Journal of Nonlinear Functional Analysis and Differential Equations
Vol. 2 No. 2 (July-December, 2017)

SENSITIVITY ANALYSIS FOR QUASIVARIATIONAL INCLUSION PROBLEMS BASED ON GENERALIZED MAXIMAL MONOTONICITY

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ABSTRACT: Sensitivity analysis for strongly monotone quasivariational inclusions based on *H-monotonicity* is discussed. *H-monotonicity* generalizes the well-known notion of maximal monotonicity, which is widely explored.

AMS Subject classification: 49J40, 47H05

Key words and phrases: Sensitive analysis, quasivariational inclusions, maximal monotone mapping, H - *monotone* mapping, generalized resolvent operator.

1. INTRODUCTION AND PRELIMINARIES

Just recently, the author [7] investigated, without using any of the differentiability assumptions on solution variables with respect to perturbation parameters by Tobin [6] and Kyparisis [4], sensitivity analysis for qusivariational inclusions involving strongly monotone mappings applying the resolvent operator technique. Variational inequality methods have been applied widely to problems arising from several fields of research, especially from model equilibria problems in economics, optimization and control theory, operations research, transportation network modeling, and mathematical programming while a considerable amount of effort to developing general methods for the sensitivity analysis for variational inequalities is made.

We intend in this paper to present the sensitivity analysis for *H-monotone* quasivariational inclusions based on the generalized resolvent operator technique. The notion of H *-monotone* mappings [2] generalizes the well-known class of maximal monotone mappings. The obtained results generalize the results on the sensitivity analysis for strongly monotone quasivariational inclusions [1, 7, 8] and others. For more details, we recommend [1 - 14].

2. H-MONOTONICITY

Fang and Huang [2] introduced the notion of *H-monotonicity* in the context of solving a system of variational inclusion problems based on the resolvent operator technique. The notion of the H - *monotonicity* generalizes the well-known concept of the maximal monotonicity.

Definition 1. [2] Let $H : X \to X$ be a nonlinear mapping on a Hilbert space X and let $M : X \to 2^X$ be a multivalued mapping on X. The map M is said to be *H - monotone* if:

(i) M is monotone.

(ii) $(H + \rho M)(X) = X$ for $\rho > 0$.

Proposition 1. [2] Let $H : X \to X$ be an *r-strongly* monotone singlevalued mapping and let $M : X \to 2^X$ be an *H-monotone* mapping. Then M is maximal monotone. The next property is helpful in shaping up the generalized resolvent operator, which is crucial to the main results on sensitivity analysis on hand.

Proposition 2. [2] Let $H : X \to X$ be an *r-strongly* monotone mapping and let $M : X \to 2^X$ be an *H-monotone* mapping. Then the operator $(H + \rho M)^{-1}$ is single-valued.

This leads to the generalized definition of the resolvent operator:

Definition 2. [2] Let $H : X \to X$ be an (r)-*strongly* monotone mapping and let $M : X \to 2^X$ be an *H - monotone* mapping. Then the generalized resolvent operator $J^M_{\rho,H} : X \to X$ is defined by

$$J^M_{\rho,H}(u) = (H + \rho M)^{-1}(u).$$

This reduces to resolvent operator J^M_ρ (when $H = I$) defined by

$$J^M_\rho(u) = (I + \rho M)^{-1}(u),$$

where I is the identity mapping.

3. MONOTONICITY/STRONG MONOTONICITY

In this section, we upgrade the notions of the monotonicity as well as strong monotonicity in the context of sensitivity analysis for nonlinear variational inclusion problems.

Definition 3. Let $A : X \to X$ and $T : X \times X \times L \to X$ be any two mappings. The map T is called:

(i) Monotone in the first argument if

$\langle T(x, u, \lambda) - T(y, u, \lambda), x - y\rangle \geq 0 \; \forall (x, y, u, \lambda) \in X \times X \times X \times L.$

(ii) (r)-*strongly* monotone in the first argument if there exists a positive constant r such that

$\langle T(x, u, \lambda) - T(y, u, \lambda), x-y\rangle \geq (r) \|x-y\|^2 \; \forall(x, y, u, \lambda) \in X \times X \times X \times L.$

(iii) (s)-*Lipschitz* continuous in the first argument if there exists a positive constant m such that

$\|T(x, u, \lambda) - T(y, u, \lambda)\| \leq (m) \|x-y\| \; \forall(x, y, u, \lambda) \in X \times X \times X \times AL$

Definition 4. The map $T : X \times X \times L \to X$ is said to be:

(i) Monotone with respect to A in the first argument if

$\langle T(x, u, \lambda) - T(y, u, \lambda), A(x) - A(y)\rangle \geq 0 \; \forall(x, y, u, \lambda) \in X \times X \times X \times L.$

(ii) (r)-*strongly* monotone with respect to A in the first argument if there exists a positive constant r such that

$$\langle T(x, u, \lambda) - T(y, u, \lambda), A(x) - A(y)\rangle \; (r) \|x - y\|^2$$

$\forall(x, y, u, \lambda) \in X \times X \times X \times L.$

4. MAIN RESULTS ON SENSITIVITY ANALYSIS

Let X denote a real Hilbert space with the norm $\| \,.\, \|$ and inner product $< .,. >$. Let $N: X \times X \times L \to X$ be a nonlinear mapping and $M : X \times X \times L \to 2^X$ be an H *-monotone* mapping with respect to first variable, where L is a nonempty open subset of X. Furthermore, let $q : X \to X$ be a nonlinear mapping such that $q(X) \cap D(M) \neq \phi$. Then the problem of finding an element $u \in X$ for a given element $f \in X$ such that

$$f \in N(q(u), q(u), \lambda) + M(q(u), q(u), \lambda), \tag{1}$$

where $\lambda \in L$ is the perturbation parameter, is called a class of generalized strongly monotone mixed quasivariational inclusion (abbreviated SMMQVI) problems.

For $q = I$ in (1), we arrive at: find an element $u \in X$ for a given element $f \in X$ such that

$$f \in N(u, u, \lambda) + M(u, u, \lambda), \tag{2}$$

where $\lambda \in L$ is the perturbation parameter. Next, another special case of the *SMMQV I* (1) problem is: for given element $f \in X$ determine an element $u \in X$ such that

$$f \in S(u, \lambda) + T(u, \lambda) + M(u, u, \lambda), \tag{3}$$

where $N(u, v, \lambda) = S(u, \lambda) + T(v, \lambda)$ for all $u, v \in X$, for $S, T : X \times L \to X$ any two nonlinear mappings, and q is the identity mapping. If $S = 0$ in (3), then (3) is equivalent to: find an element $u \in X$ such that

$$f \in T(u, \lambda) + M(u, u, \lambda). \tag{4}$$

The solvability of the *SMMQVI* problem (1) depends on the equivalence between (1) and the problem of finding the fixed point of the associated generalized resolvent operator.

Note that if M is *H-monotone,* then the corresponding generalized resolvent operator $J_{\rho,H}^{M}$ in first argument is defined by

$$J_{\rho,H}^{M(.,y,\lambda)}(u) = (H + \rho M(.,y))^{-1}(u) \forall u \in X, \tag{5}$$

where $\rho > 0$ and H is an (r) - *strongly* monotone mapping, which reduces to resolvent operator (for $H = \mathrm{I}$) as

$$J_{\rho}^{M(.,y)}(u) = (I + \rho M(.,y))^{-1}(u) \forall u \in X, \tag{6}$$

The following lemma upgrades a similar result from [2] :

Lemma 1. Let X be a real Hilbert space, let $H : X \to X$ be (r) - *strongly* monotone, and let $M : X \times X \times L \to 2^X$ be H- monotone in the first variable. Then the generalized resolvent operator associated with $M(., y, \lambda)$ for a fixed $y \in X$ and defined by

$$J_{\rho,H}^{M(.,y)}(u) = (H + \rho M(.,y))^{-1}(u)$$

$\forall u \in X,$ is $\left(\frac{1}{r}\right)$ – *Lipschitz* continuous, that is,

$$\| J_{\rho,H}^{M(.,y,\lambda)}(u) - J_{\rho,H}^{M(.,y,\lambda)}(v) \| \le \frac{1}{r} \| u - v \| \forall u, v \in X.$$

Proof. It follows from the definition of the generalized resolvent operator that

$$J_{\rho,H}^{M(.,y,\lambda)}(u) = (H + \rho M(.,y,\lambda))^{-1}(u) \forall u, v \in X,$$

$$J_{\rho,H}^{M(.,y,\lambda)}(v) = (H + \rho M(.,y,\lambda))^{-1}(v) \forall u, v \in X.$$

As a result, we have

$$\frac{1}{\rho}(u - H(J_{\rho,H}^{M(.,y,\lambda)}(u)) \in M(J_{\rho,H}^{M(.,y,\lambda)}(u)),$$

$$\frac{1}{\rho}(v - H(J_{\rho,H}^{M(.,y,\lambda)}(v)) \in M(J_{\rho,H}^{M(.,y,\lambda)}(v)).$$

Since M is monotone, it implies that

$$\frac{1}{\rho}\langle (u - H(J_{\rho,H}^{M(.,y,\lambda)}(u))$$

$$-(v - H(J_{\rho,H}^{M(.,y,\lambda)}(v))$$

$$,(J_{\rho,H}^{M(.,y,\lambda)}(u) - J_{\rho,H}^{M(.,y,\lambda)}(v)\rangle$$

$$= \frac{1}{\rho}\langle u - v(H(J_{\rho,H}^{M(.,y,\lambda)}(u) - H(J_{\rho,H}^{M(.,y,\lambda)}(v))$$

$$,(J_{\rho,H}^{M(.,y,\lambda)}(u) - J_{\rho,H}^{M(.,y,\lambda)}(v)\rangle \geq 0.$$

Therefore,

$$\| u - v \| \| J_{\rho,H}^{M(.,y,\lambda)}(u) - J_{\rho,H}^{M(.,y,\lambda)}(v) \|$$

$$\geq \langle u - v, J_{\rho,H}^{M(.,y,\lambda)}(u) - J_{\rho,H}^{M(.,y,\lambda)}(v)\rangle$$

$$\geq \langle H(J_{\rho,H}^{M(.,y,\lambda)}(u)) - H(J_{\rho,H}^{M(.,y,\lambda)}(v))$$

$$, J_{\rho,H}^{M(.,y,\lambda)}(u) - J_{\rho,H}^{M(.,y,\lambda)}(v)\rangle$$

$$\geq r \| J_{\rho,A}^{M(.,y,\lambda)}(u) - J_{\rho,A}^{M(.,y,\lambda)}(v) \|^2 .$$

Lemma 2. Let X be a real Hilbert space, let $H : X \to X$ be (r) - *strongly* monotone, and let $M : X \times X \times L \to 2^X$ be H- monotone in the first variable. Let $q : X \to X$ be any mapping such that $q(X) \cap D(M) \neq \phi$. Then the following statements are mutually equivalent:

(i) An element $u \in X$ is a solution to (1).

(ii) The map $G : X \to X$ defined by

$$q(x) = G(q(x), \lambda\ )$$

$$=: J_{\rho,H}^{M(.,q(x),\lambda)}(H(q(x)) - \rho N(q(x), q(x), \lambda) + \rho f)$$

has a fixed point $q(u) \in X$, where $J_{\rho,H}^{M(.,q(u),\lambda)} = (H + \rho M(., q(u), \lambda))^{-1}$ and $\rho > 0$.

Proof. The proof follows from the definition of the generalized resolvent operator.

Theorem 1. Let X be a real Hilbert space, let $H : X \to X$ be (r) - *strongly* monotone and (s)-*Lipschitz* continuous, and let $M : X \times X \times L \to 2^X$ be H - *monotone* in the first variable. Let $N : X \times X \times L \to X$ be (α)-*strongly* monotone (with respect to H) and (β)-*Lipschitz* continuous in the first variable, and let N be (μ) - *Lipschitz* continuous in the second variable. Furthermore, let $q : X \to X$ be such that $q(X) \cap D(M) \neq \phi$. If, in addition,

$$\| J_{\rho,H}^{M(.,q(u),\lambda)}(w) - J_{\rho,H}^{M(.,q(v),\lambda)}(w) \| \leq \eta \| q(u) - q(v) \| \, \forall (q(u), q(v), \lambda) \in X \times X \times L, \tag{7}$$

then

$$\| G(q(u),\lambda) - G(q(v),\lambda) \| \leq \theta \| q(u) - q(v) \| \, \forall (u, v, \lambda) \in X \times X \times L, \tag{8}$$

where

$$\theta = \frac{1}{r}[\sqrt{s^2 - \rho\alpha + \rho^2\beta^2} + \rho\mu] + \eta < 1,$$

$$\left| \rho - \frac{\alpha - r(1-\eta)\mu)}{\beta^2 - \mu^2} \right|$$

$$< \frac{\sqrt{(\alpha - r(1-\eta)\mu)^2 - (\beta^2 - \mu^2)(s^2 - r^2(1-\eta)^2)}}{\beta^2 - \mu^2}$$

$$\alpha > r(1-\eta)\mu r + \sqrt{(\beta^2 - \mu^2)[s^2 - r^2(1-\eta)^2]},$$

$$\beta > \mu,\ s < (1-\eta)r,\ \rho\mu < r(1-\eta),\ \eta < 1.$$

Consequently, for each $\lambda \in L$, the mapping $G(u, \lambda)$ in light of (7) has a unique fixed point $z(\lambda)$, and hence, $z(\lambda)$ is a unique solution to (1). Thus, we have

$$G(z(\lambda), \lambda) = z(\lambda).$$

When $H = I$ and $q = I$ in *Theorem*1, we arrive at:

Corollary 1. Let X be a real Hilbert space, and let $M : X \times X \times L \to 2^X$ be maximal monotone in the first variable. Let $N : X \times X \times L \to X$ be (α)-*strongly* monotone and (β)-*Lipschitz* continuous in the first variable, and let N be (μ) - *Lipschitz* continuous in the second variable. If

$$\| J_{\rho}^{M(.,u,\lambda)}(w) - J_{\rho}^{M(.,v,\lambda)}(w) \| \leq \eta \| u - v \| \, \forall (u, v, \lambda) \in X \times X \times L,$$

then

$$\| (G(u,\lambda) - G(v,\lambda) \| \leq \theta \| u - v \| \, \forall (u, v, \lambda) \in X \times X \times L, \tag{9}$$

where

$$\theta = \sqrt{1 - 2\rho\alpha + \rho^2\beta^2} + \rho\mu + \eta < 1,$$

$$\left| \rho - \frac{\alpha - (1-\eta)\mu}{\beta^2 - \mu^2} \right|$$

$$< \frac{\sqrt{(\alpha - (1-\eta)\mu)^2 - (\beta^2 - \mu^2)(2-\eta)\eta}}{\beta^2 - \mu^2}.$$

Therefore, for each $\lambda \in L,$ the mapping $G(u, \lambda)$ in light of (7) has a unique fixed point $z(\lambda)$, and hence, $z(\lambda)$ is a unique solution to (2). Thus, we have

$$G(z(l), \,) = z(l).$$

Proof of *Theorem*1. For any element $(u, v, \lambda) \in X \times X \times L,$ we have

$$G(q(u), \lambda) = J_{\rho,H}^{M(.,q(u),\lambda)}(H(q(u)) - \rho N(q(u), q(u), \lambda) + \rho f),$$

$$G(q(v), \lambda) = J_{\rho,H}^{M(.,q(v),\lambda)}(H(q(v)) - \rho N(q(v), q(v), \lambda) + \rho f).$$

It follows that

$$\begin{aligned}
\|G(q, \lambda) - G(q, \lambda)\| = \; & J_{\rho,H}^{M(.,q(u),\lambda)}(H(q(u)) - \rho N(q(u), q(u), \lambda) + \rho f) \\
& - J_{\rho,H}^{M(.,q(v),\lambda)}(H(q(v)) - \rho N(q(v), q(v), \lambda) + \rho f) \| \\
& \leq \| J_{\rho,A}^{M(.,q(u),\lambda)}(H(q(u)) - \rho N(q(u), q(u), \lambda) + \rho f) \\
& - J_{\rho,HA}^{M(.,q(u),\lambda)}(H(q(v)) - \rho N(q(v), q(v), \lambda) + \rho f) \| \\
& + \| J_{\rho,H}^{M(.,q(u),\lambda)}(H(q(v)) - \rho N(q(v), q(v), \lambda) + \rho f) \\
& - J_{\rho,H}^{M(.,q(v),\lambda)}(H(q(v)) - \rho N(q(v), q(v), \lambda) + \rho f) \|
\end{aligned}$$

$$\leq \frac{1}{r}\| H(q(u)) - H(q(v))$$

$$- \rho(N(q(u), q(u), \lambda) - N(q(v), q(v), \lambda)) \|$$

$$+ \eta \| q(u) - q(v) \|$$

$$\leq \frac{1}{r}[\| H(q(u)) - H(q(v))$$

$$- \rho(N(q(u), q(u), \lambda) - N(q(v), q(u), \lambda)) \|$$

$$+ \| \rho(N(q(v), q(u), \lambda) - N(q(v), q(v), \lambda)) \|]$$

$$+ \eta \| q(u) - q(v) \|.$$

The (r) – *strong* monotonicity and (β) – *Lipschitz* continuity of N in the first argument imply that

$$\begin{aligned} \|H(q(u)) - H(q(v)) &- \rho(N(q(u), q(u), \lambda) - N(q(v), q(u), \lambda))\|^2 \\ &= \|H(q(u)) - H(q(v))\|^2 - 2\rho\langle N(q(u), q(u), \lambda) \\ &- N(q(v), q(u), \lambda), H(q(u)) - H(q(v))\rangle \\ &+ \rho^2\|N(q(u), q(u), \lambda) - N(q(v), q(u), \lambda)\|^2 \\ &\leq (s^2 - 2\rho\alpha + \rho^2\beta^2)\|q(u) - q(v)\|^2. \end{aligned}$$

On the other hand, the (μ)–*Lipschitz* continuity of N in the second argument results

$$\|(N(q(v), q(u), \lambda)) - N(q(v), q(v), \lambda))\| \leq \mu\|q(u) - q(v)\|.$$

In light of above arguments, we infer

$$\|G(q(u), \lambda) - G(q(v), \lambda)\| \leq \theta\|q(u) - q(v)\|, \tag{10}$$

where

$$\theta = \frac{1}{r}[\sqrt{[s^2 - 2\rho\alpha + \rho^2\beta^2} + \rho\mu] + \eta.$$

Since $\theta < 1$, it concludes the proof.

Theorem 2. Let X be a real Hilbert space, let $H : X \to X$ be (r) - *strongly* monotone and (s)-*Lipschitz* continuous, and let $M : X \times X \times L \to 2^X$ be H - *monotone* in the first variable. Let $N : X \times X \times L \to X$ be (α)-*strongly* monotone (with respect to H) and (β)-*Lipschitz* continuous in the first variable, and let N be (μ) - *Lipschitz* continuous in the second variable. Furthermore, let $q : X \to X$ be such that $q(X) \cap D(M) \neq \phi$. If

$$\| J_{\rho,H}^{M(.,q(u),\lambda)}(w) - J_{\rho,H}^{M(.,q(v),\lambda)}(w) \| \leq \eta \| q(u) - q(v) \| \, \forall (u, v, \lambda) \in X \times X \times L,$$

then

$$\|G(q(u), \lambda) - G(q(v), \lambda)\| \leq \|q(u) - q(v)\| \, \forall (u, v, \lambda) \in X \times X \times L, \tag{11}$$

where

$$\theta = \frac{1}{r}[\sqrt{s^2 - 2\rho\alpha + \rho^2\beta^2} + \rho\mu] + \eta < 1,$$

$$\left| \rho - \frac{\alpha - r(1-\eta)r\mu}{\beta^2 - \mu^2} \right|$$

$$< \frac{\sqrt{(\alpha - r(1-\eta)\mu)^2 - (\beta^2 - \mu^2)(s^2 - r^2(1-\eta)^2)}}{\beta^2 - \mu^2},$$

$$\alpha > r(1-\eta)\mu r + \sqrt{(\beta^2 - \mu^2)[s^2 - r^2(1-\eta)^2]},$$

$$\beta > \mu,\ s < (1-\eta)r,\ \rho\mu < r(1-\eta),\ \eta < 1.$$

If the mappings $\lambda \to N(q(u), q(v), \lambda)$ and $\lambda \to J_{\rho,A}^{M(.,q(u),\lambda)}(w)$ both are continuous (or Lipschitz continuous) from L to X, then the solution $z(\lambda)$ of (1) is continuous (or Lipschitz continuous) from L to X.

Proof. From the hypotheses of the theorem, for any $\lambda, \lambda^* \in L$, we have

$$\begin{aligned}
& \|z(\lambda) - z(\lambda)\| \\
= \;& \|G(z(\lambda), \lambda) - G(z(\lambda^*), \lambda^*)\| \\
\leq \;& \|G(z(\lambda), \lambda) - G(z(\lambda^*), \lambda)\| + \|G(z(\lambda^*), \lambda) - G(z(\lambda^*), \lambda^*)\| \\
\leq \;& \|z(\lambda) - z(\lambda^*)\| + \|G(z(\lambda^*), \lambda) - G(z(\lambda^*), \lambda^*)\|.
\end{aligned}$$

It follows that

$$\begin{aligned}
& \|G(z(\lambda^*), \lambda) - G(z(\lambda^*), \lambda^*)\| \\
= \;& \| J_{\rho,H}^{M(.,z(\lambda^*),\lambda)}(H(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda))
\end{aligned}$$

$$- \; J_{\rho,H}^{M(.,z(\lambda^*),\lambda^*)}(H(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \,\|$$

$$\leq \; \| J_{\rho,H}^{M(.,z(\lambda^*),\lambda)}(H(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda))$$

$$- \; \| J_{\rho,H}^{M(.,z(\lambda^*),\lambda)}(H(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \,\|$$

$$+ \; J_{\rho,H}^{M(.,z(\lambda^*),\lambda)}(H(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*))$$

$$- \; J_{\rho,H}^{M(.,z(\lambda^*),\lambda^*)}(H(z(\lambda^*)) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \,\|$$

$$\leq \; \frac{\rho}{r} \| N(z(\lambda^*), z(\lambda^*), \lambda - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*) \,\|$$

$$+ \; J_{\rho,H}^{M(.,z(\lambda^*),\lambda)}(z(\lambda^*) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*))$$

$$- \; J_{\rho,H}^{M(.,z(\lambda^*),\lambda^*)}(z(\lambda^*) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \,\|$$

Hence, we have

$$\|z(\lambda) - z(\lambda^*)\|$$

$$\leq \; \frac{\rho}{r(1-\theta)} \| N(z(\lambda^*), z(\lambda^*), \lambda) - N(z(\lambda^*), z(\lambda^*), \lambda^* \,|$$

$$+ \; \frac{1}{1-\theta} \| J_{\rho,A}^{M(.,z(\lambda^*),\lambda)}(z(\lambda^*) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*))$$

$$- \; J_{\rho,A}^{M(.,z(\lambda^*),\lambda^*)}(z(\lambda^*) - \rho N(z(\lambda^*), z(\lambda^*), \lambda^*)) \,\|$$

This concludes the proof.

For $H = I$, $q = I$ in *Theorem*2, we have:

Corollary 2. Let X be a real Hilbert space, and let $M : X \times X \times L \to 2^X$ be maximal monotone in the first variable. Let $N : X \times X \times L \to X$ be (α)-*strongly* monotone and (β)-*Lipschitz* continuous in the first variable, and let N be (μ) - *Lipschitz* continuous in the second variable. If

$$\| J_{\rho,A}^{M(.,u,\lambda)}(w) - J_{\rho,A}^{M(.,v,\lambda)}(w) \| \leq \eta \| u - v \| \; \forall (u, v, \lambda) \in X \times X \times L,$$

then

$$\|G(u, \lambda) - G(v, \lambda)\| \leq \theta \|u - v\| \; \forall (u, v, \lambda) \in X \times X \times L, \tag{12}$$

where

$$\theta = \sqrt{1 - 2\rho\alpha + \rho^2\beta^2} + \rho\mu + \eta < 1,$$

$$\left| \rho - \frac{\alpha - (1-\eta)\mu}{\beta^2 - \mu^2} \right|$$

$$< \frac{\sqrt{(\alpha - (1-\eta)\mu)^2 - (\beta^2 - \mu^2)(1 - (1-\eta)^2)}}{\beta^2 - \mu^2}$$

$$\beta > \mu, \;\; \rho\mu < 1 - \eta, \; \eta < 1.$$

If the mappings $\lambda \to N(u, v, \lambda)$ and $\lambda \to J_{\rho,A}^{M(.,u,\lambda)}(w)$ both are continuous (or Lipschitz continuous) from L to X, then the solution $z(\lambda)$ of (1) is continuous (or Lipschitz continuous) from L to X.

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