Mathematical Methods, Physical Methods and Simulation Science and Technology
Vol. 3 (January-December, 2019)

Chaos Control for Periodically Forced Complex Duffing's System Based on Fuzzy Model

XIN GAO & XINGWEN LIU

Received: 23rd July 2018 Revised: 14th November 2018 Accepted: 15th December 2018

In this paper, a new chaotic control method for periodically forced complex Duffing's system is proposed by using Takagi-Sugeno fuzzy model to represent original chaotic system and adapting parallel distributed compensation (PDC) scheme. The result is represented by means of linear matrix inequalities (LMIs), thus designing controller becomes very convenient. Numerical simulations are given to illuminate the correctness of the theoretical result.

1. INTRODUCTION

In recent years, control of chaotic systems has been more and more interesting to related researchers since the pioneering work of Ott *et al.* [1]. Generally speaking, there are two ways to control chaos: feedback control methods and non-feedback control methods. Many Feedback control methods [2-6] are used to control chaos by stabilizing a desired unstable periodic solution, which is embedded in a chaotic attractor, and non-feedback methods [7-10] suppress chaotic behaviours by applying weak periodic perturbation to some control parameters or variables. In the past few years, there has been a significant discovery that a continuous nonlinear system can be well approximated by a Takagi-Sugeno (T-S) fuzzy linear model. Therefore, interest in fuzzy control for nonlinear systems is rapidly growing based on the T-S model [11-13]. Since chaotic systems are a class of typically nonlinear systems, there have existed many studies concerning the control and synchronization for the chaotic systems based on T-S fuzzy model [14-18].

Chaotic behaviour and chaos control for complex nonlinear systems have constantly been studied [8,10,19-22]. This paper considers the problem of controlling chaos of the complex Duffing's system with periodically forcing excitations [8]. We propose a new method to control the periodically forced complex system via nonlinear feedback controller using Lyapunov stability theory. First, the periodically forced complex Duffing's system is represented by T-S fuzzy model. Then, a new fuzzy control criterion is proposed in terms of linear matrix inequalities (LMIs), which are used here to achieve the parameters of controller. Finally, simulation results show that the state feedback controller designed can stabilize the periodically forced complex Duffing's chaotic system to its equilibrium.

2. FUZZY MODEL OF THE PERIODICALLY FORCED COMPLEX DUFFING'S SYSTEM

The Duffing's system with negative linear stiffness, damping and periodic excitation is often written in the form

$$\ddot{x} - x + \alpha\dot{x} + x^3 = \delta\cos\omega t \tag{1}$$

Mahmoud etc.[8] extended the Duffing's system to the complex domain in order to study strange attractors, chaotic behaviour and the problem of chaos control. The periodically forced complex Duffing's system of the form

$$\ddot{z} - z + \alpha\dot{z} + \varepsilon z|z|^2 = \gamma'\cos\omega t \tag{2}$$

where $\gamma' = \sqrt{2}\gamma\exp(i\pi/4), \gamma, \alpha, \omega, \varepsilon$ are constant parameters, $z = x + iy$ is a complex function. Eq (2) can be reduced to the famous Duffing's system (1) when $z = x$ ($y = 0, \gamma'$ is real) and ε =1. We substitute $z = x + iy$ into Eq (2), and let $x_1 = x, x_2 = \dot{x}, x_3 = y, x_4 = \dot{y}$, (2) can be written as the following model:

$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= x_1 - \alpha x_2 - \varepsilon x_1(x_1^2 + x_3^2) + \gamma\cos\omega t, \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= x_3 - \alpha x_4 - \varepsilon x_3(x_1^2 + x_3^2) + \gamma\cos\omega t,
\end{aligned} \tag{3}$$

when $\alpha = 0.13, \varepsilon = 1, \gamma = 0.18$ system (1) exhibits chaotic activity, which is shown in Fig.1.

In this section, we will construct the T-S fuzzy model of (3) using the method introduced by [13]. (3) can be rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1-\varepsilon(x_1^2+x_3^2) & -\alpha & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1-\varepsilon(x_1^2+x_3^2) & -\alpha \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \gamma\cos\omega t \\ 0 \\ \gamma\cos\omega t \end{bmatrix}. \tag{4}$$

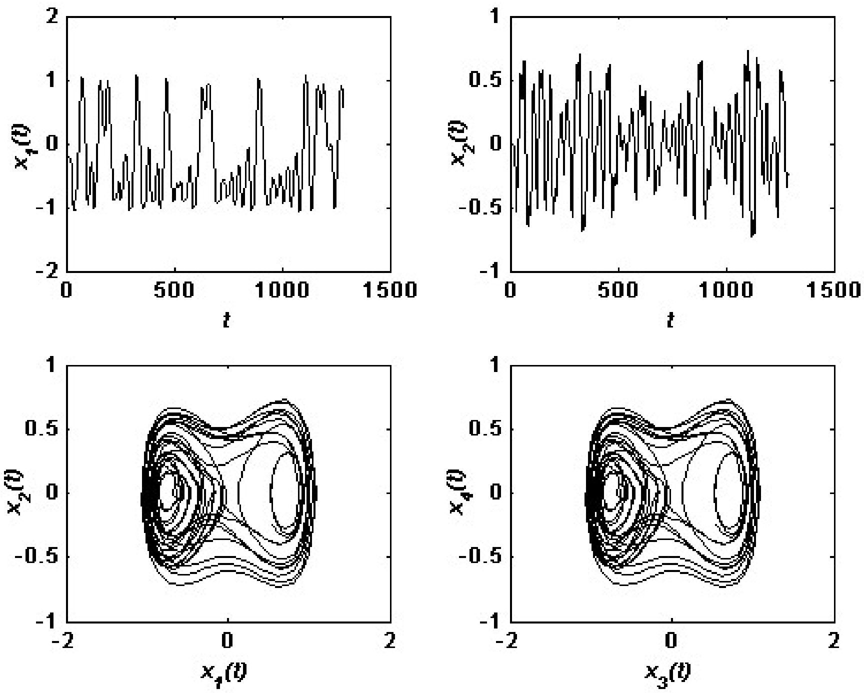


Figure 1. Periodically Forced Complex Duffing System's Attractor

Define $z(t) = 1 - \varepsilon(x_1^2 + x_3^2),$ and assume that $x_1 \in [-d, d],$ where *d* is a constant and *d* = 2 in this paper. It follows that

$$\max_{x_1, x_3} z(t) = 1, \quad \min_{x_1, x_3} z(t) = 1 - 2\varepsilon d^2,$$

Let $z(t) = M_1(z(t)) \times 1 + M_2(z(t)) \times (1 - 2\varepsilon d^2)$ and $M_1(z(t)) + M_2(z(t)) = 1,$ we have

$$M_1(z(t)) = 1 - \frac{1 - z(t)}{2\varepsilon d^2}, \quad M_2(z(t)) = \frac{1 - z(t)}{2\varepsilon d^2}.$$

Thus we can construct the following fuzzy model of (1):

Rule $i\ (i = 1, 2)$: IF $z(t)$ is M_i, THEN $\dot{x}(t) = A_i\, x(t) + v(t)$, where $x = \left[x_1, x_2, x_3, x_4\right]^T$, $v(t)$ = $\left[0, \gamma \cos \omega t, 0, \gamma \cos \omega t\right]^T$, and

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & -\alpha & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -\alpha \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1-2\varepsilon d^2 & -\alpha & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1-2\varepsilon d^2 & -\alpha \end{bmatrix}.$$

Then the fuzzy system can be represented as

$$\dot{x} = \sum_{i=1}^{2} M_i\, z(t))\, A_i x + v(t), \tag{5}$$

whose phase plots are shown in Fig.2. Comparing Fig.1 and Fig.2, we can see that system (5) is identical with (3) if $x_1 \in [-d, d]$ and $x_3 \in [-d, d]$.

To control (5) we introduce the control input $u(t) \in R^4$. Thus (5) with control input term is

$$\dot{x} = \sum_{i=1}^{2} M_i\,(z(t)) A_i\, x + v(t) + u(t) = \sum_{i=1}^{2} M_i\,(z(t)) A_i\, x + \bar{u}(t), \tag{6}$$

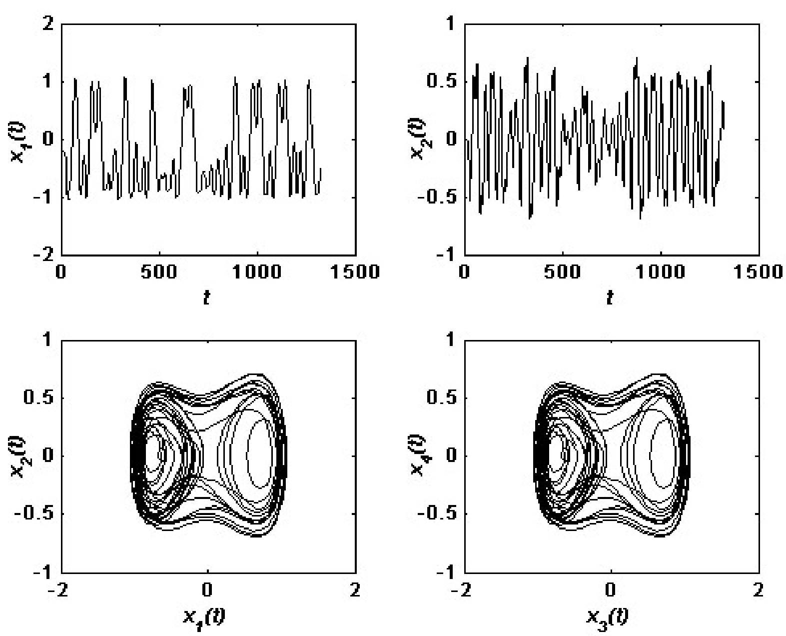


Figure 2. Fuzzy Periodically Forced Complex Duffing System's Attractor

where $\overline{u}(t) = u(t) + v(t)$.

We will use the parallel distributed compensation (PDC) design method, each control rule is designed from the corresponding rule of a T-S model in the premise parts. So, we construct the following fuzzy controller via the PDC:

Control Rule i (i = 1,2): IF $z(t)$ is M_i, THEN $\overline{u}(t) = -F_i x(t)$, where $F_i = [F_{i1}, F_{i2}, F_{i3}, F_{i4}]^T \in R^{4\times 4}$ is a constant gain matrix to be determined later. $F_{il}(l = 1,2,3,4) \in R^4$, $F_{il} \neq 0$ means that there is a control input added on the *l*th equation of (5) whereas $F_{il} = 0$ means not. Then the overall fuzzy controller is

$$\overline{u}(t) = -\sum_{i=1}^{2} M_i(z(t)) F_i x. \tag{7}$$

Applying (7) to (6), we have

$$\dot{x} = \sum_{i=1}^{2} M_i(z(t))(A_i - F_i)x. \tag{8}$$

Obviously, once $F_i\,(i = 1,2)$ is obtained, the control input $u(t)$ can be computed by $u(t) = \overline{u}(t) - v(t)$.

3. MAIN RESULT

Theorem 1: System (8) is stable if there exist matrix G_i (i = 1, 2) and a positive definite symmetrical matrix *P*, such that the following inequality holds

$$A_i P + P A_i^T - G_i - G_i^T < 0, \quad i = 1,2, \tag{9}$$

and the gain matrix $F_i = G_i P^{-1}$.

Proof: Let $\overline{P} = P^{-1}$, and choose the Lyapunov function as

$$V(t) = x^T \overline{P} x,$$

Take the derivative of $V(t)$ along the trajectories of (8), we have

$$\begin{aligned} \dot{V}(t) &= 2x^T \overline{P} \dot{x} \\ &= 2\sum_{i=1}^{2} M_i(z(t)) x^T \overline{P}(A_i - F_i)x \\ &= \sum_{i=1}^{2} M_i(z(t)) x^T (\overline{P} A_i + A_i^T \overline{P} - \overline{P} F_i - F_i^T \overline{P})x \end{aligned} \tag{10}$$

According to (9), we get

$$\bar{P}(A_i P + P A_i^T - G_i - G_i^T)\bar{P} = \bar{P}A_i + A_i^T \bar{P} - \bar{P}F_i - F_i^T \bar{P} < 0, \ \ i = 1,2. \tag{11}$$

The proof is completed therefore.

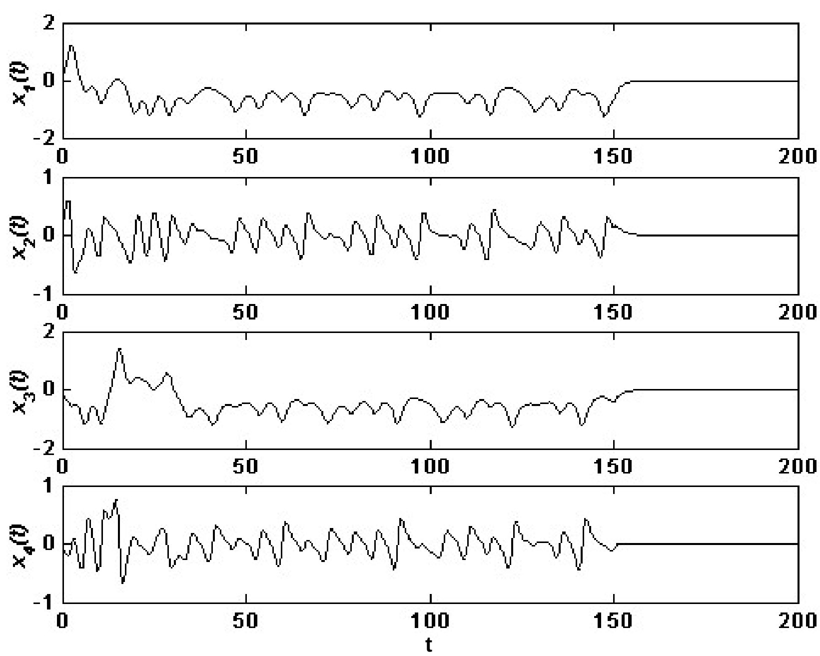
4. NUMERICAL SIMULATIONS

In this section, we consider the following cases:

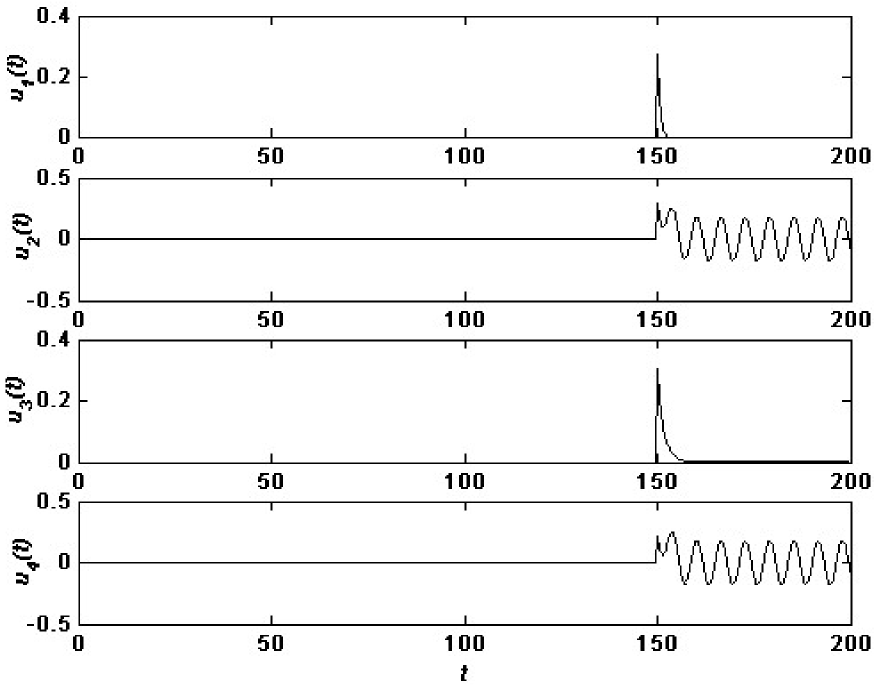
Case 1: Each equation of (5) is added a control input. Applying Theory 1, we can get gain matrixes:

$$F_1 = \begin{bmatrix} 0.5000 & 1.0000 & 0 & 0 \\ 1.0000 & 0.3700 & 0 & 0 \\ 0 & 0 & 0.5000 & 1.0000 \\ 0 & 0 & 1.0000 & 0.3700 \end{bmatrix}, F_2 = \begin{bmatrix} 0.5000 & -3.0000 & 0 & 0 \\ -3.0000 & 0.3700 & 0 & 0 \\ 0 & 0 & 0.5000 & -3.0000 \\ 0 & 0 & -3.0000 & 0.3700 \end{bmatrix}$$

denote t_s the time at which the control input starts. Simulation runs under condition x_0=[0.2, 0.1 ,-0.2, -0.1]^T , with the waveforms and control input shown in Fig.3. (*a*), (*b*)



(*a*) Closed loop state responses ***x***



(*b*) Control input *u* (*t*)

Figure 3: $t_s = 150$, x_0=[0.2, 0.1 ,-0.2, -0.1]^T

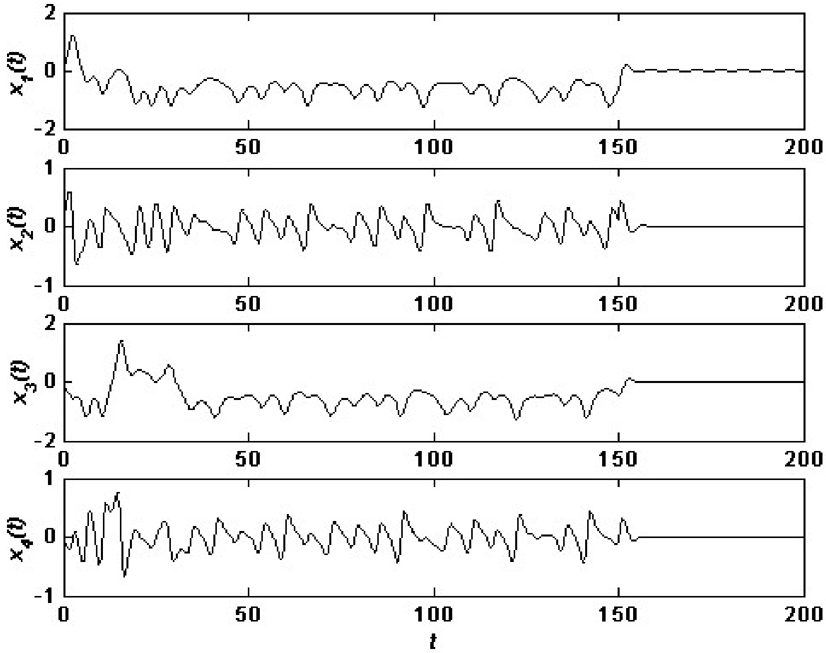
Case 2: 3rd equation of (5) isn' t added control input. Applying Theory 1, we can get gain matrixes:

$$F_1 = \begin{bmatrix} 0.5000 & 0.0000 & 0 & 0 \\ 2.0000 & 0.3700 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2.4286 & 0.9414 \end{bmatrix}, F_2 = \begin{bmatrix} 0.5000 & -7.0000 & 0 & 0 \\ 1.0000 & 0.3700 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -5.5714 & 0.9414 \end{bmatrix}$$

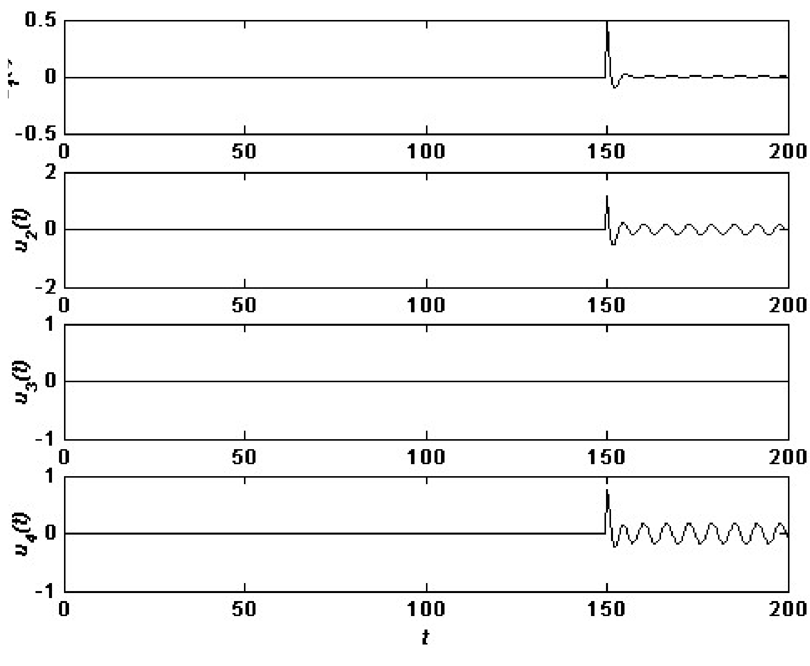
the waveforms and control input shown in Fig.4. (*a*), (*b*)

Case 3: 2nd and 4th equations of (5) are added control input, and gain matrixes:

$$F_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 2.4286 & 0.9414 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2.4286 & 0.9414 \end{bmatrix}, F_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -5.5714 & 0.9414 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -5.5714 & 0.9414 \end{bmatrix}$$



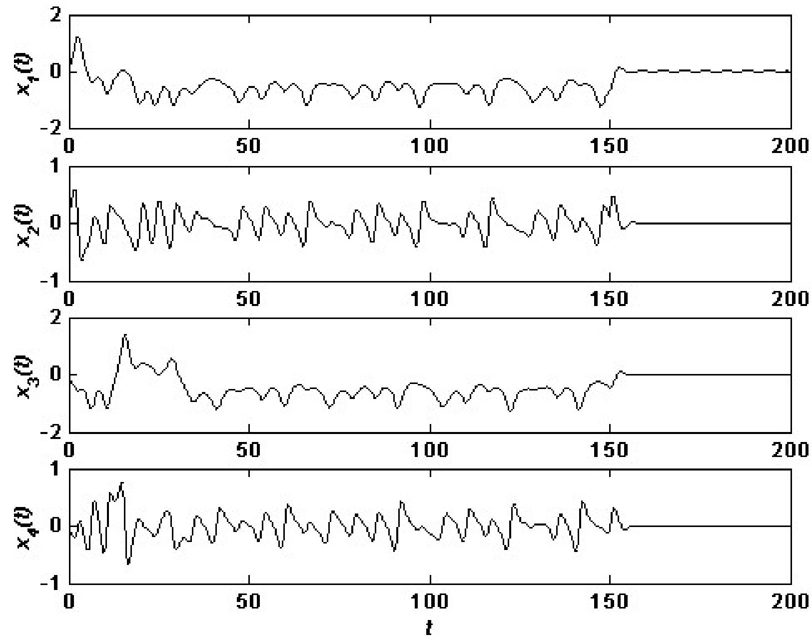
(a) Closed loop state responses *x*



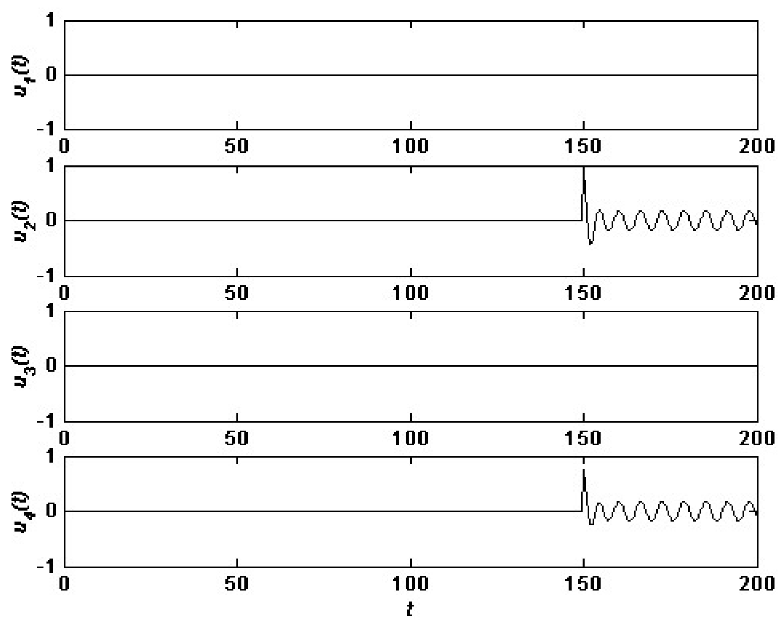
(*b*) Control input *u* (*t*)

Figure 4. $t_s = 150$, x_0=[0.2, 0.1 ,-0.2, -0.1]^T

the waveforms and control input shown in Fig.5. (*a*), (*b*).



(*a*) Closed loop state responses *x*



(*a*) Closed loop state responses *x*(*b*) Control input *u* (*t*)

Figure 5. $t_s = 150$, x_0=[0.2, 0.1 ,-0.2, -0.1]^T

Simulation results show that the designed fuzzy controller can stabilize the periodically forced complex Duffing's chaotic system to its equilibrium point, and one can easily see that the control trajectories of full state feedback in Fig.3 is much better than those of partial state feedback in Fig.4 (or Fig.5).

5. CONCLUSIONS

In this paper, we have explored the chaotic control problem of the periodically forced complex Duffing's system based on its T-S fuzzy model via state feedback, and proposed a control criterion. One can see that from numerical simulations, the trajectory of chaotic system can be driven to the zero equilibrium by designed fuzzy controller, and numerical simulations show the effectiveness and feasibility of the proposed controller.

ACKNOWLEDGEMENT

Project supported by the Natural Science Foundation of the State Nationalities Affairs Commission of China. (Grant No. 05XN07)

REFERENCES

[1] Ott E., Grebogi C., York J.A. Controlling chaos. Phys. Rev. Lett., **64**:1196-99, 1990.

[2] Hwang C.C., Hsieh J.Y., Lin R.S. A linear continuous feedback control of Chua's circuit. Chaos, Solitons & Fractals. **8**:1507–15, 1997.

[3] Lu J.H., Lu J.A. Controlling uncertain $L\ddot{u}$ system using linear feedback. Chaos, Solitons & Fractals. **17**:127–33, 2003.

[4] Chen M., Han Z. Controlling and synchronizing chaotic Genesio system via nonlinear feedback control. Chaos, Solitons & Fractals. **17**: 709–16, 2003.

[5] Zhang H., Ma Xk, Xue BL. A novel boundedness-based linear and nonlinear approach to control chaos. Chaos, Solitons & Fractals. **22**: 433–42, 2004.

[6] Park J.H., Controlling chaotic systems via nonlinear feedback control. Chaos, Solitons & Fractals. **23**: 1049–54,2005.

[7] Lima R., Pettini M. Suppression of chaos by resonant parametric perturbations. Phys Rev A. **41**: 726-33,1990.

[8] Mahmoud G., Mohamed A., Aly S. Strange attractors and chaos control in periodically forced complex Duffing's oscillators. Physica A. **292**: 193–206, 2001.

[9] Ramesh M., Narayanan S. Chaos control by nonfeedback methods in the presence of noise. Chaos, Solitons & Fractals. **10**: 1473–89, 1999.

[10] Xu Y., Mahamoud G.M., Xu W., Lei Y. Suppressing chaos of a complex Duffing's system using a random phase. Chaos, Solitons & Fractals. **23**: 265–73, 2005.

[11] Zhang H.B., Yu J.B. LMI-based stability analysis of fuzzy large-scale systems with time delays. Chaos, Solitons & Fractals. **25**:1193–07, 2005.

[12] Liu X.W., Zhang H.B., Zhang F.L. Delay-dependent stability of uncertain fuzzy large-scale systems with time delays. Chaos, Solitons & Fractals. **26**: 147-58, 2005.

[13] Tanaka K., Wang H.O. Fuzzy control systems design and analysis, a linear matrix inequality approach. New York: Wiley, 2001.

[14] Tanaka K., Ikeda T., Wang H.O. A unified approach to controlling chaos via an LMI-based fuzzy control system design. IEEE Trans CAS. **45**: 1021–40, 1998.

[15] Wang Y.W., Guan Z.H., Wang H.O. LMI-based fuzzy stability and synchronization of Chen's system. Phys Lett A. **320**: 154–9, 2003.

[16] Xue Y.,Yang S. Synchronization of generalized Henon map by using adaptive fuzzy controller. Chaos, Solitons & Fractals. **17**: 717-22, 2003.

[17] Zhang H.B., Liao X.F., Yu J.B. Fuzzy modeling and synchronization of hyperchaotic systems. Chaos, Solitons & Fractals, **26**: 835-43, 2005.

[18] Park J.H., Kwon O.M. LMI optimization approach to stabilization of time-delay chaotic systems. Chaos, Solitons & Fractals. **23**: 445-50, 2005.

[19] Mahmoud GM, Farghaly AA. Chaos control of chaotic limit cycles of real and complex van der Pol oscillators. Chaos, Solitons & Fractals. **21**: 915–24, 2004.

[20] Mahmoud G.M., Abdusalam H.A., Farghaly AAMA. Chaotic behavior and chaos control for a class of complex partial differential equations. Int J. Mod Phys C. **12**(6): 889–99, 2001.

[21] Rauh A., Hannibal L., Abraham N. Global stability properties of the complex Lorenz model. Physica D. **99**: 45–58, 1996.

[22] Fowler A.C., Gibbon J.D., McGuinness M.J. The real and complex Lorenz equations and their relevance to physical systems. Physica D. **7**: 126–34, 1983.

Xin Gao & Xingwen Liu
School of Electrical and Information Engineering
Southwest University for Nationalities, Chengdu
Sichuan 610041, PR China