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Application of Burrxii- Dal and Weibull-Dal Distribution: A Case Study of Rice Yield in Gujranwala and Sheikhupura Pakistan

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Abstract

Background: In statistical theory; the development of new models is necessary that are helpful in describing and producing innovative distributions. BurrXII-DAL and Weibull-DAL distribution are the two models that are used for estimating the failure rate, dependability, and acceptability sampling plans along with the distribution of the data.

Objective:"The objective of the study was to modified new models, investigate the properties of the models along with the estimation of the maximum likelihood of the parameters".

Methods: The study was conducted to determined the property of BurrXII-DAL and Weibull-DAL distribution with the help of five parameters according to the maximum likelihood and the distributions of the two sets of the real data were inspected against the three and two parameters. Exponential Generalized Power Weibull (EGPW), Nadarajah Haghighi (NH), and Kumaraswamy Generalized Power Weibull (KwGPW) distributions were compared with the proposed model. The real data set of rice yield per acre was analyzed by using BURXII-DAL and Weibull-DAL models for the further description of parameters. The analysis was done by using R packages and the square mean was used for the analysis of the approximation of the shape of parameters.

Result: The results of the study showed that BXII-DAL performs better than the other models having maximizing the distribution with maximum MLE's and α (0.0300), β (2.7272) and λ 5.7469. Similarly; The statistics W^*, A^*, K-S, p-value, AIC, BIC and l^ of BXII-DAL showed that AIC has the final value of 861.3306 while BIC with final value of 872.7139. MLE and standard error of W-DAL model showed better and smaller leading towards smaller variation among values with α (0.1343), β (4.7917) and λ (0.0991). The value of AIC, A^*and W^* for W-DAL statistic showed that AIC had the final value of 505.395 while BIC had final value of 516.778.

Conclusion: BurrXII-DAL and Weibull-DAL models are good proposed models among competitive models to the data set and can be highly effective for future analysis as both MLE values showed smaller variation as compared to other models with smaller standard errors.

Keywords: Exponentiated Generalized Power Weibull Distribution, Maximum likelihood estimation, Weibull distribution, Weibull Dagum Weibull-DAL.

INTRODUCTION

Lifetime models with growing and decreasing risk rates have been established during the last several decades. Exponentiated Generalized Power Weibull Distribution (EG) is a distribution that described as two-parameter lifetime with a reduced rate of failure that was developed by Adamidis and Loukas (1998) by merging exponential and geometric distributions. A broad variety of investigations, including dependability, failure time modelling, and acceptability sampling plans, have motivate the development of Burr XII distribution in the last twenty years. This distribution was initially introduced by Burr in 1942. Burr created twelve possibilities for x, which led in twelve possible distributions that could be used to fit data.

Kus (2007) and Tahmasbi and Razavi (2009) established the Exponentiated Power Distribution (EP) and Exponentiated Least Power (ELP) distribution, respectively. In 2009, Chahkandi and Ganjali (2009) introduced a new category of distributions called exponential power by merging exponential and power series distributions (EPS).

When Wang and Keats (1996) calculated BurrXII distribution parameters, they used a maximum likelihood method. Software dependability was studied by Abdel-Ghaly et al. (1997) using the Burr XII distribution, and the proper quality and reliability of the software for analysis was determined. The form parameter of the Burr XII distribution must be evaluated in order to get a confidence range under a failure censored plan by Wu and Yu (2005). For the Burr XII distribution under the LINEX loss function, Li et al. (2007) developed experimental estimators of dependability performance Maximum likelihood analysis was used to estimate the Burr XiI distribution parameters (2007). Kumar (2017) studied the statistical properties of Burr XII distribution, while Kumar (2016) studied Marshall Olkin's extended Burr XII distribution. Abdel-Hamid examined life tests for Burr XII distributions with progressive type II censoring (2009). In addition, Cordeiro et al. (2018) offered a double BXII model with forty exceptional situations. Yousof et al. (2017) proposed and examined the Topp Leone produced Burr XII distribution as maximum estimation of likelihood methodology and two Bayesian estimation techniques were used to determine accelerated life test (ALT) sample's maximum likelihood of the in which there is constant-stress value that collected from the Burr XII distribution having three-parameter. Due to the difficulty of computing the maximum likelihood estimation technique, two Bayesian methods were proposed for estimating parameters and quantiles in optimal conditions.

Guerra et al. (2018) studied the Nadarajah Haghighi distribution, Cordeiro et al. (2019) studied the Nadarajah-Haghighi Lindley distribution, and Oluyede et al. (2019) studied the exponential generalized power series distributions (2020). Weibull Generalized G family with two additional positive shape parameters, suggested by Yousof et al. (2018), expands numerous well-known models and is named the Weibull Generalized G family by the authors. Weibull-DAL distributions with five parameters are studied in Shahbaz et al. (2021). The Dimitrakopoulou Adamidis Loukas (DAL) Weibull distribution, as developed by Dimitrakopoulou et al. in 2007 and has the potential to describe more complicated failure mechanisms in lifespan modelling. According to Waloddi Weibull (Weibull, 1939), the Weibull distribution is a continuous probability distribution that was pioneered to a particular field data (Weibull, 1951), Weibull distribution is mostly implemented for the better understanding of the physical characteristics of structures or parts of structures, especially failure times.

The purpose of the study is to development of the DAL distribution for the comparative analysis of BurrXII-DAL and Weibull-DAL. Previous studies focused on the innovative distribution techniques but the comparative analysis along with the investigation of the properties of the models on the distribution and estimation of likelihood is not determined.

MATERIALS AND METHODS

This study was designed to determine the property of BurrXII-DAL and Weibull-DAL distribution. To conduct this study; there are the five parameters that appraised according to the maximum likelihood and the distributions of the two sets of the real data which were inspected against the three and two parameters. Exponential Generalized Power Weibull (EGPW), Nadarajah Haghighi (NH), and Kumaraswamy Generalized Power Weibull (KwGPW) distributions were compared with the BurrXII-DAL distributions. The analysis was done by using R packages; mathematical models and statistical assessments.

"In this article, real data set of rice yield per acre was analyzed for the description by using BURXII-DAL and Weibull-DAL models". "The maximum likelihood estimates (MLEs) and AIC, BIC, A^*,W^*, K-S and p-value (K-S) are analyzed". The BFGS (Broyden–Fletcher–Goldfarb–Shanno) provide better results of MLEs by using MaxLik R- language and package and Adequacy Model. After the analysis, the square mean was used for the analysis of the approximation of the shape of parameters. The analysis was done by finding the mean squared error.

BurrXII-DAL distribution

"The model variables are assessed using the strategy of maximum likelihood and 4 goodness-of-fit statistics". "The BurrXII-DAL model is associated with KwGPW, BENH, EGPW, GPW and NH distributions which were established to delineate the probability of the BurrXII-DAL model". The collective distribution function of DAL distribution is known as

$$G(x) = 1 - exp\exp\left[1 - \left(1 + \lambda x^{\beta}\right)^{\alpha}\right] \qquad ; \quad x > 0$$

The usefulness of the models was assessed by analyzing its AIC. The goodness of fit of the model can be determined by inserting the parameters that can provide maximum likelihood of the function. Thus, MLE is more statistically significance while the standard errors are estimated for each parameter value to examine variation among original and fitted values.

The densities of the BurrXII-DAL model are respectively, given by

$$\begin{aligned} f_{KwGPW}(x) \;&=\; ab\alpha\beta\lambda x^{\beta-1}(1 \\ &+ \lambda x^{\beta})^{\alpha}\exp\left(1 - \left[1 + \lambda x^{\beta}\right]^{\alpha}\right)\left(1 - \exp\left(1 - \left[1 + \lambda x^{\beta}\right]^{\alpha}\right)\right)^{\alpha-1} \\ &\times\; [1 - \{1 - \exp(1 - [1 + \lambda x^{\beta}]^{\alpha})\}^{a}]^{b-1} \qquad ; \; a, b, \alpha, \beta, \lambda > 0. \qquad \dots\dots\dots (i) \end{aligned}$$

$$f_{BENH}(x) \quad = \quad \frac{\alpha\beta\lambda}{\beta(a,b)}\left(1 + \lambda x^{\beta}\right)^{\alpha}\exp\left(1 - \left[1 + \lambda x^{\beta}\right]^{\alpha}\right)\left\{1 - \exp\left(1 - \left[1 + \lambda x^{\beta}\right]^{\alpha}\right)\right\}^{\alpha\beta-1} \times$$

$$\left[1 - \left\{1 - \exp\left(1 - \left[1 + \lambda x^{\beta}\right]^{\alpha}\right)\right\}^{\beta-1}\right]^{b-1} ss \quad ; \quad a, b, \alpha, \beta, \lambda > 0. \qquad \dots\dots.(ii)$$

$$l(\theta) = nlog(\alpha\lambda) - nlog[B(a,b)] + \left(1 - \frac{1}{\alpha}\right)\sum \log((1+\lambda x)^{\alpha}) + (a-1)\sum \log\left(1 - e^{1-(1+\lambda x)^{\alpha}}\right)$$
$$+ b\sum 1 - (1+\lambda x)^{\alpha}$$

The MLE for all parameters are included below by differentiating above equation.

$$U_a = \frac{\partial l}{\partial a} = -n\psi(a) + n\psi(a+b) + \sum \log(1 - e^{1-(1+\lambda x)^{\alpha}})$$

$$U_b = \frac{\partial l}{\partial b} = -n\psi(b) + n\psi(a+b) + \sum 1 - (1+\lambda x)^{\alpha}$$

$$U_{\alpha} = \frac{n}{\alpha} - \frac{1}{\alpha^2}\sum \log((1+\lambda x)^{\alpha})$$
$$-\left(1 - \frac{1}{\alpha}\right)\sum \frac{(1-(1+\lambda x)^{\alpha})^{\alpha}}{(1+\lambda x)^{\alpha}} + (1-\alpha)\sum \frac{(1-(1+\lambda x)^{\alpha})^{\alpha} e^{1-(1+\lambda x)^{\alpha}}}{1 - e^{1-(1+\lambda x)^{\alpha}}}$$
$$+ b\sum (1-(1+\lambda x)^{\alpha})^{\alpha}$$

$$U_{\lambda} = \frac{\partial l}{\partial \lambda} = \frac{n}{\lambda} - \left(1 - \frac{1}{\alpha}\right)\sum \frac{(1-(1+\lambda x)^{\alpha})^{\lambda}}{(1+\lambda x)^{\alpha}} + (1-a)\sum \frac{(1-(1+\lambda x)^{\alpha}) e^{1-(1+\lambda x)^{\alpha}}}{1 - e^{1-(1+\lambda x)^{\alpha}}}$$
$$+ b\sum (1-(1+\lambda x)^{\alpha})^{\lambda}$$

$$f_{EGPW}(x) = \alpha\beta\lambda\theta x^{\beta-1}(1+\lambda x^{\beta})^{\alpha} \exp\left(1 - \left[1+\lambda x^{\beta}\right]^{\alpha}\right)\left\{1 - \exp\left(1 - \left[1+\lambda x^{\beta}\right]^{\alpha}\right)\right\}^{\theta-1}$$

$$a, b, \alpha, \beta, \lambda > 0. \qquad \dots\dots\dots(iii)$$

$$f_{GPW}(x) = \alpha\beta\lambda x^{\beta-1}(1+\lambda x^{\beta})^{\alpha} \exp\left(1 - \left[1+\lambda x^{\beta}\right]^{\alpha}\right) \quad ; \quad x, \alpha, \beta, \sigma > 0 \quad \dots\dots\dots(iv)$$

$$f_{NH}(x) = \alpha\lambda(1+\lambda x)^{\alpha} \exp(1 - [1+\lambda x]^{\alpha}) \quad ; \quad x, \alpha, \lambda > 0 \quad \dots\dots\dots(v)$$

Weibull-DAL (W-DAL) distribution

In this subsection, fits of the W-DAL were compared with WDa, WPF, WLx, GPW, NH, BW and KwW distributions (Yousof, 2017). The competitive models had the following densities:

$$f_{WDa}(x)$$

$$= ab\alpha\beta\lambda x^{\beta-1}\left[\frac{\left(1+\lambda x^{-\beta}\right)^{-ab-1}}{\{1-(1+\lambda x^{-\beta})^{-\alpha}\}^{b+1}}\right]\exp\left[-a\left\{\frac{\left(1+\lambda x^{-\beta}\right)^{-\alpha}}{1-(1+\lambda x^{-\beta})^{-\alpha}}\right\}^{b}\right] \quad \ldots\ldots\ldots..(vi)$$

$$a,b,\alpha,\beta,\lambda,x>0.$$

$$f_{WPF}(x)=ab\alpha^{-\beta}\beta x^{\beta-1}\left[\frac{\left\{\left(\frac{x}{\alpha}\right)^{\beta}\right\}^{b-1}}{\left\{1-\left(\frac{x}{\alpha}\right)^{\beta}\right\}^{b+1}}\right]\exp\left[-a\left\{\frac{\left(\frac{x}{\alpha}\right)^{\beta}}{1-\left(\frac{x}{\alpha}\right)^{\beta}}\right\}^{b}\right] \quad , \quad a,b,\alpha,\beta,x$$

$$>0.\ldots\ldots(vii)$$

$$f_{WLx}(x)=ab\alpha\beta^{-1}(1$$

$$+\frac{x}{\beta})^{-\alpha-1}\left[\frac{\left\{1-\left(1+\frac{x}{\beta}\right)^{-\alpha}\right\}^{b-1}}{\left\{\left(1+\frac{x}{\beta}\right)^{-\alpha-1}\right\}^{b+1}}\right]\exp\left[-a\left\{\frac{1-\left(1+\frac{x}{\beta}\right)^{-\alpha}}{\left(1+\frac{x}{\beta}\right)^{-\alpha}}\right\}^{b}\right] \quad ,\ldots.(viii)$$

$$a,b,\alpha,\beta,x>0.$$

$$f_{GPW}(x)=\ \alpha\beta\lambda x^{\beta-1}\left(1+\lambda x^{\beta}\right)^{\alpha}\exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right) \quad , \quad x,\alpha,\beta,\lambda>0\ldots\ldots\ldots(ix)$$

$$f_{NH}(x)=\ \alpha\lambda(1+\lambda x)^{\alpha}exp(1-[1+\lambda x]^{\alpha}) \quad , \quad x,\alpha,\lambda>0.\ldots\ldots\ldots\text{(xx)}$$

$$f_{BW}(x)=\ \frac{\alpha\beta x^{\beta-1}\exp\left(-\alpha x^{\beta}\right)}{\beta(a,b)}\left[1-\exp\left(-\alpha x^{\beta}\right)\right]^{\alpha-1}\left[\exp\left(\alpha x^{\beta}\right)\right]^{b-1} \quad ,\ldots\ldots(xi)$$

$$x,a,b,\alpha,\beta,\lambda>0$$

$$f_{KwW}(x)=ab\alpha\beta x^{\beta-1}\exp\left(-\alpha x^{\beta}\right)\left(1-\exp\left[-\alpha x^{\beta}\right]\right)^{a-1}\Big[1$$

$$-\left\{1-\exp\left[-\alpha x^{\beta}\right]\right\}^{a}\Big]^{b-1} \quad ,\ldots\ldots\ldots(xii)$$

$$x,a,b,\alpha,\beta>0$$

RESULTS AND DISCUSSION

This study was designed to investigate, explores and to develop the best fitted model among the proposed models. Thus, the production of rice yield in pounds per acre from Gujranwala and Sheikhupura were used to test the models. The data was retrieved from Bureau of Statistics, Punjab, Pakistan. (Punjab Bureau of Statistics, 2010).

The data set has been obtained from a small sample of 73 units. The data are:

8.90	8.59	8.89	8.23	9.92	16.70
9.13	9.02	8.71	8.64	9.41	15.86
8.65	8.13	8.23	8.58	9.64	15.46
8.69	9.24	9.20	9.23	10.51	15.61
13.01	14.74	14.80	15.71	15.93	15.65
16.43	14.59	15.49	15.83	15.72	15.98
16.34	15.99	16.35	16.35	17.56	18.48
17.62	16.91	16.78	15.85	17.08	16.41
18.56	19.35	18.92	19.51	20.74	20.44
18.58	20.36	19.93	20.18	21.41	21.32
22.38	23.74	24.15	20.63	24.24	24.26
21.03	24.51	25.12	25.44	25.98	25.93
23.98					

Applications of BurrXII-DAL distribution:

The rice production(yield) data set was used to illustrate the performance of BurrXII-DAL distribution, Weibull-DAL (W-DAL) distribution and other competitive models. Table 1 showed MLEs of parameters and Table 2 showed value for the fitted models of goodness of fit statistics .

Table 1: Parentheses for MLEs along standard errors:

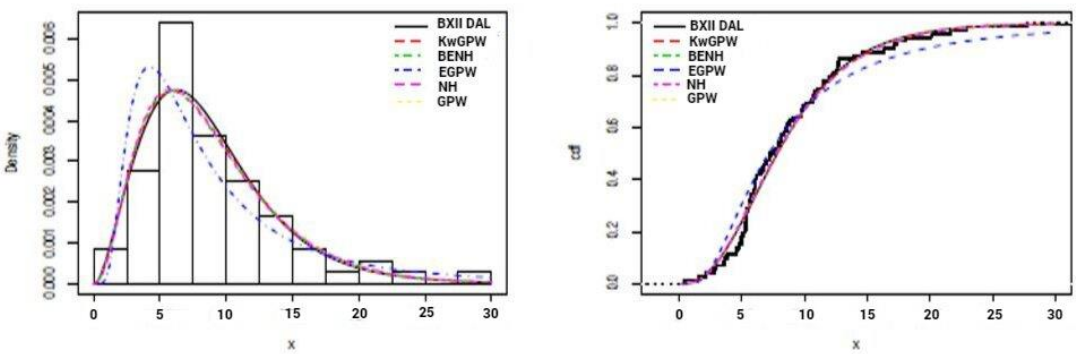
Distribution	a	b	α	β	λ	Θ
BXII-DAL	7.5565 (2.7228)	3.6672 (4.6936)	0.0310 (0.0002)	2.7272 (0.0708)	5.7469 (0.2274)	- -
KwGPW	3.3259 (0.8084)	0.9414 (1.7563)	1.2953 (1.4121)	0.9590 (0.2669)	0.0092 (0.0041)	- -
BENH	2.1615 (1.7161)	0.5271 (0.8944)	1.3655 (0.8190)	1.4763 (1.3146)	0.0096 (0.0036)	- -
EGPW	- -	- -	0.0433 (0.0246)	8.0992 (4.2118)	0.0035 (0.0039)	21.9928 (6.8421)
GPW	- -	- -	0.0083 (0.0037)	18.7243 (8.6113)	9.3543 (-)	- -
NH	- -	- -	2.2994 (0.2130)	- -	0.0020 (0.0001)	- -

Table 1 shows the MLE and standard error of BurrXII-DAL (BXII-DAL) model with other comparative models. Based on this table, it shows that BXII-DAL performs better than the other models. This is because the parameters for maximizing the distribution are larger than the others. Here, MLE's are indicating the values of parameters for which probability of obtaining the observed data is maximum. Hence, the values of parameters for obtaining the maximum probability of data are maximum for BXII-DAL. Standard error for each parameter indicates that specific parameter of the model takes amount of variation as compared to real data set.

Table 2: W^*, A^*, K-S, p-value, AIC, BIC and l^values:

Distribution	W^*	A^*	K-S	p-value(K-S)	AIC	BIC	l^
BXII-DAL	0.0873	0.5393	0.0858	0.6651	831.3306	876.7139	455.6653
KwGPW	0.0895	0.5585	0.0934	0.5785	831.3854	876.7687	455.6927
BENH	0.0882	0.556	0.0955	0.5305	831.3591	876.7425	455.6796
EGPW	0.2O85	1.4496	0.1463	0.0507	857.8558	883.9624	424.9279
GPW	0.1366	0.9455	0.5578	0.0002	1556.815	1O23.645	515.4075
NH	0.1532	0.91O7	0.2732	0.0001	839.422	885.9753	467.7110

Furthermore, in Table 2, BXII-DAL statistic produced the smallest value of AIC, A^*,W^*, and K-S for the model. As, we knew that the smallest K.S values refers to the fact that the data follows normal distribution. Largest value of current parameters indicated data do not follow distribution pattern thus smaller values seems that the data. Thus, smaller value would be the best one. AIC calculated for this data set using BXII DAL was reduced from the earlier value of 879.422 to the final value of 861.33O6. Moreover, BIC of BXII-DAL was reduced from the earlier value of 1O63.645 to the final value of 872.7139, which indicate significant improvement in the overall model fit for BXII-DAL. Hence it is concluded that the proposed BXII-DAL model is best model among the fitted models.



(a) (b)

Figure 1: Comparison of pdf and cdf estimated values of BXII-DAL with KWGPW, EGPW and ENH for data

Figure 1 shows the graphs of probability and cumulative distributions function of proposed BXII-DAL model along with KWPGW, EGPW and ENH. Based on Figure 1, it can be seen that for cumulative distribution, the curve BXII-DAL shows little variations in rice production data as compared to the other models. It is following an increasing pattern in estimating the real data of rice production.

Applications of Weibull-DAL distribution:

“In this subsection, W-DAL values were compared with Weibull Dagum , Weibull power function, Weibull Lomax, generalized power Weibull , Nadarajah Haghighi, Beta Weibull and Kumaraswamy Weibull distributions by using the rice yield data sets to illustrate the usefulness of the W-DAL model”.

"Table 3 provides the MLE estimates of the model parameter for the data, the corresponding SEs (given in parenthesis) and the goodness of fit statistics".

Table 3: "MLEs and their standard errors (in parentheses) for data"

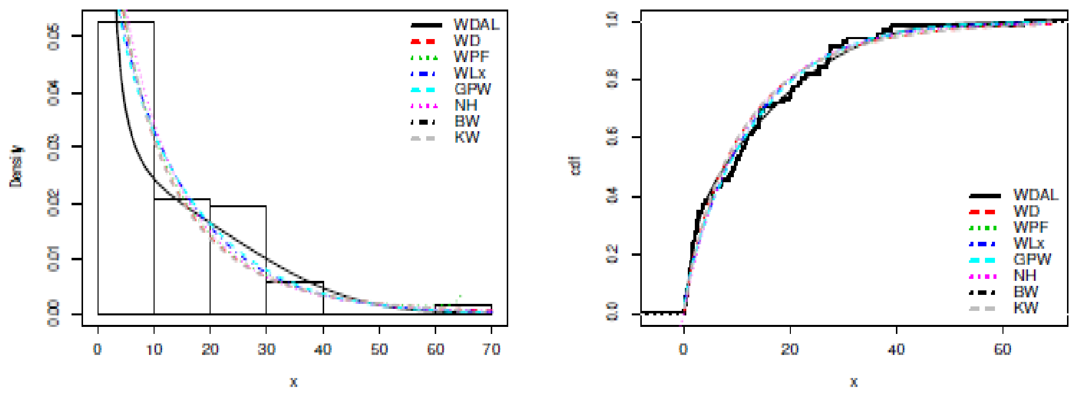
Distribution	α	β	a	b	λ
W-DAL	0.1343	4.7917	O.4469	0.2892	0.0991
	(0.0469)	(2.5790)	(0.1389)	(0.1481)	(O.3023)
BW	0.0495	1.2931	0.5518	0.4976	-
	(0.1235)	(0.45O4)	(0.2766)	(0.6416)	-
KwW	1.1833	0.824O	1.3778	0.1196	-
	(0.4547)	(0.04O8)	(0.3988)	(0.0020)	-
WDa	0.09584	2.8688	0.1487	0.2999	1.0119
	(1.8256)	(4.0376)	(0.3122)	(0.4283)	(7.9938)
WPF	65	10.8374	4.5937	0.0820	-
	-	(9.7206)	(O.7439)	(0.0723)	-
WLx	1.2757	3.1889	0.2320	0.7920	-
	(1.2514)	(3.4718)	(0.6218)	(0.3270)	-
GPW	2.9113	0.7511	-	-	0.0403
	(4.05O3)	(0.1359)	-	-	(0.0540)
NH	0.8412	-	-	-	0.1094
	(0.2600)	-	-	-	(0.0597)

Table 3 shows the MLE and standard error of W-DAL model with other comparative models. Based on this Table 3, W-DAL perform the best as compared to all other models. This is because W-DAL produced better MLE values for all coefficients of model and the standard deviations are also smaller leading towards smaller variation among values. Here, parameters of model estimating MLE using W-DAL are significantly not enough while their standard errors are also explaining smaller variation among original and fitted values of model in presence of these parameters.

Table 4: "The statistics W^*,A^*, K-S, p-value, AIC, BIC and l^for data"

Distribution	W^*	A^*	K-S	p-value (K-S)	AIC	BIC	l^
W-DAL	0.027	0.211	0.161	0.823	515.395	536.778	247.397
BW	0.123	0.624	0.103	0.776	519.959	539.066	250.580
KwW	0.137	0.876	0.095	0.667	519.878	538.984	250.639
Wda	0.143	0.631	0.087	0.813	521.399	542.783	250.210
WPF	0.111	0.724	0.104	0.634	516.469	533.299	250.710
WLx	0.146	0.710	0.106	0.577	520.792	539.899	251.196
GPW	0.116	0.648	0.103	0.793	517.831	534.661	250.316
NH	0.124	0.857	0.126	0.515	517.975	532.528	251.287

Table 4 shows the value of AIC, A^*and W^* for W-DAL statistic is the smallest among the other fitted models that leads to model accuracy. AIC of this model was reduced from the earlier value of 511.399 to the final value of 505.395. Moreover, BIC of W-DAL model was reduced from the earlier value of 522.783 to the final value of 516.778, which indicate significant improvement in the overall model fit. Hence it is concluded that the proposed study W-DAL model is best model among the fitted models.



(a) (b)

Figure 2: "Plots of estimated pdf (a) and cdf (b) of W-DAL"

"Figure 2 shows the graphical representation of probability distribution function and cumulative distribution function of proposed model W-DAL". The W-DAL is the best fitted curve with the values of this distribution are nearly equal to observed values, although its curve is deeper as compared to other models. Further for cumulative distribution, the curve of W-DAL predicts smaller fluctuations among probabilities for f rice per acre which follows an increasing pattern estimating the real data set of rice production.

CONCLUSION

It is concluded that BurrXII-DAL and W-DAL models are good models among competitive models to the data set. The usefulness of these models appeared for this data set because they can predict the failure rate more appropriately than other distributions due to presence of scale parameter which was lacking in previous distributions. Both suggested distributions have the ability to use for the greater applications in further analysis of reliability.

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