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Dynamic Modeling and Forecasting Market Share of a Business Organization

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ABSTRACT

In poly-business environment a business organization is always keen to enhance its customer base by minimizing the switching over of loyalty of its existingcustomers to other businesses and offering incentives to attract the new ones. To understand this phenomenon, the market researchers, usually formulate it by static models, which in general, fail to capture the upheavals of the dynamics of business environment. To cope with this problem, a linear dynamic system model is proposed. This model , apart from, providing insight in to the existing business environment is expected to assist the decision makers to foresee short, medium and long term future of their business.

Key words: *Customers loyalty, Market share, Static Models, Linear Dynamic Market Share Model*

1. INTRODUCTION

The importance of forecasting in managerial decision making cannot be overlooked. It is through forecasting that management steps in to the future with confidence. Nearly every decision depends on some sort of forecasting for its resolution. For example, in the light of forecasts, the management of a business organization can decide whether to introduce a new product or stick to the old one and improve its market strategy to enhance its profit.

Since the introduction of Markov Chains and, in general, the Markov Processes by Andrey Markov (Markov A, 1906), his work is widely used in studying the evolution in transition of many systems over multiple and repeated trials in successive time periods. His concept led the statisticians to develop dynamic system and state space models, such as, that of Harrison-Akram (Harrison P. J. - Akram M, 1983) and Akram (Akram M, 1987).

In marketing, Markov chains are frequently used to describe consumers' behavior in relation to their loyalty towards a brand which in turn is used to determine market share of a business organization for planning of business strategies.

In past, some work has been done by some researchers, such as, Terui (Terui N, 1997), Kumara (Kumara V., 2002), Quagrainie (Quagrainie K. Kwamena, 2004) and Dura (Dura C, 2006).

Most of their work is related to the steady state probabilities, which is static in nature. Their work is useful, for systems whereby, interest is to look in to far flung future by making use of transition probabilities, where the probability of being in a particular state at any one time period depends only on the state in immediate preceding time period, without updating.

Today, we are living in an era of ever changing dynamic world, where, microprocessors have become part and parcel of our lives. The products based on microprocessors, such as, digital cameras, mobiles and computers, are being updated so fast that we can't wait for a long time to enjoy the fruit of new technologies. We wish to acquire them at the earliest possible.

From business point of view, specially to estimate and forecast market share of these products, therefore, we need dynamic system models as static models can't capture ups and downs of market share, as these models are not short sighted.

To cope with this problem, LDMS model is introduced. This model, dynamic in nature, is fully capable of forecasting short to long term market share of a product in an optimum manner.

2. LDMS MODEL

For observationsy$_1$ and y$_2$, at time t, on loyalty to the businesses B$_1$ and B$_2$respectively the LDMS (Linear Dynamic Market share model) is defined as follows.

$$y_t = F\theta_t + V_t$$
$$\theta_t = G\theta_{t-1} + W_t$$

Where:

$$y_t = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_t \text{ a Vector of Observations}$$

$$F = \begin{bmatrix} f_1 & f_2 & 0 & 0 \\ 0 & 0 & f_3 & f_4 \end{bmatrix} \textit{a Matrix of Known Functions}$$

$$\theta_t = \begin{pmatrix} \theta_{10} \\ \theta_{11} \\ \theta_{20} \\ \theta_{21} \end{pmatrix} \textit{(State Vector of Parameters)}$$

$$G = \begin{bmatrix} G_{11} & 0 \\ 0 & G_{22} \end{bmatrix} \text{ (Transition Matrix)}$$

$$G_{11} = \begin{bmatrix} 1 & 0 \\ 0 & \lambda_1 \end{bmatrix}$$

$$G_{22} = \begin{bmatrix} 1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

λ_1 and λ_2 are eigen values of the Loyalty Matrix

$$L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}$$

$$V = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix}\right]$$

$$w = \begin{pmatrix} w_{10} \\ w_{11} \\ w_{20} \\ w_{21} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} w_{10} & 0 & 0 & 0 \\ 0 & w_{11} & 0 & 0 \\ 0 & 0 & w_{20} & 0 \\ 0 & 0 & 0 & w_{21} \end{pmatrix}\right]$$

$$W_{10} = (1-\beta)\ (1 + \lambda_1)(\lambda_1 - \beta)V_t / (\lambda_1\beta)$$
$$W_{11} = (1-\beta)(\lambda_1 - \beta)(1-\lambda_1\beta)(\lambda_1^2 - \beta)\ V_t / \lambda_1\ \beta^2$$
$$W_{20} = (1-\beta)\ (1 + \lambda_2)(\lambda_2 - \beta)V_t / (\lambda_2\beta)$$
$$W_{21} = (1-\beta)(\lambda_2 - \beta)(1-\lambda_2\beta)(\lambda_2^2 - \beta)\ V_t / \lambda_2\ \beta^2$$

β is a smoothing coefficient, such that $\mathbf{0 < \beta < min}\ (\lambda_1^2, \lambda_2^2)$

3. ESTIMATION OF THE PARAMETERS OF THE MODEL

At time t-1, For known data $\mathbf{D_{t-1} = \{y_1,\ldots, y_{t-1}\}}$ the prior $(\boldsymbol{\theta_{t-1}}, \mid \mathbf{D_{t-1}}) \sim \mathbf{N[m_{t-1}; C_{t-1}]}$

the posterior $(\theta_t \mid D_t) \sim N[m_t; C_t]$ is obtained through the following recurrence equations.

$$\boxed{\begin{aligned} R_t &= G\ C_{t-1}\ G' + W_t \\ A_t &= R_t\ F\ [I + F\ R_t\ F']^{-1} \\ C_t &= [I - A_t F]\ R_t \\ m_t &= G\ m_{t-1} + A_t[y_t - F\ G\ m_{t-1}] \\ e_t &= y_t - F\ G\ m_t \end{aligned}}$$

Where at time t:

R is a system matrix

C is a Covariance matrix

A is an updating or gain vector

I is an identity matrix.

F and **G** are as defined earlier.

All vectors and matrices are assumed to be compatible in dimensions with their associated vectors and matrices of the system. Analogous to linear control theory these stochastic difference equations cluster themselves into an ensemble of a closed loop of linear system.

V_1 and V_2, the observation variances of market share, are found from the past observed data and used for computation of $W_1,\ldots, W_4$, the components of the W matrix.

For more discussion see Akram (Akram M ,1990 &1994).

4. FORECASTING THE MARKET SHARE

On the basis of the above updated estimate m_t of the parameter θ, the K = 1,2..., steps ahead forecasts $\Im_{t+k}$ of market share are generated through the forecast function:

$$\Im_t(k) = \Im_{t+k} = FG^k\ m_t$$

5. SPECIAL CASES

(i) Generally, for non seasonal and non cyclical data the components of the F- vector: $f_1 = f_2 = f_3 = f_4 = 1$ are considered.

(ii) The model is written in a diagonal form. However, if required, it may be transformed in to a canonical form using transformation procedure of Akram (Akram M , 1988).

6. PRACTICAL IMPLICATIONS

To apply the model and estimate its parameters the prior information is provided as follows.

$$m_o = \begin{pmatrix} m_{10} \\ m_{11} \\ m_{20} \\ m_{21} \end{pmatrix}$$

Where:

m_{10} and m_{11} are prior informationon the parameters θ_{10} and θ_{11}, the level and the growth state vector parameters of the model.

m_{20} and m_{21} are prior information on the parameters θ_{20} and θ_{21}, the level and the growth state vector parameters of the model.

$$\mathbf{C}_0 = \begin{pmatrix} c_{10} & 0 & 0 & 0 \\ 0 & c_{11} & 0 & 0 \\ 0 & 0 & c_{20} & 0 \\ 0 & 0 & 0 & c_{21} \end{pmatrix}$$

Where:

c_{10} and c_{11} are the prior information on the variances of the state vector parameters θ_{10} and θ_{11} of the model

c_{20} and c_{21} are the prior information on the variances of the state vector parameters θ_{20} and θ_{21} of the model

Here, it is assumed that the priors are known. If unknown then flat priors may be considered. In this case, affect of flat priors eliminated after about 30 iterations of the parameter updating mechanism. For more discussion see (Akram M, 1994).

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