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OSCILLATION OF SECOND ORDER DYNAMIC EQUATIONS ON TIME SCALES*

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ABSTRACT: In this paper, by using the Riccati transformation technique, some oscillatory criteria for the second-order dynamic equation on time scales are given.

Keywords: Second-order dynamic equation, time scale, oscillation, Riccati transformation technique.

AMS 2000 Subject Classification: 34K11; 34C10; 93C70.

1. INTRODUCTION

This paper concerns the oscillation of solutions to the following second order dynamic equation

$$(r(t)|x^{\Delta}(t)|^{\alpha-1}x^{\Delta}(t))^{\Delta} + p(t)|x^{\beta-1}(t-\tau)|x(t-\tau) = 0,\ t \in [t_0, \infty), \tag{1.1}$$

where $\beta \geq \alpha > 1$ are positive integers; τ is a positive constant such that the delay function $\tau(t) = t - \tau < t$, $\tau(t) : \mathbb{T} \to \mathbb{T}$; r, p are positive, real-valued *rd*-continuous functions defined on the time scales interval $[t_0, \infty)$ and

$$\int_{t_0}^{\infty} \frac{1}{r^{\frac{1}{\alpha}}(t)} \Delta t = \infty. \tag{1.2}$$

Recall that a solution of (1.1) is a nontrivial real-valued function $x(t)$ such that $x(t) \in C_{rd}^1[t_x, \infty)$, and $r(t)|x^{\Delta}(t)|^{\alpha-1}x^{\Delta}(t) \in C_{rd}^1[t_x, \infty)$ for $t_x \geq t_0$ and satisfying Eq. (1.1) for $t \geq t_x$.

We restrict our attention to solutions $x(t)$ of Eq. (1.1) which exist on some half-line $[t_x, \infty)$ and nontrivial for all large t, that is, satisfy $\sup\{|x(t)|, t \geq t_1\} > 0$ for any $t_1 \geq t_x$. It is tacitly assumed that such solutions exist.

As is customary, the solution $x(t) \in C_{rd}^1[t_x, \infty)$ of Eq. (1.1) is said to be eventually positive (negative) if there exists a sufficiently large positive number t_μ such that the

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inequality $x(t) > 0$ ($x(t) < 0$) holds for $t \geq t_\mu$. A solution $x(t)$ of Eq. (1.1) is said to be oscillatory if it is neither eventually positive nor eventually negative, otherwise it is nonoscillatory. Eq. (1.1) is said to be oscillatory if all its solutions are oscillatory.

In 1988, Hilger [1] introduced the theory of time scales in order to unify continuous and discrete analysis. Bohner and Peterson [2] summarizes and organizes much of time scales calculus. From then on, there has been much research activity concerning the oscillation and nonoscillation of solutions of different dynamic equations on time scales, we refer the reader to the papers [3-7].

When $\alpha = \beta$, $\tau(t) = t$, Eq. (1.1) reduces to second order half-linear dynamic equation

$$(r(t)(x^\Delta(t))^\alpha)^\Delta + p(t)x^\alpha(t) = 0,\ t \in [t_0, \infty), \tag{1.3}$$

which has been discussed in the literature [7].

We note if $\mathbb{T} = \mathbb{R}$, then $\sigma(t) = t$, $\mu(t) = 0$, $f^\Delta(t) = f'(t)$ and (1.1) becomes the second order differential equation

$$(r(t)|x'(t)|^{\alpha-1}x'(t))' + p(t)|x^{\beta-1}(\tau(t))\ |x(\tau(t))| = 0,\ t \in [t_0, \infty). \tag{1.4}$$

In 1997, Agarwal *et al.* [6] showed the oscillatory behavior of the retarded differential equation (1.4).

If $\mathbb{T} = \mathbb{Z}$, then $\sigma(t) = t + 1$, $\mu(t) = 1$,

$$y^\Delta(t) = \Delta y(t) = y(t+1) - y(t),$$

and (1.1) becomes the second order difference equation

$$\Delta(r(t)(\Delta(|x(t)|))^{\alpha-1}\Delta(x(t))) + p(t)|x(t)^{\beta-1}x(t).$$

2. SOME PRELIMINARIES

Since we are interested in oscillatory behavior, we suppose that the time scale under consideration is not bounded above, i.e., it is a time scale interval of the form $[t_0; \infty)$. We define the forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ by

$$\sigma(t) = \inf\{s \in \mathbb{T} | s > t\},\ \rho(t) = \sup\{s \in \mathbb{T} | s < t\}. \tag{2.1}$$

In this definition we put $\inf \phi = \sup \mathbb{T}$ (i.e., $\sigma(t) = t$ if $\mathbb{T}$ has a maximum t) and $\sup \phi = \inf \mathbb{T}$ (i.e., $\rho(t) = t$ if $\mathbb{T}$ has a minimum t), where ϕ denotes the empty set. If $\sigma(t) > t$, we say that t is right-scattered, while if $\rho(t) < t$ we say that t is left-scattered. Points that are right-scattered and left-scattered at the same time are called isolated. Also, if $t < \sup \mathbb{T}$ and $\rho(t) = t$, then t is called right-dense, and if $t > \inf \mathbb{T}$ and $\rho(t) =$

t, then t is called left-dense. Points that are right-dense and left-dence at the same time are called dence. Finally, the graininess function $\mu(t) : \mathbb{T} \to [0, \infty)$ is defined by

$$\mu(t) := \sigma(t) - t.$$

A function $f : \mathbb{T} \to \mathbb{R}$ is said to be rd-continuous if it is continuous at each right-dense point and if there exist a finite left limit in all left-dense points. The derivative (dealta) f^{Δ} of f is defined by

$$f^{\Delta}(t) = \frac{f(\sigma(t)) - f(t)}{\sigma(t) - t}. \tag{2.2}$$

if f is continuous at t and t is right-scattered. If t is not right-scattered then the derivative is defined by

$$f^{\Delta}(t) = \lim_{s \to t} \frac{f(t) - f(s)}{t - s}, \tag{2.3}$$

provided this limit exists. The derivative and the forward jump operator are related by the useful formula

$$f^{\sigma} = f + \mu f^{\Delta}, \text{ where } f^{\sigma} = f \circ \sigma \tag{2.4}$$

We will also make use of the following product and quotient rules for the derivative of the product fg and the quotient f/g (where $gg^{\sigma} \neq 0$) of two differentiable functions f and g

$$(fg)^{\Delta} = f^{\Delta} g + f^{\sigma} g^{\Delta} \text{ and } \left(\frac{f}{g}\right)^{\Delta} = \frac{f^{\Delta} g - f g^{\Delta}}{g g^{\sigma}}. \tag{2.5}$$

For $a, b \in \mathbb{T}$ and a differentiable function f, the Cauchy integral of f^{Δ} is defined by

$$\int_a^b f^{\Delta}(s) \Delta s = f(b) - f(a) \tag{2.6}$$

The integration by parts formula reads

$$\int_a^b f^{\Delta}(s) g(s) \Delta s = \left[f(s) g(s)\right]_a^b - \int_a^b f^{\sigma}(s) g^{\Delta}(s) \Delta s \tag{2.7}$$

and infinite integrals are defined as

$$\int_a^{\infty} f^{\Delta}(s) \Delta s = \lim_{t \to \infty} \int_a^t f(s) \Delta s. \tag{2.8}$$

3 MAIN RESULTS

In this section, we give some oscillation criteria for (1.1). We suppose that the time scale under consideration is a time scale interval of the form $[t_0, \infty)$. Also, we will use the following formula

$$(x^{\Delta}(t-\tau))^{\Delta} > x^{\beta-1}(t-\tau)x^{\Delta}(t-\tau).$$

which can been obtained by using the definition of derivatives on time scales.

Lemma 3.1 [7]: If A and B are positive constants, then

$$A^{\lambda} + (\lambda - 1)B^{\lambda} - \lambda B^{\lambda-1} \geq 0, \lambda > 1. \tag{3.1}$$

Theorem 3.1: Suppose that (1.2) holds. Furthermore, assume that there exists a positive Δ-differentiable function $\rho(t)$ such that

$$\limsup_{t\to\infty}\int_a^t\left[\rho(s)p(s)-\frac{\alpha^{\alpha}}{(\alpha+1)^{\alpha+1}}\frac{r(s)\left(\rho^{\Delta}\right)^{\alpha+1}}{\rho^{\alpha}(s)}\right]\Delta s=\infty, \tag{3.2}$$

Then every solution of Eq. (1.1) is oscillatory on $[t_0, \infty)$.

Proof: Suppose that $x(t)$ is a nonoscillatory solution of Eq.(1.1). Without loss of generality, let $x(t - \tau)$ be an eventually positive solution($x(t - \tau)$ is an eventually negative solution can be similarly proved). In view of (1.1), we have

$$(r(t)|x^{\Delta}(t)|^{\alpha-1}x^{\Delta}(t))^{\Delta} = -p(t)|x^{\beta-1}(t-\tau)|\, x(t-\tau) < 0, \tag{3.3}$$

for all $t \geq t_0$, and so $\{r(t)|x^{\Delta}(t)|^{\alpha-1}x^{\Delta}(t)\}$ is an eventually decreasing function. We first show that $\{r(t)|x^{\Delta}(t)|^{\alpha-1}x^{\Delta}(t)\}$ is eventually nonnegative. Since $r(t)$ is a positive function, it implies $x^{\Delta}(t)$ is eventually nonnegative. Indeed, since $p(t)$ is a positive function, the decreasing function $\{r(t)|x^{\Delta}(t)|^{\alpha-1}x^{\Delta}(t)\}$ is either eventually positive or eventually negative. Suppose there exists an integer $t_1 \geq t_0$ such that

$$r(t_1)|x^{\Delta}(t_1)|^{\alpha-1}x^{\Delta}(t_1) = c < 0,$$

then from (3.3) we have $r(t)|x^{\Delta}(t)|^{\alpha-1}x^{\Delta}(t) < r(t_1)|x^{\Delta}(t_1)|^{\alpha-1}x^{\Delta}(t_1) = c < 0$ for $t \geq t_1$, hence

$$x^{\Delta}(t)\leq c^{\frac{1}{\alpha}}\left(\frac{1}{r(t)}\right)^{\frac{1}{\alpha}}$$

Using (2.6) and (1.2), we have

$$x(t) \le x(t_1) + c^{\frac{1}{\alpha}} \int_{t_1}^{t} \left(\frac{1}{r(s)} \right)^{\frac{1}{\alpha}} \Delta s \to -\infty \text{ as } t \to \infty \tag{3.4}$$

which contradicts the fact that $x(t) > 0$ for all $t \ge t_0$. Hence $\{r(t)|x^\Delta(t)|^{\alpha-1}x^\Delta(t)\}$ is eventually nonnegative. Therefore, we see that there is some t_0 such that

$$x(t) > 0,\ x^\Delta(t) \ge 0,\ (r(t)|x^\Delta(t)|^{\alpha-1}x^\Delta(t))^\Delta < 0,\ t \ge t_0. \tag{3.5}$$

Defining the function $w(t)$ by

$$w(t) = \frac{\rho(t) r(t) \left| x^\Delta(t) \right|^{\alpha-1} x^\Delta(t)}{\left| x^{\beta-1}(t-\tau) \right| x(t-\tau)},\ t \ge t_0.$$

Then $w(t) > 0$, and using (2.5) we obtain

$$\begin{aligned}
w^\Delta(t) &= \frac{\rho(t)}{x^\beta(t-\tau)} \left(r\left(x^\Delta\right)^\alpha \right)^\Delta + \left(r\left(x^\Delta\right)^\alpha \right)^\sigma \left(\frac{\rho(t)}{x^\beta(t-\tau)} \right)^\Delta \\
&= \frac{\rho(t)}{x^\beta(t-\tau)} \left(r\left(x^\Delta\right)^\alpha \right)^\Delta + \left(r\left(x^\Delta\right)^\alpha \right)^\sigma \left(\frac{x^\beta(t-\tau)\rho^\Delta(t) - \rho(t)\left(x^\beta(t-\tau)\right)^\Delta}{x^\beta(t-\tau)x^\beta(\sigma(t)-\tau)} \right) \\
&= -\rho(t)p(t) + \frac{\rho^\Delta(t)}{\rho^\sigma} w^\sigma - \frac{\rho(t) w^\sigma \left(x^\beta(t-\tau)\right)^\Delta}{\rho^\sigma x^\beta(t-\tau)}
\end{aligned} \tag{3.6}$$

In view of (2.2), we get

$$\begin{aligned}
(x^\beta(t-\tau))^\Delta &= \frac{x^\beta(\sigma(t-\tau)) - x^\beta(t-\tau)}{\sigma(t-\tau) - (t-\tau)} \\
&\ge \frac{\beta x^{\beta-1}(t-\tau)}{\sigma(t-\tau)-(t-\tau)} \left(x(\sigma(t-\tau)) - x(t-\tau) \right) \\
&= \beta x^{\beta-1}(t-\tau) x^\Delta(t-\tau) \\
&> x^{\beta-1}(t-\tau) x^\Delta(t-\tau).
\end{aligned} \tag{3.7}$$

It follows from (3.6) and (3.7), we obtain

$$w^{\Delta}(t) < -\rho(t)p(t) + \frac{\rho^{\Delta}(t)}{\rho^{\sigma}}w^{\sigma} - \frac{\rho(t)w^{\sigma}\left(x^{\Delta}(t-\tau)\right)}{\rho^{\sigma}(t-\tau)}. \tag{3.8}$$

Since

$$\left(\frac{r(t)\left(x^{\Delta}(t)\right)^{\alpha}}{x^{\beta}(t-\tau)}\right)^{\sigma} \leq \frac{r(t)\left(x^{\Delta}(t)\right)^{\alpha}}{x^{\beta}(t-\tau)} \leq \frac{r(t)\left(x^{\Delta}(t)\right)^{\alpha}}{x^{\alpha}(t-\tau)},$$

we have

$$\left(\frac{r(\sigma(t))\left(x^{\Delta}(\sigma(t))\right)^{\alpha}}{x^{\beta}(\sigma(t)-\tau)}\right)^{\frac{1}{\alpha}} \leq \left(\frac{r(t)\left(x^{\Delta}(t)\right)^{\alpha}}{x^{\beta}(t-\tau)}\right)^{\frac{1}{\alpha}} \leq \left(\frac{r(t)\left(x^{\Delta}(t)\right)^{\alpha}}{x^{\alpha}(t-\tau)}\right)^{\frac{1}{\alpha}}$$

i.e.

$$\left(\frac{r(\sigma(t))\left(x^{\Delta}(\sigma(t))\right)^{\alpha}}{x^{\beta}(\sigma(t)-\tau)}\right)^{\frac{1}{\alpha}} \leq \left(\frac{r(t)\left(x^{\Delta}(t)\right)^{\alpha}}{x^{\beta}(t-\tau)}\right)^{\frac{1}{\alpha}} \leq \frac{r^{\frac{1}{\alpha}}(t)x^{\Delta}(t)}{x(t-\tau)}. \tag{3.9}$$

Substituting (3.9) in (3.8), we find that

$$\begin{aligned} w^{\Delta}(t) &\leq -\rho(t)p(t) + \frac{\rho^{\Delta}(t)}{\rho^{\sigma}}w^{\sigma} - \left(\frac{r(\sigma(t))\left(x^{\Delta}(\sigma(t))\right)^{\alpha}}{x^{\beta}(\sigma(t)-\tau)}\right)^{\frac{1}{\alpha}} \frac{\rho(t)w^{\sigma}}{\rho^{\sigma}r^{\frac{1}{\alpha}}(t)} \\ &= -\rho(t)p(t) + \frac{\rho^{\Delta}(t)}{\rho^{\sigma}}w^{\sigma} - \frac{\rho(t)}{\left(\rho^{\sigma}\right)^{\lambda} r^{\frac{1}{\alpha}}(t)}\left(w^{\sigma}\right)^{\lambda}, \end{aligned} \tag{3.10}$$

where $\lambda = \dfrac{\alpha+1}{\alpha}$. Set

$$A = \left[\frac{\rho(t)}{(\rho^\sigma)^\lambda r^{\lambda-1}(t)} \right]^{\frac{1}{\lambda}} w^\sigma,\ B = \left[\frac{\rho^\Delta(t)}{\lambda \rho^\sigma} \left(\frac{\rho(t)}{(\rho^\sigma)^\lambda r^{\lambda-1}(t)} \right)^{\frac{-1}{\lambda}} \right]^{\frac{1}{\lambda-1}}$$

Using inequality (3.1), we have

$$\frac{\rho^\Delta(t)}{\rho^\sigma} w^\sigma - \frac{\rho(t)}{(\rho^\sigma)^\lambda r^{\lambda-1}(t)} (w^\sigma)^\lambda \le (\lambda-1)\lambda^{\frac{-\lambda}{\lambda-1}} \left(\frac{\rho^\Delta(t)}{\rho^\sigma} \right)^{\frac{\lambda}{\lambda-1}} \left(\frac{\rho(t)}{(\rho^\sigma)^\lambda r^{\lambda-1}(t)} \right)^{\frac{-1}{\lambda-1}}$$

$$= \frac{\alpha^\alpha}{(\alpha+1)^{\alpha+1}} \frac{r(t)(\rho^\Delta)^{\alpha+1}}{\rho^\alpha(t)} \tag{3.11}$$

Thus, we obtain

$$w^\Delta(t) \le -\left[\rho(t) p(t) - \frac{\alpha^\alpha}{(\alpha+1)^{\alpha+1}} \frac{r(t)(\rho^\Delta)^{\alpha+1}}{\rho^\alpha(t)} \right] \tag{3.12}$$

Integrating (3.12) from t_0 to t, we get

$$-w(t_0) \le w(t) - w(t_0) \le -\int_{t_0}^{t} \left[\rho(s) p(s) - \frac{\alpha^\alpha}{(\alpha+1)^{\alpha+1}} \frac{r(s)(\rho^\Delta)^{\alpha+1}}{\rho^\alpha(s)} \right] \Delta s \tag{3.13}$$

which yields

$$\int_{t_0}^{t} \left[\rho(s) p(s) - \frac{\alpha^\alpha}{(\alpha+1)^{\alpha+1}} \frac{r(s)(\rho^\Delta)^{\alpha+1}}{\rho^\alpha(s)} \right] \Delta s \le w(t_0)$$

for all large t. This is contrary to (3.2). The proof is complete.

From Theorem 3.1, we can obtain different conditions for oscillation of all solutions of (1.1) by different choices of $\rho(t)$. For instance, let $\rho(t) = t$, $t \ge t_0$. By Theorem 3.1, we have the following results.

Corollary 3.1: Assume that

$$\limsup_{t\to\infty}\int_a^t\left[sp(s)-\frac{\alpha^\alpha}{(\alpha+1)^{\alpha+1}}\frac{r(s)}{s^\alpha}\right]\Delta s=\infty, \tag{3.14}$$

Then every solution of Eq. (1.1) is oscillatory on $[t_0, \infty)$.

Let $\rho(t) = 1$, $t \geq t_0$. Now Theorem 3.1 yields the following well-known result (Leighton-Wintner theorem).

Corollary 3.2: If

$$\int_a^\infty p(s)\Delta s=\infty, \tag{3.15}$$

Then every solution of Eq. (1.1) is oscillatory on $[t_0, \infty)$.

Remark 3.1: From Theorem 3.1, we can give sufficient conditions for oscillations of (1.1) on different type of time scales, for example, we can deduce that

$$\int_{t_0}^\infty\frac{1}{r^\alpha(s)}ds=\infty \text{ and } \limsup_{t\to\infty}\int_{t_0}^t\left[\rho(s)p(s)-\frac{\alpha^\alpha}{(\alpha+1)^{\alpha+1}}\frac{r(s)(\rho')^{\alpha+1}}{\rho^\alpha(s)}\right]ds=\infty$$

are sufficient conditions for oscillation of second order differential equation

$$(r(t)|x'(t)|^{\alpha-1}x'(t))' + p(t)|x^{\beta-1}(t-\tau)|x(t-\tau) = 0,\ t \in [t_0, \infty),$$

$$\sum_{i=n_0}^\infty\left[\frac{1}{r^\alpha(i)}\right]=\infty \text{ and } \limsup_{n\to\infty}\sum_{i=n_0}^{n-1}\left[\rho(i)p(i)-\frac{\alpha^\alpha}{(\alpha+1)^{\alpha+1}}\frac{r(i)(\Delta\rho(i))^{\alpha+1}}{\rho^\alpha(i)}\right]=\infty$$

are sufficient conditions for oscillation of the second order difference equation

$$\Delta(r(n)(\Delta x(n))^\alpha) + p(n)x^\beta(n-\tau) = 0,\ n \in [n_0, \infty).$$

$$\sum_{i=\frac{n_0}{h}}^\infty\left[\frac{1}{r^\alpha(i)}\right]=\infty \text{ and } \limsup_{n\to\infty}\sum_{i=\frac{n_0}{h}}^{\frac{n}{n-1}}\left[\rho(i)p(i)-\frac{\alpha^\alpha}{(\alpha+1)^{\alpha+1}}\frac{r(i)(\Delta_h\rho(i))^{\alpha+1}}{\rho^\alpha(i)}\right]=\infty$$

are sufficient conditions for oscillation of the general second order difference equation

$$\Delta_h(r(n)(\Delta_h x(n))^\alpha) + p(n)x^\beta(n-\tau) = 0,\ n \in [n_0, \infty).$$

Theorem 3.2: Assume that (1.2) holds. Furthermore, let $\rho(t)$ be defined in Theorem 3.1. If

$$\limsup_{t\to\infty}\frac{1}{t^m}\int_a^t (t-s)^m\left[\rho(s)p(s)-\frac{\alpha^\alpha}{(\alpha+1)^{\alpha+1}}\frac{r(s)\left(\rho^\Delta\right)^{\alpha+1}}{\rho^\alpha(s)}\right]\Delta s=\infty, \quad (3.16)$$

for an odd positive integer m. Then every solution of Eq. (1.1) is oscillatory on $[t_0, \infty)$.

The proof is similar to that of the proof of Theorem 3.1 by using the inequality (3.12) and hence is omitted.

Remark 3.2: Note that when $\rho(t) = 1$, we get

$$\lim_{t\to\infty}\frac{1}{t^m}\int_a^t (t-s)^m\, p(s)\Delta s=\infty,$$

Then, (3.12) can be considered as the extension of Kamenev-type oscillation criteria for second order differential equations.

Also, from Theorem 3.2, we have the similar oscillation criteria for the equations as mentioned.

Example 3.1: Consider the following dynamic equation

$$(|x^\Delta(t)|^3 x^\Delta(t))^\Delta + t|x^4(t-\tau)|x(t-2) = 0,\ t \in [1, \infty), \quad (3.17)$$

where $\mathbb{T} = [1, \infty)$ is a time scale (where all the points are right scattered), and $\tau = 2$. In (3.17), $\alpha = 4$, $\beta = 5$, $r(t) = 1$, $p(t) = t$. Then, by Corollary 3.1, we have

$$\limsup_{t\to\infty}\int_1^t\left[sp(s)-\frac{\alpha^\alpha}{(\alpha+1)^{\alpha+1}}\frac{r(s)}{s^\alpha}\right]\Delta s=\limsup_{t\to\infty}\int_1^t\left[s^2-\frac{4^4}{5^5}\frac{1}{s^4}\right]\Delta s=\infty.$$

Therefore (3.17) is oscillatory.

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