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GRACEFUL AND SKOLEM GRACEFUL LABELINGS IN EXTENDED DUPLICATE TWIG GRAPHS

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ABSTRACT: In this paper, we show that the class of Extended Duplicate Graph of a Twig is Graceful and Skolem-graceful.

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KEYWORDS: Graph labeling, Graceful labeling, Skolem-graceful labeling, Duplicate Graphs, Twigs.

1. INTRODUCTION

The concept of graph labeling was introduced by Rosa in 1967 [8]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs serve as useful models for a broad range of applications such as circuit design, radar, astronomy, coding theory, communication network addressing and data base management [3]. Hence in the intervening years various labeling of graphs such as graceful labeling, harmonious labeling, magic labeling, antimagic labeling, bimagic labeling, prime labeling, cordial labeling, mean labeling, arithmetic labeling etc., have been studied in over 1100 papers [3].

The study of graceful graphs and graceful labeling methods was introduced by Rosa [8]. Rosa defined a β-valuation of a graph G with e edges as an injection from the vertices of G to the set {0, 1, 2,…, e} such that when each edge xy is assigned the label $|f(x) - f(y)|$ the resulting edge labels are distinct. β-valuations are functions that produce graceful labelings.

However, the term graceful labeling was not used until Golomb [5] studied such labelings several years later. When studying graceful labelings we need only simple graphs, or graphs without loops or parallel edges. The gracefulness of several classes of graphs has already been established. For example, all paths P_n are graceful. Cycles C_n are graceful only when $n \equiv 0$ or 3 (mod 4).

Amutha and Kathiresan [1] proved that the graph obtained by attaching a pendent edge to each vertex of $\overline{K_n} + 2K_2$ is graceful. Huang and Rosa [6] conjectured that all trees are graceful. Gallian and Ropp [4] conjectured that every graph obtained by adding a single pendent edge to one or more vertices of a cycle is graceful. Lee has introduced Skolem-graceful labeling. Lee and Wui [7] have shown that a connected graph is Skolem if and only if it is graceful tree. Choudum and Kishore [2] proved that all 5-stars are Skolem graceful. Youssef [11] proved that if G is Skolem graceful, then $G + \overline{K_n}$ is graceful. In this paper, we prove that the Extended Duplicate Graph of Twigs are Graceful and Skolem graceful.

2. PRELIMINARIES

In this section we give the basic notions relevant to this paper. Let G = G (V, E) be a finite, simple and undirected graph with p vertices and q edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers, called labels. In this paper we deal labeling with domain as the set of all vertices.

Definition 2.1 (Graceful): A function f is said to be Graceful of a graph G with 'q' edges if f is 1 – 1 from $V \to \{0, 1, 2, 3, \ldots, q\}$ such that for each edge uv is assigned the label $|f(u) - f(v)|$, the resulting edge labels are distinct numbers $\{1, 2, 3, \ldots, q\}$.

Definition 2.2 (Skolem-Graceful): Let G (p, q) be a graph. If there exists 1 – 1 map $f : V \to \{1, 2, \ldots, p\}$ such that the edge labels induced by $|f(x) - f(y)|$ for each edge xy are (1, 2, 3, …, q). This is also called Node – Graceful. A necessary condition for a graph to be Skolem graceful is that $p > q + 1$.

Definition 2.3 (Twig): A graph G (V, E) obtained from a path by attached exactly two pendent edges to each internal vertices of the path is called a Twig graph. Generally, a Twig T_m with m internal vertices, has 3m + 1 edges and 3m + 2 vertices. We classify the Twigs into two categories such as T_{2n} and T_{2n-1}, $n \in N$.

Definition 2.4: Let G (V, E) be a simple graph. A duplicate graph of G is $DG = (V_1, E_1)$ where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f : V \to V'$ is bijective, (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is defined as follows: The edge ab is in E if and only if both ab' and $a'b$ are edges in E_1. Clearly the duplicate graph of the twig graph is disconnected. In order to view as a connected graph we give the following definition from [9, 10].

Definition 2.5 EDG (T_m): Let G (V, E) be a Twig graph Tm and let $DG = (V_1, E_1)$ where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ be a duplicate graph of a Twig T_m. Add an edge between any one vertex from V to any other vertex in V', except the terminal vertices of V and V'. For convenience, let us take $v_2 \in V$ and $v_2' \in V'$ and thus the edge (v_2, v_2') is formed. We call this new graph as the Extended Duplicate Graph of the Twig T_m and it is denoted as EDG (T_m). Clearly this EDG (T_m) has (6m + 4) vertices and (6m + 3) edges.

3. MAIN RESULTS

In this section, we present an algorithm and prove the existence of Graceful labeling for EDG (T_m).

Algorithm:

Input: EDG (T_m) with $(6m + 4)$ vertices and $(6m + 3)$ edges.

Step 1: Denote the $(6m + 4)$ vertices as

$$V = \{v_1, v_2, ..., v_{3m+1}, v_{3m+2}, v_1, v_2, ..., v_{3m+1}, v_{3m+2}\}.$$

Step 2: If 'm' is even, then $m = 2n$; $n \in N$.

If $n \in N$, then the EDG ( ) m T is of the form EDG (T_{2n}); $n \in N$.

Define $f : V \rightarrow \{0, 1, 2, 3, \ldots, 6m + 3\}$ such that for $0 < i < \frac{m-2}{2}$.

(i) $f(v_{6i+3}) = 3i + 1$, $f(v_{6i+4}) = 3i + 2$, $f(v_{6i+5}) = 3i + 3$

(ii) $f(v_{6i+6}) = 3(m - i)$, $f(v_{6i+7}) = 3(m - i) - 1$, $f(v_{6i+8}) = 3(m - i) - 2$

(iii) $f(v_{6i+3}) = 3(m + i) + 3$, $f(v_{6i+4}) = 3(m + i) + 4$, $f(v_{6i+5}) = 3(m + i) + 5$

(iv) $f(v_{6i+6}) = 3(2m - i) + 2$, $f(v_{6i+7}) = 3(2m - i) + 1$, $f(v_{6i+8}) = 3(2m + i)$

and also $f(v_1) = 0$, $f(v_2) = 3m + 1$, $f(v_1) = 3m + 2$, $f(v_2) = 6m + 3$.

Step 3: If m is odd, then $m = 2n - 1$; $n \in N$.

The EDG (T_m) is of the form EDG (T_{2n-1}); $n \in N$.

Define $f : V \rightarrow \{0, 1, 2, 3, \ldots, 6m + 3\}$ for the function defined as in Step 2 for (i) to (iv) with the limits $0 < i < \frac{m-1}{2}$, $0 < i < \frac{m-3}{2}$, $0 < i < \frac{m-1}{2}$ and $0 < i < \frac{m-3}{2}$ respectively.

Step 4: Define $f^* : E \rightarrow N$ such that $f^*(v_i, v_j) = |f(v_i) - f(v_j)|$.

Output: Labeled EDG (T_m).

Theorem: The Extended Duplicate Graph of Twig (T_m), $m > 1$ is graceful.

Proof: Let EDG (T_m) be a Extended duplicate graph of the Twig (T_m). Clearly EDG (T_m) has $(6m + 4)$ vertices and $(6m + 3)$ edges.

Denote the set of vertices as

$$V = \{v_1, v_2, ..., v_{3m+1}, v_{3m+2}, v_1, v_2, ..., v_{3m+1}, v_{3m+2}\}.$$

Case (1): Let T_m be a Twig where $m = 2n$, $n \in N$.

Consider the Twig of the Type T_{2n}, $n \in N$.

In this case we get Twig graphs $T_2, T_4, T_6, T_8, \ldots$

To label the vertices of V, define a map $f : V \to \{0, 1, 2, 3, \ldots, 6m + 3\}$ as given in Step 2 of the above algorithm.

The vertices v_1 and v_2 are labeled as 0 and $3m + 1$; the vertices, (v_3, v_4, v_5), (v_9, v_{10}, v_{11}), ..., $(v_{3m-3}, v_{3m-2}, v_{3m-1})$, $(v_{3m+2}, v_{3m+1}, v_{3m})$, $(v_{3m-4}, v_{3m-5}, v_{3m-6})$, ..., (v_8, v_7, v_6) receive consecutive numbers such as $1, 2, 3, \ldots, 3m - 2, 3m - 1, 3m$ as labels; the vertices v_1 and v_2 are labeled as $3m + 2$ and $6m + 3$; the vertices (v_3, v_4, v_5), (v_9, v_{10}, v_{11}), ..., $(v_{3m-3}, v_{3m-2}, v_{3m-1})$, $(v_{3m+2}, v_{3m+1}, v_{3m})$, $(v_{3m-4}, v_{3m-5}, v_{3m-6})$, ..., (v_8, v_7, v_6) receive consecutive numbers $3m + 3, 3m + 4, 3m + 5, \ldots, 6m, 6m + 1, 6m + 2$ as labels.

Thus all the $6m + 4$ vertices are labeled.

From the definition of EDG (T_m), the $(6m + 3)$ edges of EDG (T_m) are of the form (v_2, v_2), (v_1, v_2), (v_2, v_1), (v_{2+3i}, v_{3i+3+j}), (v_{2+3i}, v_{3i+3+j}) for $0 < j < 2$, $0 < i < m - 1$.

The induced function $f^* : E \to N$ is defined such that

$$f^*(v_i, v_j) = |f(v_i) - f(v_j)|.$$

Thus the edges (v_1, v_2), (v_2, v_3), (v_2, v_4), (v_2, v_5), (v_5, v_6), (v_5, v_7), (v_5, v_8), ..., (v_{3m-1}, v_{3m}), (v_{3m-1}, v_{3m+1}), (v_{3m-1}, v_{3m+2}) receive consecutive numbers such as $1, 2, 3, 4, \ldots, 3m - 1, 3m, 3m + 1$ as labels; the edge (v_2, v_2) receive the number $3m + 2$ as label; the edges (v'_{3m+2}, v_{3m-1}), (v_{3m+1}, v_{3m-1}), (v_{3m}, v_{3m-1}), (v_{3m-1}, v_{3m-4}), (v_{3m-2}, v_{3m-4}), (v_{3m-3}, v_{3m-4}), ..., (v_5, v_2), (v_4, v_2), (v_3, v_2), (v_2, v_1) receive consecutive numbers such as $3m + 3, 3m + 4, 3m + 5, \ldots, 6m, 6m + 1, 6m + 2, 6m + 3$ as labels.

Thus all the edges are labeled as $\{1, 2, 3, 4, \ldots, 6m + 3\}$ and all are distinct. (The whole idea of the proof is illustrated in Fig. (2)).

Hence EDG (T_m), where $m = 2n$; $n \in N$ is graceful.

Case 2: Let T_m be a Twig where $m = 2n - 1$, $n \in N$.

Consider the Twig of the Type T_{2n-1}, $n \in N$.

In this case we get Twig graphs $T_1, T_3, T_5, T_7, \ldots$

To label the vertices of V, define a map $f : V \longrightarrow \{0, 1, 2, 3, \ldots, 6m + 3\}$ as given in Step 3 of the above algorithm.

The vertices v_1 and v_2 are labeled as 0 and 3m + 1; the vertices (v_3, v_4, v_5), (v_9, v_{10}, v_{11}), ..., $(v_{3m}, v_{3m+1}, v_{3m+2})$, $(v_{3m-1}, v_{3m-2}, v_{3m-3})$, ..., (v_8, v_7, v_6) receive consecutive numbers such as 1, 2, 3, …, 3m – 2, 3m – 1, 3m as labels; the vertices v_1 and v_2 are labeled as 3m + 2 and 6m + 3; the vertices (v_3, v_4, v_5), (v_9, v_{10}, v_{11}), ..., $(v_{3m}, v_{3m+1}, v_{3m+2})$, $(v_{3m-1}, v_{3m-2}, v_{3m-3})$, ..., (v_8, v_7, v_6) receive consecutive numbers such as 3m + 3, 3m + 4, 3m + 5, …, 6m, 6m + 1, 6m + 2 as labels.

Thus all the 6m + 4 vertices are labeled.

From the definition of EDG (T_m), the (6m + 3) edges of EDG (T_m) are of the form (v_2, v_2), (v_1, v_2), (v_2, v_1), (v_{2+3i}, v_{3i+3+j}), (v_{2+3i}, v_{3i+3+j}) for $0 < j < 2$, $0 < i < m - 1$.

The induced function $f^* : E \longrightarrow N$ is defined such that

$$f^*(v_i, v_j) = |f(v_i) - f(v_j)|.$$

Thus the edges (v_1, v_2), (v_2, v_3), (v_2, v_4), (v_2, v_5), (v_5, v_6), (v_5, v_7), (v_5, v_8), ..., (v_{3m-1}, v_{3m}), (v_{3m-1}, v_{3m+1}), (v_{3m-1}, v_{3m+2}) receive consecutive numbers such as 1, 2, 3, 4, …, 3m – 1, 3m, 3m + 1 as labels; the edge (v_2, v_2) receive the number 3m + 2 as label; the edges (v_{3m+2}, v_{3m-1}), (v_{3m+1}, v_{3m-1}), (v_{3m}, v_{3m-1}), (v_{3m-1}, v_{3m-4}), (v_{3m-2}, v_{3m-4}), (v_{3m-3}, v_{3m-4}), ..., (v_5, v_2), (v_4, v_2), (v_3, v_2), (v_2, v_1) receive consecutive numbers such as 3m + 3, 3m + 4, 3m + 5, …, 6m, 6m + 1, 6m + 2, 6m + 3 as labels.

Thus all the edges are labeled as {1, 2, 3, 4, …, 6m + 3} and all are distinct. (The above concept is illustrated in Fig. (1)).

Hence EDG (T_m), where m = 2n – 1; $n \in N$ is graceful.

SKOLEM–GRACEFUL LABELING FOR EDG (T_m)

In this section, we present an algorithm and prove the existence of Skolem-Graceful labeling for EDG (T_m).

Algorithm:

Input: EDG (T_m) with (6m + 4) vertices and (6m + 3) edges.

Step 1: Denote the (6m + 4) vertices as

$$V = \{v_1, v_2, \ldots, v_{3m+1}, v_{3m+2}, v_1, v_2, \ldots, v_{3m+1}, v_{3m+2}\}.$$

Step 2: If 'm' is even, then $m = 2n$; $n \in N$.

If $n \in N$, then the EDG (T_m) is of the form EDG (T_{2n}); $n \in N$.

Define $f : V \rightarrow \{1, 2, 3, \ldots, 6m + 4\}$ such that for

(i) $f(v_{6i+3}) = 3i + 2$, $f(v_{6i+4}) = 3i + 3$, $f(v_{6i+5}) = 3i + 4$

(ii) $f(v_{6i+6}) = 3(m - i) + 1$, $f(v_{6i+7}) = 3(m - i)$, $f(v_{6i+8}) = 3(m - i) - 1$

(iii) $f(v_{6i+3}) = 3(m + i) + 4$, $f(v_{6i+4}) = 3(m + i) + 5$, $f(v_{6i+5}) = 3(m + i) + 6$

(iv) $f(v_{6i+6}) = 3(2m - i) + 3$, $f(v_{6i+7}) = 3(2m - i) + 2$, $f(v_{6i+8}) = 3(2m + i) + 1$

and also $f(v_1) = 1$, $f(v_2) = 3m + 2$, $f(v_1) = 3m + 3$, $f(v_2) = 6m + 4$.

Step 3: If m is odd, then $m = 2n - 1$; $n \in N$.

The EDG (T_m) is of the form EDG (T_{2n-1}); $n \in N$.

Define $f : V \rightarrow \{1, 2, 3, \ldots, 6m + 4\}$ for the functions defined as in Step 2 for (i) to (iv) with limits $0 < i < \frac{m-1}{2}$, $0 < i < \frac{m-3}{2}$, $0 < i < \frac{m-1}{2}$ and $0 < i < \frac{m-3}{2}$ respectively.

Step 4: Define $f^* : E \rightarrow N$ such that

$$f^*(v_i, v_j) = \left| f(v_i) - f(v_j) \right|.$$

Output: Labeled EDG (T_m).

Theorem: The Extended Duplicate Graph of Twig (T_m), $m > 1$ is Skolem-graceful labeling.

Proof: Let EDG (T_m) be a Extended duplicate graph of the Twig (T_m). Clearly EDG (T_m) has $(6m + 4)$ vertices and $(6m + 3)$ edges.

Denote the set of vertices as

$$V = \{v_1, v_2, \ldots, v_{3m+1}, v_{3m+2}, v_1, v_2, \ldots, v_{3m+1}, v_{3m+2}\}.$$

Case (1): Let T_m be a Twig where $m = 2n$, $n \in N$.

Consider the Twig of the Type T_{2n}, $n \in N$.

In this case we get Twig graphs $T_2, T_4, T_6, T_8, \ldots$

To label the vertices of V, define a map $f : V \rightarrow \{1, 2, 3, \ldots, 6m + 4\}$ as given in Step 2 of the above algorithm.

The vertices v_1 and v_2 are labeled as 1 and $3m + 2$; the vertices (v_3, v_4, v_5), (v_9, v_{10}, v_{11}), $\ldots$, $(v_{3m-3}, v_{3m-2}, v_{3m-1})$, $(v_{3m+2}, v_{3m+1}, v_{3m})$,

$(v_{3m-4}, v_{3m-5}, v_{3m-6})$, ..., (v_8, v_7, v_6) receive consecutive numbers such as 2, 3, 4, …, 3m – 1, 3m, 3m + 1 as labels; the vertices v_1 and v_2 are labeled as 3m + 3 and 6m + 4; the vertices (v'_3, v'_4, v'_5), (v'_9, v'_{10}, v'_{11}), ..., $(v'_{3m-3}, v'_{3m-2}, v'_{3m-1})$, $(v'_{3m+2}, v'_{3m+1}, v'_{3m})$, $(v'_{3m-4}, v'_{3m-5}, v'_{3m-6})$, ..., (v'_8, v'_7, v'_6) receive consecutive numbers 3m + 4, 3m + 5, 3m + 6, …, 6m + 1, 6m + 2, 6m + 3 as labels.

Thus all the 6m + 4 vertices are labeled.

From the definition of EDG (T_m), the (6m + 3) edges of EDG (T_m) are of the form (v_2, v'_2), (v_1, v_2), (v'_2, v'_1), (v_{2+3i}, v_{3i+3+j}), (v'_{2+3i}, v'_{3i+3+j}) for $0 < j < 2$, $0 < i < m - 1$.

The induced function $f^* : E \to N$ is defined such that

$$f^*(v_i, v_j) = |f(v_i) - f(v_j)|.$$

Thus the edges (v_1, v_2), (v_2, v_3), (v_2, v_4), (v_2, v_5), (v_5, v_6), (v_5, v_7), (v_5, v_8), ..., (v_{3m-1}, v_{3m}), (v_{3m-1}, v_{3m+1}), (v_{3m-1}, v_{3m+2}) receive consecutive integers such as 1, 2, 3, 4, …, 3m – 1, 3m, 3m + 1 as labels; the edge (v_2, v'_2) receive the number 3m + 2 as labels; the edges (v'_{3m+2}, v'_{3m-1}), (v'_{3m+1}, v'_{3m-1}), (v'_{3m}, v'_{3m-1}), (v'_{3m-1}, v'_{3m-4}), (v'_{3m-2}, v'_{3m-4}), (v'_{3m-3}, v'_{3m-4}), ..., (v'_5, v'_2), (v'_4, v'_2), (v'_3, v'_2), (v'_2, v'_1) receive consecutive integers such as 3m + 3, 3m + 4, 3m + 5 …, 6m, 6m + 1, 6m + 2, 6m + 3 as labels.

Thus all the edges are labeled as (1, 2, 3, 4,…, 6m + 3) and all are distinct. (The above concept is illustrated in Fig. (4)).

Hence EDG (T_m) where m = 2n; $n \in N$ is Skolem-graceful.

Case (2): Let T_m be a Twig where m = 2n – 1, $n \in N$.

Consider the Twig of the Type T_{2n-1}, $n \in N$.

In this case we get Twig graphs T_1, T_3, T_5, T_7, ...

To label the vertices of V, define a map $f : V \to \{1, 2, 3, …, 6m + 4\}$ as given in Step 3 of the above algorithm.

The vertices v_1 and v_2 are labeled as 1 and 3m + 2; the vertices (v_3, v_4, v_5), (v_9, v_{10}, v_{11}), ..., $(v_{3m}, v_{3m+1}, v_{3m+2})$, $(v_{3m-1}, v_{3m-2}, v_{3m-3})$, ..., (v_8, v_7, v_6) receive consecutive integers such as 2, 3, 4, …, 3m – 1, 3m, 3m + 1 as labels; the vertices v'_1 and v'_2 are labeled as 3m + 3 and 6m + 4; the vertices (v'_3, v'_4, v'_5), (v'_9, v'_{10}, v'_{11}), ..., $(v'_{3m}, v'_{3m+1}, v'_{3m+2})$, $(v'_{3m-1}, v'_{3m-2}, v'_{3m-3})$, ..., (v'_8, v'_7, v'_6) receive consecutive integers such as 3m + 4, 3m + 5, 3m + 6, …, 6m + 1, 6m + 2, 6m + 3 as labels.

Thus all the 6m + 4 vertices are labeled.

From the definition of EDG (T_m), the (6m + 3) edges of EDG (T_m) are of the form (v_2, v_2), (v_1, v_2), (v_2, v_1), (v_{2+3i}, v_{3i+3+j}), (v_{2+3i}, v_{3i+3+j}) for $0 < j < 2$, $0 < i < m - 1$.

The induced function $f^* : E \rightarrow N$ is defined such that

$$f^*(v_i, v_j) = \left| f(v_i) - f(v_j) \right|.$$

Thus the edges (v_1, v_2), (v_2, v_3), (v_2, v_4), (v_2, v_5), (v_5, v_6), (v_5, v_7), (v_5, v_8), ..., (v_{3m-1}, v_{3m}), (v_{3m-1}, v_{3m+1}), (v_{3m-1}, v_{3m+2}) receive consecutive integers such as 1, 2, 3, 4, …, 3m – 1, 3m, 3m + 1 as labels; the edge (v_2, v_2) receive the number 3m + 2 as label; the edges (v_{3m+2}, v_{3m-1}), (v_{3m+1}, v_{3m-1}), (v_{3m}, v_{3m-1}), (v_{3m-1}, v_{3m-4}), (v_{3m-2}, v_{3m-4}), (v_{3m-3}, v_{3m-4}), ..., (v_5, v_2), (v_4, v_2), (v_3, v_2), (v_2, v_1) receive consecutive integers such as 3m + 3, 3m + 4, 3m + 5, …, 6m, 6m + 1, 6m + 2, 6m + 3 as labels.

Thus all the edges are labeled as (1, 2, 3, 4, …, 6m + 3) and all are distinct. (The whole idea of the proof is illustrated in Fig. (3)).

Hence EDG (T_m) where m = 2n – 1; $n \in N$ is Skolem-graceful.

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