]$ such that $\sum_{\upsilon \in N(\upsilon)} f(\upsilon) \geq 1$ for all $\upsilon \in V$, where $N[\upsilon] = \{u \in V/u \text{ is adjacent with } v\} \cup \{v\}$. A DF f is called a minimal dominating function (MDF) if there is no function $g : V \to [0, 1]$ such that $g < f$ and g is a DF. Where $g < f$ if $g(u) \leq f(u)$ for all $\upsilon \in V$ and $g(\upsilon_0) < f(\upsilon_0)$ for some $\upsilon_0 \in V$. For a real-valued function $f : V(D) \to R$ the weight of f is $|f| = \sum_{\upsilon \in V} f(\upsilon)$ and for $S \subseteq V$, we define $f(S) = \sum_{\upsilon \in S} f(\upsilon)$ so $|f| = f(V)$.

The boundary set B_f and the positive set P_f of a DF f are defined by

$$B_f = \{v \in V : f(N[v]) = 1 \quad \text{and} \quad P_f = \{v \in V : f(v) > 0\}.$$

Let A and B be subsets of V. We say that A dominates B and write $A \to B$ if every vertex in B\A is adjacent to some vertex in A. The following theorem gives a necessary and sufficient condition for a DF to be an MDF.

Theorem 1.1 [3]:

A DF f of G is an MDF if and only if $B_f \to P_f$.

For any DF f, the fractional domination number $\gamma_f(G)$ is defined by

$$\gamma_f(G) = \min\{|f| : f \text{ is an MDF of G}\}.$$

Here we transfer this concept to digraphs, called directed fractional vertex dominating function and directed fractional vertex dominating number $a_\alpha(D)$.

2. DIRECTED VERTEX DOMINATING FUNCTION

Let D be a finite simple digraph with vertex set V (D) = V and arc set A (D) = A. If (u, v) is an arc of D, we say that v is adjacent from u or u is adjacent to v. The outdegree od (v) of a point v is the number of points adjacent from it, and the indegree id (v) is the number adjacent it. Let $N^+(v)$ denote the set of all vertices of D which are adjacent from v. Let $N^+[v] = N^+(v) \cup \{v\}$.

A directed vertex dominating function (DVDF) of a digraph D = (V, A) is a function $f : V \to [0, 1]$ such that $\sum_{v \in N^+(v)} f(v) \geq 1$ for all $v \in V$. A DVDF is called minimal DVDF if there is no function $g : V \to [0, 1]$ such that g < f and g is a DVDF.

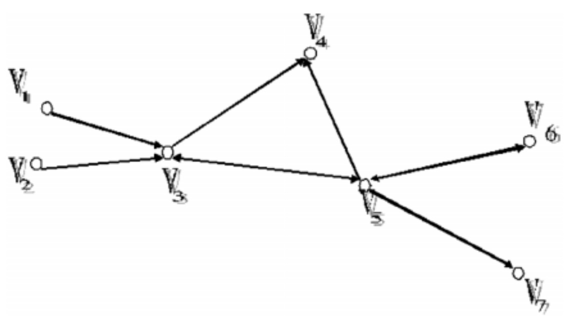
The directed fractional vertex domination number $a_\alpha(D)$ is defined as

$a_\alpha(D) = \text{Min}\{|f| : f \text{ is a minimal directed vertex dominating function on } D\}$.

Notation 2.1:

1. $f(N^+[v]) = \sum_{v \in N^+[v]} f(v)$,
2. $B_f^+ = \{v \in V / f(N^+[v]) = 1\}$,
3. $P_f^+ = \{v \in V / f(v) > 0\}$.

Example 2.2: Consider a digraph D = (V, E) where $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and $E = \{(v_1, v_3), (v_2, v_3), (v_3, v_4), (v_3, v_5), (v_5, v_3), (v_5, v_4), (v_5, v_6), (v_6, v_5), (v_5, v_7)\}$.



Let $(f(v_1), f(v_2), f(v_3), f(v_4), f(v_5), f(v_6), f(v_7)) = (1, 1, 3/10, 1, 5/10, 6/10, 1)$

$N^+[v_1] = \{v_1, v_3\}$,	$f(N^+[v_1]) = 13/10$
$N^+[v_2] = \{v_2, v_3\}$,	$f(N^+[v_2]) = 13/10$
$N^+[v_3] = \{v_3, v_4, v_5\}$,	$f(N^+[v_3]) = 18/10$

$N^+[v_4] = \{v_4\}$, $f(N^+[v_4]) = 1$

$N^+[v_5] = \{v_5, v_3, v_4, v_7, v_6\}$, $f(N^+[v_5]) = 34/10$

$N^+[v_6] = \{v_6, v_5\}$, $f(N^+[v_6]) = 11/10$

$N^+[v_7] = \{v_7\}$, $f(N^+[v_7]) = 1$

Therefore, $f(N^+[v]) \geq 1$ for all $v \in V$. So f is a directed vertex dominating function.

Definition 2.3: Let A and B be two subsets of V. We say that A dominates B and write $A \to B$ if every vertex $u \in B \backslash A$ is adjacent from some vertex in A ie there exists $v \in A$ such that $(v, u) \in V$.

Lemma 2.4: Let f be a directed vertex dominating function of a digraph D. Let v be a vertex of D such that $f(v) > 0$ and $f(N^+[v]) \geq 1$. Then $id(v) \geq 1$.

Proof: Suppose $id(v) = 0$.

Let $f(N^+[v]) = 1 + s$

Let $x = \min(s, f(v))$

Define g : by $V \to [0, 1]$

$$g(u) = \begin{cases} f(u), & u \neq v \\ f(v) - x, & u = v \end{cases}$$

$$g(N^+[v]) = f(N^+[v]) - x$$

$$= 1 + s - x$$

$$g(N^+[v]) \geq 1 \quad (\because \ s - x \geq 0)$$

Also $g(N^+[u]) = f(N^+[u])$ for all $u \neq v$ ($\because$ $id(v) = 0$)

$\therefore$ $g(N^+[u]) \geq 1$ for all $u \neq v$

$\therefore$ g is a directed vertex dominating function and $g < f$

$\Rightarrow \Leftarrow$ ($\therefore$ f is a minimal directed vertex dominating function)

$\therefore$ $id(v) \geq 1$.

Theorem 2.5: A directed vertex dominating function f of D is a minimal directed vertex dominating function iff $B_f^+ \to P_f^+$.

Proof: Assume that f is a minimal directed vertex dominating function.

If $P_f^+ \setminus B_f^+ = \emptyset$ there is nothing to prove

Otherwise, let $v \in P_f^+ \setminus B_f^+$

$\therefore$ $f(v) > 0$ and $f(N^+[v]) > 1$

$\therefore$ By Lemma $id(v) \geq 1$

To prove that v is adjacent from some vertex in B_f^+

Suppose not, the vertices adjacent to v are not in B_f^+

Let $v_1, v_2, \ldots, v_n$ be the vertices adjacent to v.

Therefore, $f(N^+[v_1]) > 1, f(N^+[v_2]) > 1, \cdots, f(N^+[v_n]) > 1$.

Let $f(N^+[v]) = 1 + s, f(N^+[v_1]) = 1 + s_1, \cdots, f(N^+[v_n]) = 1 + s_n$, where $s, s_1, \cdots, s_n > 0$.

Let $x = \min(s, s_1, \cdots, s_n, f(v))$

Define $g : V \to [0, 1]$ by

$$g(u) = \begin{cases} f(u), & u \neq v \\ f(v) - x, & u = v \end{cases}$$

$$g(N^+[v]) = f(N^+[v]) - x$$

$$= 1 + s - x, \text{ where } s - x \geq 0$$

$$g(N^+[v_1]) = f(N^+[v_1]) - x$$

$$= 1 + s_1 - x, \text{ where } s_1 - x \geq 0$$

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$$g(N^+[v_n]) = g(N^+[v_n]) - x$$

$$= 1 + s_n - x, \text{ where } s_n - x \geq 0$$

Also $g(N^+[u]) = f(N^+[u]) \geq 1$ for all $u \in V \setminus \{v_1, v_2, \ldots, v_n\}$

Hence $g(N^+[u]) \geq 1$ for all $u \in V$

Therefore, g is a directed vertex dominating function and $g < f$

This is a contradiction.

Therefore, v is adjacent from some vertex in B_f^+

i.e., $B_f^+ \to P_f^+$

Conversely, assume that $B_f^+ \to P_f^+$.

Suppose f is not a minimal directed vertex dominating function.

Therefore, there exists a directed vertex dominating function $g : V \to [0, 1]$ such that $g < f$.

i.e., there exists $u_0 \in P_f^+$ such that $g(u_0) < f(u_0)$

If $f(N^+[u_0]) = 1$ then $g(N^+[u_0]) < 1$ (since $g < f$ and $g(u_0) < f(u_0)$).

This is a contradiction (since g is a directed vertex dominating function).

If $f(N^+[u_0]) > 1$ then $u_0 \in P_f^+ \setminus B_f^+$

Since $B_f^+ \to P_f^+$, there exists $v_0 \in B_f^+$ such that u_0 is adjacent from v_0

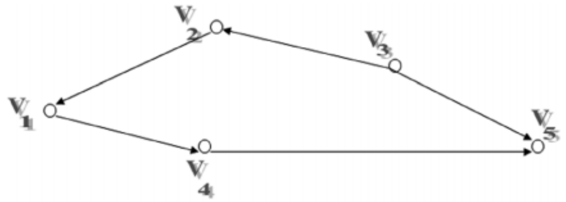
i.e., $f(N^+[v_0]) = 1$

Therefore $g(N^+[v_0]) < 1$ (since $g < f$ and $g(u_0) < f(u_0)$)

This is a contradiction (since g is a directed vertex dominating function).

Therefore f is a minimal directed vertex dominating function.

Example 2.6: Consider a digraph $D = (V, A)$ where $V = \{v_1, v_2, v_3, v_4, v_5\}$ $A = \{(v_2, v_1), (v_1, v_4), (v_3, v_2), (v_3, v_5), (v_3, v_4), (v_4, v_5)\}$.



Let $(f(v_1), f(v_2), f(v_3), f(v_4), f(v_5)) = (1/2, 1/2, 0, 1/2, 1)$

$N^+[v_1] = \{v_1, v_4\}$ $\quad f(N^+[v_1]) = 1$

$N^+[v_2] = \{v_2, v_1\}$ $\quad f(N^+[v_2]) = 1$

$N^+[v_3] = \{v_3, v_2, v_5, v_4\}$ $\quad f(N^+[v_3]) = 2$

$N^+[v_4] = \{v_4, v_5\}$ $\quad f(N^+[v_4]) = 3/2$

$N^+[v_5] = \{v_5\}$ $\quad f(N^+[v_5]) = 1$

$P_f^+ = \{v_1, v_2, v_4, v_5\}$ $B_f^+ = \{v_1, v_2, v_5\}$

$P_f^+ \setminus B_f^+ = \{v_4\}$

$(v_1, v_4) \in A$ and $f(N^+[v_1]) = 1.$

Therefore, f is a minimal directed vertex dominating function.

3. CIRCULAR TOURNAMENT CT (n)

It is defined in [7]. That is, the vertex set of CT (n) is $\{u_0, u_1, \ldots, u_{n-1}\}$. For each i, the arcs are going from u_i to $u_{i+1}, u_{i+2}, \ldots, u_{i+r}$, where the indices are taken modulo n, $1 \le r \le n-1$. When r = 1, CT (n) becomes unidirected cycle $\overrightarrow{C_n}$. $\overrightarrow{C_n}$ is defined in [5] ie the vertex set and the arc set of $\overrightarrow{C_n}$ are $\{v_1, v_2, \ldots, v_n\}$ and $\{(v_1, v_2), (v_2, v_3), \ldots, (v_n, v_1)\}$ respectively.

Theorem 3.1: For $n \ge 3$, $a_\alpha(CT(n)) = \frac{n}{r+1}$, $1 \le r \le n-1$

Proof: Let $V(CT(n)) = \{u_0, u_1, \ldots, u_{n-1}\}$

Then $N^+[u_i] = \{u_i, u_{i+1}, \ldots, u_{i+r}\}$, $i = 0, 1, \ldots, n-1$

Let $f(u_i) = \frac{1}{r+1}$, $i = 0, 1, \ldots, n-1$

Clearly $f(N^+[u_i]) = 1$, $i = 0, 1, \ldots, n-1$

Also $P_f^+ \setminus B_f^+ = \emptyset$

$\therefore$ f is a minimal directed vertex dominating function

$$\therefore \quad a_\alpha(CT(n)) \le \frac{1}{r+1} + \frac{1}{r+1} + \ldots \text{ n terms}$$

$$= \frac{n}{r+1}$$

i.e., $$a_\alpha(CT(n)) \le \frac{n}{r+1} \qquad (1)$$

Let f be a minimal directed vertex dominating function and $a_\alpha(CT(n)) = |f|$

i.e., $$a_\alpha(CT(n)) = f(u_0) + f(u_1) + \ldots + f(u_{n-1}) \qquad (2)$$

Also $f(N^+[u_i]) \ge 1$, $i = 0, 1, \ldots, n-1$

$f(u_i) + f(u_{i+1}) + \ldots + f(u_{i+r}) \ge 1$, $i = 0, 1, \ldots, n-1$

Adding these n inequalities, we get $(f(u_0) + f(u_1) + \ldots + f(u_{n-1})) + (f(u_1) + f(u_2) + \ldots + f(u_0)) + \ldots + ((f(u_r) + f(u_{r+1}) + \ldots + f(u_{r+n-1})) \geq n$ where $f(u_r) + f(u_{r+1}) + \ldots + f(u_{r+n-1})$ is nothing but $f(u_0) + f(u_1) + \ldots + f(u_{n-1})$

$\therefore$ $a_\alpha(CT(n)) + a_\alpha(CT(n)) + \ldots$ (r + 1) terms $\geq n$ (By (2))

$\therefore$ $(r + 1)\, a_\alpha(CT(n)) \geq n$

i.e., $$a_\alpha(CT(n)) \geq \frac{n}{r+1} \qquad (3)$$

From (1) and (3) we get

$$a_\alpha(CT(n)) = \frac{n}{r+1}$$

Corollary 3.2: For a unidirected cycle $\overrightarrow{C_n}$ with n vertices $a_\alpha(\overrightarrow{C_n}) = \frac{n}{2}$

Proof: When $r = 1$, $CT(n)$ becomes $\overrightarrow{C_n}$

$\therefore$ By Theorem 3.1, $a_\alpha(\overrightarrow{C_n}) = \frac{n}{2}$.

4. UNIDIRECTED CATERPILLER

A source in a digraph is a point which can reach all others. An out-tree is a digraph with a source having no semicycle.

Unidirected Caterpiller is defined as an out-tree with the property that the removal of its points with out-degree 0 leaves an unidirected path $\overrightarrow{P_n} = (v_1, v_2, \ldots, v_n)$. Also $od(v_i) = 3$, $i = 1, 2, \ldots, n - 1$ and $od(v_n) = 2$.

That is, the number of vertices in this Unidirected Caterpiller is 3n.

Theorem 4.1: Let $\vec{C}$ be an unidirected caterpillar with 3n vertices. Then $a_\alpha(\vec{C}) = 2n$.

Proof: Let $\overrightarrow{P_n} = (v_1, v_2, \ldots, v_n)$ be the unidirected path got by removing the end vertices of the caterpillar. Let s_i, v_i be the vertices adjacent from v_i, $i = 1, 2, \ldots, n$.

Define $f : V \to [0, 1]$ by

$f(s_i) = f(t_i) = 1$, $\quad i = 1, 2, \ldots, n$. and

$f(v_i) = 0$, $\quad i = 1, 2, \ldots, n$.

Then $N^+[s_i] = \{s_i\}$, $N^+[t_i] = \{t_i\}$, $i = 1, 2, \ldots, n$.

$N^+[v_i] = \{v_i, v_{i+1}, t_i, s_i\}$, $i = 1, 2, \ldots, n-1$.

$N^+[v_n] = \{v_n, t_n, s_n\}$

Clearly $f(N^+[v]) \geq 1 \; \forall \; v \in V$

Also $P_f^+ \setminus B_f^+ = \emptyset$

$\therefore$ f is a directed vertex dominating function.

$$\therefore \quad a_\alpha(\vec{C}) \leq |f| = \sum_{i=1}^{n} f(s_i) + \sum_{i=1}^{n} f(t_i) + \sum_{i=1}^{n} f(v_i)$$

$$= n + n + 0$$

i.e., $a_\alpha(\vec{C}) \leq 2n$ (1)

Let f be a minimal directed vertex dominating function and $a_\alpha(\vec{C}) = |f|$.

Since $od(t_i) = od(s_i) = 0$, $i = 1, 2, \ldots, n$,

$f(t_i) = f(s_i) = 1$, $i = 1, 2, \ldots, n$

$$\therefore \quad a_\alpha(\vec{C}) = \sum_{i=1}^{n} f(s_i) + \sum_{i=1}^{n} f(t_i) + \sum_{i=1}^{n} f(v_i)$$

$$= n + n + \sum_{i=1}^{n} f(v_i)$$

$$a_\alpha(\vec{C}) \geq 2n \quad \left(\because \sum_{i=1}^{n} f(v_i) \geq 0 \right) \qquad (2)$$

From (1) and (2) we get

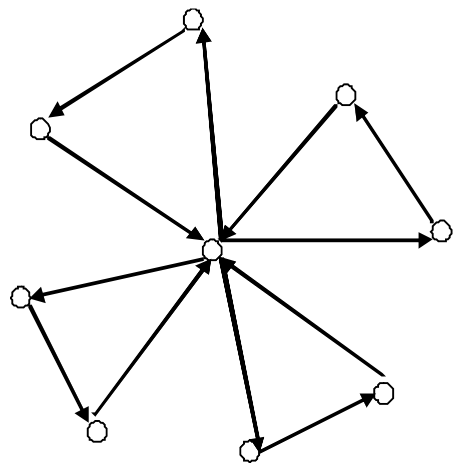
$$a_\alpha(\vec{C}) = 2n.$$

5. THE DIGRAPH

Unidirected cycle $\overrightarrow{C_m}$ is used in [5] i.e., $\overrightarrow{C_m}$ is a digraph with vertex set $\{v_1, v_2, \ldots, v_n\}$ and arc set $\{(v_1, v_2), (v_2, v_3), \ldots, (v_n, v_1)\}$.

The digraph $\overrightarrow{C_m^n}$ is defined as the n copies of unidirected cycles of length m with one vertex in common and all copies in the same direction.

Example 5.1: $\overrightarrow{C_3^4}$



Theorem 5.1: For

$$m \geq 3, \quad a_\alpha(\overrightarrow{C_m^n}) = \frac{(m-2)\,n}{2} + 1 \qquad \text{if m is even}$$

$$= \frac{(m-2)\,n}{2} + \frac{1}{n+1} \qquad \text{if m is odd}$$

Proof: Let u_0 be the common vertex and (u_0, u_{i1}), (u_{i1}, u_{i2}), …, $(u_{i\,(m-1)}, u_0)$ be the arcs in the i^{th} copy of $\overrightarrow{C_m^n}$, where i = 1, 2, …, n.

Case 1: m is even.

If $f(u_0) = 1$, $f(u_{i\,(m-1)}) = 0$, $f(u_{i\,(m-2)}) = 1$, $f(u_{i\,(m-3)}) = 0$, …, $f(u_{i2}) = 1$, $f(u_{i1}) = 0$, i = 1, 2, …, n then

$$f(N^+[u_{i\,(m-1)}]) = f(u_{i\,(m-1)}) + f(u_0) = 1$$

$$f(N^+[u_{i\,(m-2)}]) = f(u_{i\,(m-2)}) + f(u_{i\,(m-1)}) = 1$$

…

…

…

$$f(N^+[u_{i1}]) = f(u_{i1}) + f(u_{i2}) = 1$$

where $i = 1, 2, \ldots, n$ and

$$f(N^+[u_0]) = f(u_0) + f(u_{11}) + f(u_{21}) + f(u_{31}) + \ldots + f(u_{n1}) = 1$$

Also $\quad P_f^+ \setminus B_f^+ = \emptyset$

Hence f is a minimal vertex dominating function on $\overrightarrow{C_m^n}$.

Let g be a minimal vertex dominating function on $\overrightarrow{C_m^n}$ and $|g| = a_\alpha(\overrightarrow{C_m^n})$.

Then

$$g(N^+[u_{i\,(m-1)}]) = g(u_{i\,(m-1)}) + g(u_0) = 1$$

$$g(N^+[u_{i\,(m-2)}]) = g(u_{i\,(m-2)}) + g(u_{i\,(m-1)}) = 1$$

$$\ldots$$

$$\ldots$$

$$\ldots$$

$$g(N^+[u_{i1}]) = g(u_{i1}) + g(u_{i2}) = 1$$

where $i = 1, 2, \ldots, n$

$$g(N^+[u_0]) = g(u_0) + g(u_{11}) + g(u_{21}) + g(u_{31}) + \ldots + g(u_{n1}) = 1$$

Solving these equations,

Let $\quad g(u_0) = x$

$$g(u_{i\,(m-1)}) + g(u_0) = 1 \Rightarrow g(u_{i\,(m-1)}) = 1 - x$$

$$g(u_{i\,(m-2)}) + g(u_{i\,(m-1)}) = 1 \Rightarrow g(u_{i\,(m-2)}) = x$$

$$\ldots$$

$$\ldots$$

$$\ldots$$

$$g(u_{i1}) + g(u_{i2}) = 1 \Rightarrow g(u_{i1}) = 1 - x$$

$$g(N^+[u_0]) = g(u_0) + g(u_{11}) + g(u_{21}) + g(u_{31}) + \ldots + g(u_{n1})$$

$$= x + (1 - x) + (1 - x) + \ldots n \quad \text{times}$$

$$= x + n(1 - x)$$

$$= x(1 - n) + n$$

$$\therefore \quad g(N^+[u_0]) = 1 \Rightarrow x(1 - n) + n = 1$$

$$\therefore \quad x = 1$$

Hence, $g(u_0) = 1$, $g(u_{i(m-1)}) = 0$, $g(u_{i(m-2)}) = 1$, $g(u_{i(m-3)}) = 0$, …, $g(u_{i2}) = 1$, $g(u_{i1}) = 0$, $i = 1, 2, \ldots, n$

$$\therefore \quad a_\alpha(\overrightarrow{C_m^n}) = n \sum_{j=1}^{m-1} g(u_{ij}) + g(u_0)$$

$$\therefore \quad a_\alpha(\overrightarrow{C_m^n}) = |g| = \frac{(m-2)\,n}{2} + 1$$

Case 2: m is odd.

If $f(u_0) = \frac{1}{n+1}$, $f(u_{i(m-1)}) = \frac{1}{n+1}$, $f(u_{i(m-2)}) = \frac{1}{n+1}$, $f(u_{i(m-3)}) = \frac{1}{n+1}$, …, $f(u_{i2}) = \frac{1}{n+1}$, $f(u_{i1}) = \frac{1}{n+1}$ then

$$f(N^+[u_0]) = f(u_0) + f(u_{11}) + f(u_{21}) + f(u_{31}) + \ldots + f(u_{n1})$$

$$= \frac{1}{n+1} + \frac{1}{n+1} + \ldots,\ (n+1) \text{ terms}$$

$$= 1$$

$$f(N^+[u_{i(m-1)}]) = f(u_{i(m-1)}) + f(u_0) = \frac{1}{n+1} + \frac{1}{n+1} = 1$$

$$f(N^+[u_{i(m-2)}]) = f(u_{i(m-2)}) + f(u_{i(m-1)}) = \frac{1}{n+1} + \frac{1}{n+1} = 1$$

…

…

…

$$f(N^+[u_{i1}]) = f(u_{i1}) + f(u_{i2}) = \frac{1}{n+1} + \frac{1}{n+1} = 1$$

$i = 1, 2, \ldots, n$

Also $\quad P_f^+ \setminus B_f^+ = \emptyset$

Hence f is a minimal vertex dominating function on $\overrightarrow{C_m^n}$.

Let g be a minimal vertex dominating function on $\overrightarrow{C_m^n}$ and $|g| = a_\alpha(\overrightarrow{C_m^n})$.

Then $g(N^+[u_{i\,(m-1)}]) = g(u_{i\,(m-1)}) + g(u_0) = 1$

$g(N^+[u_{i\,(m-2)}]) = g(u_{i\,(m-2)}) + g(u_{i\,(m-1)}) = 1$

…

…

…

$g(N^+[u_{i1}]) = g(u_{i1}) + g(u_{i2}) = 1$

where i = 1, 2, …, n

$g(N^+[u_0]) = g(u_0) + g(u_{11}) + g(u_{21}) + g(u_{31}) + \ldots + g(u_{n1}) = 1$

Solving these equations,

Let $g(u_0) = x$

$g(u_{i\,(m-1)}) + g(u_0) = 1 \Rightarrow g(u_{i\,(m-1)}) = 1 - x$

$g(u_{i\,(m-2)}) + g(u_{i\,(m-1)}) = 1 \Rightarrow g(u_{i\,(m-2)}) = x$

…

…

…

$g(u_{i1}) + g(u_{i2}) = 1 \Rightarrow g(u_{i1}) = x$

$g(N^+[u_0]) = g(u_0) + g(u_{11}) + g(u_{21}) + g(u_{31}) + \ldots + g(u_{n1})$

$= x + x + x + \ldots + (n + 1)$ terms

$= (n + x)\,x$

$\therefore\quad g(N^+[u_0]) = 1 \Rightarrow (n + 1)\,x = 1$

$$\therefore\quad x = \frac{1}{n+1}$$

Hence $g(u_0) = \frac{1}{n+1}$, $g(u_{i\,(m-1)}) = \frac{n}{n+1}$, $g(u_{i\,(m-2)}) = \frac{1}{n+1}$, $g(u_{i\,(m-3)}) = \frac{n}{n+1}$,

…, $g(u_{i2}) = \frac{n}{n+1}$, $g(u_{i1}) = \frac{1}{n+1}$, i = 1, 2 … n

$$\therefore \quad a_{\alpha}(\overrightarrow{C_m^n}) = n\sum_{j=1}^{m-1} g(u_{ij}) + g(u_0)$$

$$\therefore \quad a_{\alpha}(\overrightarrow{C_m^n}) = |g| = \frac{(m-1)\,n}{2} + \frac{1}{n+1}.$$

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