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Comparative Study of population Mean Using Ratio Estimator for equal Mean and Variance in Simple Random Sampling

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Abstract: This paper deals with the estimation of population mean of study variable by using ratio estimator equal mean and variance in sample random sampling. In simple random sampling we made an attempt to compare all the existing ratio estimators and in simple random sampling about filthy two ratio estimators are proposed by different way time to time. These all estimator are presented along with their Mean Square Error. For the comparison purpose we use simulation study using different cases when mean is equal to variance. The key is obtaining the best estimators of the population parameter that are not known. Here some estimators are discussed by using single auxiliary variable. For a large sample of size fourth efficient estimator is$\widehat{\mathrm{m}}_{16}$, which is the combination of classical ratio estimator and simple mean. Fifth ranking is obtained by $\widehat{\mathrm{m}}_{15}$ for sample size 30. For sample size 40, 50 and 60 efficient estimator is found to be$\widehat{\mathrm{m}}_{38}$. These estimators are utilized by using the correlation coefficient and regression-cum ratio estimator.

Keywords: Ratio Estimator, Simple Random Sampling, population Mean, Mean square Error, Auxiliary

INTRODUCTION

A predetermined number that are taken from larger population to observe, is known as sampling. Large population, sampling the methodology used to sample, depends upon the type of analysis to be performed, but it must include simple random sampling. There are two different type of sampling viz random sampling and non random sampling. Non random sampling includes person judgment and not be much supported by using comparing tool. Random sampling is suitable for random selection and this technique can be compared in term of mean square error or relative efficiency. Random sampling includes different sampling techniques one of them named as simple random sampling.

Simple random sampling

The sampling procedure is known as simple random sampling if every population units has the same chance of being selected in the sample. The sample thus obtained is termed a simple random sample. In practice to select simple random sampling, one by one unit from population is drawn. If in population any particular draw is replacement back before drawing the next unit, the phenomenon is called replacement sample of size *n*. In such procedure of selection, it may be possible that there is one or more population units are selected and we can ignore are the repetition, that is called simple random sampling without replacement. Before executing the next draw where the selected are not replacement back in the population, this procedure is equivalent to this method.

From the population under study for drawing a simple random sampling is not as trivial as it appears. By decision if the investigator selects a sample, the representative of the population is claimed as the sample, it is subjected to investigator bias. The properties of which sample cannot be evaluated is estimated by such sampling. Therefore, every population unit should have equal probability of being selected in a sample, in such sampling methods with replacement simple random sampling.

Auxiliary Variable

The working of auxiliary information has broadly discussed in the sampling theory. Auxiliary variables used in survey sampling to get improved sampling designs and to get bigger precision of the estimates of some population parameter i-e mean and variance of the study variable. Simply it is known, when the helping information is to be used at the estimation stage, the ratio, product, and regression methods of estimation are largely available in many directions.

Ratio Estimator

The ratio estimator is a statistical parameter and is defined to be the ratio of means of two random variables. Ratio estimators are biased and correlations must be made when be made when they are used in experimental of research work. A ratio estimator is a ratio of the means of two random variables that is commonly used in survey sampling.

Objective of Study

1. To obtain the best ratio estimators among all available ratio estimators in simple random sampling for population mean estimation.

LITERATURE REVIEW

Prasad (1989) proposed in this article a ratio in the finite population sample (WOR) simple random sampling design and ratio type estimators of population mean. MSE were compared with MSE of the usual ratio estimator estimators$\bar{y}$R of $\bar{Y}$ population mean of y and evaluated MSE of these estimators. They showed that under some conditions these estimator were more efficient than these estimator were more efficient than $\bar{y}$R. Under some conditions $\bar{y}$R was less efficient then that of $\bar{y}1$. They proposed another estimator $\bar{y}2$ of $\bar{Y}$ when the value of coefficient of variation CY of y we as known.

Prased (1989) proposed auxiliary variate X positively correlated with the main variate y was available in a class of ratio type estimators of the population mean and ratio in a finite population sample surveys with WOR simple random sampling design .Ratio estimator $\bar{Y}R$of $\bar{Y}$ population mean of Y were compared with MSE .These estimators were more efficient than $\bar{y}$R .They proposed another estimator, say $\bar{y}_2$of $\bar{y}$, which were always better than $\bar{y}R$as far as the effectiveness were concerned .A prior knowledge of$\rho, cx$and $cy$was noted that construction of $\bar{y}_1$small fraction of the full budget allocated for the current survey. By conducting a preliminary survey utilizing a prior value of$\rho, cx$and cy can be guessed by utilizing appropriate information

Singh and Tailor (2005) proposed the general family of estimators for estimating the population mean by using known value of some population parameters (S) which led to some product and ratio estimators. It was proposed that all the modified estimators performed better in the family than that of earlier proposed estimators.

Abdullahi and Yahaya (2017) estimated that population mean was one of the tricky aspect in population study and in sampling theory, and also strongly employed to improve the precision of estimates. The expression of MSE and bias of proposed estimators find out under large sample approximation (AOE) asymptotic optimum estimator was identified with its MSE.

Bii, Onyango and Odhiambo (2020) proposed a finite population estimator like variance, mean and asymptotic MSE was individual of the main objective of sample survey theory and practice. To adopt the better one, sample survey practitioners were required to access the properties of these estimators. Optimality of the estimators and interference of finite population was affected by the accordance non response parameter

Iqbal et at., (2020) To differentiate the performance of the proposed evaluators, an empirical study was conducted by including quantitative and qualitative characteristics simultaneously in the form of attributes and auxiliary variables. Comparisons were made with single-phase mixed regression ratio estimators in simple samples. Mixed regression ratio estimators using multiple auxiliary variables and attributes together in stratified random sampling have been found to be more effective than estimating mixed regression ratios in simple random sampling.

MATERIAL AND METHODS

In this chapter, present under study estimators of population mean for comparative study. The estimators are considered using single auxiliary variable in case of simple random sampling. Mean square error of each estimator is also presented along with the estimator.

The standard ratio estimator due to Cochran (1940) in simple random sampling is given below

$$\widehat{m}_1 = \frac{\bar{y}}{\bar{x}}\bar{X}$$

For the above estimator the mean square error is presented below

$MES(\widehat{m}_1) \cong \frac{1-f}{n}\left(R^2\, S_X^2 - 2RS_{XY} + S_Y^2\right)$, Where,$R = \frac{\bar{Y}}{\bar{X}}$

Kadilar and Cingi (2004) presented the mean estimator for population mean by using the Ray and Singh (1981). The estimator is expressed below

$$\widehat{m}_6 = \frac{\bar{y} + b_{yx}(\bar{X} - \bar{x})}{\bar{x}}\bar{X}$$

For the above estimator the mean square error given below

$MSE(\widehat{m}_6) \cong \frac{(1-f)}{n}[R^2 S_X^2 + S_Y^2(1-\rho^2)]$, Where$b_{yx} = \frac{S_{xy}}{S_x^2}$

Swain (2014) proposed an alternative ratio-type exponential estimator suggested as

$$\hat{m}_{33} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{1/2}$$

The MSE

MSE ($\hat{m}_{33}$)= $\frac{1-f}{n}\bar{Y}^2\left[c_y^2 + \frac{C_x^2}{4} - \rho C_y C_x\right]$.

Proposed estimator Abdullahi & yahaya (2017) Motivated by Housila and Neha –Agnihotri (2008)

$$\hat{m}_{43} = \frac{\bar{y}}{2}\left[\left(1+\frac{\hat{K}}{\theta}\right)\left(\frac{\bar{X}C_x + M_d}{\bar{x}C_x + M_d}\right) + \left(1+\frac{\hat{K}}{\theta}\right)\left(\frac{\bar{x}C_x + M_d}{\bar{X}C_x + M_d}\right)\right]$$

For the above estimator the mean square error presented below

MSE($\hat{m}_{43}$) $= \frac{(1-f)}{n}S_y^2(1-\rho^2)$

Noor-ul-Amin (2019)

$$\hat{m}_{52} = \frac{Z_y}{Z_x}\bar{X}$$

MSE ($\hat{m}_{52}$)= $\frac{1-f}{n}\frac{\lambda}{2-\lambda}\left[R^2S_x^2 - 2RS_{xy} + S_y^2\right]$

Where

$$Z_Y = \lambda\bar{y} + (1-\lambda)Z_{y-1}$$

$$Z_X = \lambda\bar{x} + (1-\lambda)Z_{x-1}$$

RESULTS AND DISCUSSIONS

In this chapter we tried to analyze our study related analysis. As our study is concerned with data analysis of ratio estimator in simple random sampling, for this purpose we use fifty-two ratio estimator proposed by various authors from time to time. Duration of this study was from 1940 to 2019. First estimator is classical ratio estimator proposed by Cochran in 1940 and last ratio estimator is proposed by Noor-ul-Amin in 2019.

For detail analysis we studied all the cases by simulation study to support our analyses i.e. to choose the best ratio estimator among the existing. In order to see the impact of sample size we use different sample size, i.e. 30, 40,50, 60, 100 and 200.

For the simulation study following step we considered for the calculation of MSE and RE of all the estimators under study. Simulation study is conducted to evaluate the performance of all the comparative ratio estimators used in our study. The presented *MSE*s are based on 50,000 replicates for simple random sampling schemes. The *MSE* is computed for each scheme by using the formula

$$MSE\,(\omega) = \frac{1}{50,000}\sum_{i=1}^{50000}(\omega_i - \mu_y)^2$$

$\mu_x = 1, \mu_x = 1, \sigma_x^2 = 1, \sigma_y^2 = 1, \rho_{xy} = 0.50$

Table 1: MSE for sample size 30,40,50,60,100 and 200

Estimators	n=30	n=40	n=50	n=60	n=100	n=200
$\hat{\mathbf{m}}_1$	0.02136	0.01485	0.01162	0.00954	0.00533	0.00232
$\hat{\mathbf{m}}_2$	0.01677	0.01234	0.00970	0.00800	0.00460	0.00202
$\hat{\mathbf{m}}_3$	0.02069	0.01539	0.01213	0.01000	0.00575	0.00255
$\hat{\mathbf{m}}_4$	0.01669	0.01206	0.00948	0.00782	0.00446	0.00196
$\hat{\mathbf{m}}_5$	0.02069	0.01539	0.01213	0.01000	0.00575	0.00255
$\hat{\mathbf{m}}_6$	0.05231	0.03498	0.02729	0.02225	0.01208	0.00527
$\hat{\mathbf{m}}_7$	0.02398	0.01726	0.01367	0.01135	0.00644	0.00285
$\hat{\mathbf{m}}_8$	0.01828	0.01337	0.01056	0.00877	0.00504	0.00223
$\hat{\mathbf{m}}_9$	0.03431	0.02407	0.01900	0.01568	0.00874	0.00385
$\hat{\mathbf{m}}_{10}$	0.01828	0.01337	0.01056	0.00877	0.00504	0.00223
$\hat{\mathbf{m}}_{11}$	0.35832	0.20113	0.13095	0.09632	0.03440	0.00992
$\hat{\mathbf{m}}_{12}$	0.08542	0.05939	0.04646	0.03816	0.02131	0.00930
$\hat{\mathbf{m}}_{13}$	0.09622	0.05851	0.04448	0.03547	0.01843	0.00786
$\hat{\mathbf{m}}_{14}$	0.01669	0.01206	0.00948	0.00782	0.00446	0.00196
$\hat{\mathbf{m}}_{15}$	0.01629	0.01192	0.00937	0.00773	0.00444	0.00195
$\hat{\mathbf{m}}_{16}$	0.01683	0.01207	0.00945	0.00777	0.00442	0.00194
$\hat{\mathbf{m}}_{17}$	0.01727	0.01240	0.00969	0.00800	0.00461	0.00203
$\hat{\mathbf{m}}_{18}$	0.02518	0.01702	0.01325	0.01083	0.00597	0.00259

$\hat{m}_{19}$	0.03059	0.02293	0.01812	0.01495	0.00857	0.00382
$\hat{m}_{20}$	0.02243	0.01547	0.01209	0.00991	0.00551	0.00240
$\hat{m}_{21}$	0.02518	0.01702	0.01325	0.01083	0.00597	0.00259
$\hat{m}_{22}$	0.01650	0.01195	0.00939	0.00775	0.00443	0.00194
$\hat{m}_{23}$	0.01842	0.01364	0.01074	0.00885	0.00509	0.00225
$\hat{m}_{24}$	0.01690	0.01245	0.00978	0.00807	0.00464	0.00204
$\hat{m}_{25}$	0.01690	0.01245	0.00978	0.00807	0.00464	0.00204
$\hat{m}_{26}$	0.02350	0.01693	0.01341	0.01114	0.00632	0.00280
$\hat{m}_{27}$	0.02350	0.01693	0.01341	0.01114	0.00632	0.00280
$\hat{m}_{28}$	0.01751	0.01293	0.01017	0.00839	0.00483	0.00213
$\hat{m}_{29}$	0.01626	0.01189	0.00934	0.00771	0.00442	0.00194
$\hat{m}_{30}$	0.01690	0.01244	0.00978	0.00806	0.00464	0.00204
$\hat{m}_{31}$	0.03370	0.02527	0.01998	0.01648	0.00944	0.00421
$\hat{m}_{32}$	0.03739	0.02805	0.02218	0.01828	0.01046	0.00466
$\hat{m}_{33}$	0.01688	0.01242	0.00976	0.00804	0.00462	0.00204
$\hat{m}_{34}$	0.02778	0.02104	0.01674	0.01387	0.00804	0.00360
$\hat{m}_{35}$	0.02836	0.01992	0.01559	0.01281	0.00712	0.00311
$\hat{m}_{36}$	0.01263	0.00948	0.00742	0.00616	0.00354	0.00157
$\hat{m}_{37}$	0.01218	0.00915	0.00716	0.00595	0.00343	0.00152
$\hat{m}_{38}$	0.01634	0.01189	0.00934	0.00771	0.00442	0.00194
$\hat{m}_{39}$	0.04979	0.03464	0.02686	0.02181	0.01191	0.00512
$\hat{m}_{40}$	0.03368	0.02366	0.01869	0.01543	0.00861	0.00379
$\hat{m}_{41}$	0.02398	0.01726	0.01367	0.01135	0.00644	0.00285
$\hat{m}_{42}$	0.02547	0.01826	0.01447	0.01200	0.00679	0.00300
$\hat{m}_{43}$	0.01717	0.01251	0.00984	0.00813	0.00466	0.00205
$\hat{m}_{44}$	0.01715	0.01251	0.00984	0.00814	0.00467	0.00205
$\hat{m}_{45}$	0.12446	0.09048	0.07112	0.05847	0.03303	0.01501
$\hat{m}_{46}$	0.02709	0.01935	0.01532	0.01270	0.00716	0.00317
$\hat{m}_{47}$	0.02709	0.01935	0.01532	0.01270	0.00716	0.00317

$\hat{m}_{48}$	0.02145	0.01554	0.01230	0.01022	0.00583	0.00258
$\hat{m}_{49}$	0.03783	0.02630	0.02073	0.01707	0.00946	0.00416
$\hat{m}_{50}$	0.01747	0.01282	0.01011	0.00840	0.00484	0.00214
$\hat{m}_{51}$	0.03368	0.02366	0.01869	0.01543	0.00861	0.00379
$\hat{m}_{52}$	0.00048	0.00037	0.00027	0.00024	0.00013	0.00006

Table 2: RE for sample size 30,40,50,60,100 and 200

Estimators	n=30	n=40	n=50	n=60	n=100	n=200
$\hat{m}_1$	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
$\hat{m}_2$	1.27370	1.20340	1.19794	1.19250	1.15870	1.14852
$\hat{m}_3$	1.03238	0.96491	0.95796	0.95400	0.92696	0.90980
$\hat{m}_4$	1.27981	1.23134	1.22574	1.21995	1.19507	1.18367
$\hat{m}_5$	1.03238	0.96491	0.95796	0.95400	0.92696	0.90980
$\hat{m}_6$	0.40833	0.42453	0.42580	0.42876	0.44123	0.44023
$\hat{m}_7$	0.89074	0.86037	0.85004	0.84053	0.82764	0.81404
$\hat{m}_8$	1.16849	1.11070	1.10038	1.08780	1.05754	1.04036
$\hat{m}_9$	0.62256	0.61695	0.61158	0.60842	0.60984	0.60260
$\hat{m}_{10}$	1.16849	1.11070	1.10038	1.08780	1.05754	1.04036
$\hat{m}_{11}$	0.05961	0.07383	0.08874	0.09904	0.15494	0.23387
$\hat{m}_{12}$	0.25006	0.25004	0.25011	0.25000	0.25012	0.24946
$\hat{m}_{13}$	0.22199	0.25380	0.26124	0.26896	0.28920	0.29517
$\hat{m}_{14}$	1.27981	1.23134	1.22574	1.21995	1.19507	1.18367
$\hat{m}_{15}$	1.31123	1.24581	1.24013	1.23415	1.20045	1.18974
$\hat{m}_{16}$	1.26916	1.23032	1.22963	1.22780	1.20588	1.19588
$\hat{m}_{17}$	1.23683	1.19758	1.19917	1.19250	1.15618	1.14286
$\hat{m}_{18}$	0.84829	0.87250	0.87698	0.88089	0.89280	0.89575
$\hat{m}_{19}$	0.69827	0.64762	0.64128	0.63813	0.62194	0.60733
$\hat{m}_{20}$	0.95230	0.95992	0.96112	0.96266	0.96733	0.96667
$\hat{m}_{21}$	0.84829	0.87250	0.87698	0.88089	0.89280	0.89575

$\hat{m}_{22}$	1.29455	1.24268	1.23749	1.23097	1.20316	1.19588
$\hat{m}_{23}$	1.15961	1.08871	1.08194	1.07797	1.04715	1.03111
$\hat{m}_{24}$	1.26391	1.19277	1.18814	1.18216	1.14871	1.13726
$\hat{m}_{25}$	1.26391	1.19277	1.18814	1.18216	1.14871	1.13726
$\hat{m}_{26}$	0.90894	0.87714	0.86652	0.85637	0.84335	0.82857
$\hat{m}_{27}$	0.90894	0.87714	0.86652	0.85637	0.84335	0.82857
$\hat{m}_{28}$	1.21987	1.14849	1.14258	1.13707	1.10352	1.08920
$\hat{m}_{29}$	1.31365	1.24895	1.24411	1.23735	1.20588	1.19588
$\hat{m}_{30}$	1.26391	1.19373	1.18814	1.18362	1.14871	1.13726
$\hat{m}_{31}$	0.63383	0.58765	0.58158	0.57888	0.56462	0.55107
$\hat{m}_{32}$	0.57128	0.52941	0.52390	0.52188	0.50956	0.49785
$\hat{m}_{33}$	1.26540	1.19565	1.19057	1.18657	1.15368	1.13726
$\hat{m}_{34}$	0.76890	0.70580	0.69415	0.68782	0.66294	0.64444
$\hat{m}_{35}$	0.75317	0.74548	0.74535	0.74473	0.74860	0.74598
$\hat{m}_{36}$	1.69121	1.56646	1.56604	1.54870	1.50565	1.47771
$\hat{m}_{37}$	1.75370	1.62295	1.62291	1.60336	1.55394	1.52632
$\hat{m}_{38}$	1.30722	1.24895	1.24411	1.23735	1.20588	1.19588
$\hat{m}_{39}$	0.42900	0.42870	0.43261	0.43741	0.44752	0.45313
$\hat{m}_{40}$	0.63420	0.62764	0.62172	0.61828	0.61905	0.61214
$\hat{m}_{41}$	0.89074	0.86037	0.85004	0.84053	0.82764	0.81404
$\hat{m}_{42}$	0.83863	0.81325	0.80304	0.79500	0.78498	0.77333
$\hat{m}_{43}$	1.24403	1.18705	1.18089	1.17343	1.14378	1.13171
$\hat{m}_{44}$	1.24548	1.18705	1.18089	1.17199	1.14133	1.13171
$\hat{m}_{45}$	0.17162	0.16412	0.16339	0.16316	0.16137	0.15456
$\hat{m}_{46}$	0.78848	0.76744	0.75849	0.75118	0.74441	0.73186
$\hat{m}_{47}$	0.78848	0.76744	0.75849	0.75118	0.74441	0.73186
$\hat{m}_{48}$	0.99580	0.95560	0.94472	0.93346	0.91424	0.89922
$\hat{m}_{49}$	0.56463	0.56464	0.56054	0.55888	0.56342	0.55769
$\hat{m}_{50}$	1.22267	1.15835	1.14936	1.13571	1.10124	1.08411

$\hat{\mathbf{m}}_{51}$	0.63420	0.62764	0.62172	0.61828	0.61905	0.61214
$\hat{\mathbf{m}}_{52}$	44.50000	40.13514	43.03704	39.75000	41.00000	38.66667

Table 3: Relative Position of Ratio Estimators

Estimators Ranking	n=30	n=40	n=50	n=60	n=100	n=200
1	$\hat{\mathbf{m}}_{52}$	$\hat{\mathbf{m}}_{52}$	$\hat{\mathbf{m}}_{52}$	$\hat{\mathbf{m}}_{52}$	$\hat{\mathbf{m}}_{52}$	$\hat{\mathbf{m}}_{52}$
2	$\hat{\mathbf{m}}_{37}$	$\hat{\mathbf{m}}_{37}$	$\hat{\mathbf{m}}_{37}$	$\hat{\mathbf{m}}_{37}$	$\hat{\mathbf{m}}_{37}$	$\hat{\mathbf{m}}_{37}$
3	$\hat{\mathbf{m}}_{36}$	$\hat{\mathbf{m}}_{36}$	$\hat{\mathbf{m}}_{36}$	$\hat{\mathbf{m}}_{36}$	$\hat{\mathbf{m}}_{36}$	$\hat{\mathbf{m}}_{36}$
4	$\hat{\mathbf{m}}_{29}$	$\hat{\mathbf{m}}_{29}$	$\hat{\mathbf{m}}_{29}$	$\hat{\mathbf{m}}_{29}$	$\hat{\mathbf{m}}_{16}$	$\hat{\mathbf{m}}_{16}$
5	$\hat{\mathbf{m}}_{15}$	$\hat{\mathbf{m}}_{38}$	$\hat{\mathbf{m}}_{38}$	$\hat{\mathbf{m}}_{38}$	$\hat{\mathbf{m}}_{29}$	$\hat{\mathbf{m}}_{22}$
6	$\hat{\mathbf{m}}_{38}$	$\hat{\mathbf{m}}_{15}$	$\hat{\mathbf{m}}_{15}$	$\hat{\mathbf{m}}_{15}$	$\hat{\mathbf{m}}_{38}$	$\hat{\mathbf{m}}_{29}$
7	$\hat{\mathbf{m}}_{22}$	$\hat{\mathbf{m}}_{22}$	$\hat{\mathbf{m}}_{22}$	$\hat{\mathbf{m}}_{22}$	$\hat{\mathbf{m}}_{22}$	$\hat{\mathbf{m}}_{38}$
8	$\hat{\mathbf{m}}_{4}$	$\hat{\mathbf{m}}_{4}$	$\hat{\mathbf{m}}_{16}$	$\hat{\mathbf{m}}_{16}$	$\hat{\mathbf{m}}_{15}$	$\hat{\mathbf{m}}_{15}$
9	$\hat{\mathbf{m}}_{14}$	$\hat{\mathbf{m}}_{14}$	$\hat{\mathbf{m}}_{4}$	$\hat{\mathbf{m}}_{4}$	$\hat{\mathbf{m}}_{4}$	$\hat{\mathbf{m}}_{4}$
10	$\hat{\mathbf{m}}_{2}$	$\hat{\mathbf{m}}_{16}$	$\hat{\mathbf{m}}_{14}$	$\hat{\mathbf{m}}_{14}$	$\hat{\mathbf{m}}_{14}$	$\hat{\mathbf{m}}_{14}$
11	$\hat{\mathbf{m}}_{16}$	$\hat{\mathbf{m}}_{2}$	$\hat{\mathbf{m}}_{17}$	$\hat{\mathbf{m}}_{2}$	$\hat{\mathbf{m}}_{2}$	$\hat{\mathbf{m}}_{2}$
12	$\hat{\mathbf{m}}_{33}$	$\hat{\mathbf{m}}_{17}$	$\hat{\mathbf{m}}_{2}$	$\hat{\mathbf{m}}_{17}$	$\hat{\mathbf{m}}_{17}$	$\hat{\mathbf{m}}_{17}$
13	$\hat{\mathbf{m}}_{24}$	$\hat{\mathbf{m}}_{33}$	$\hat{\mathbf{m}}_{33}$	$\hat{\mathbf{m}}_{33}$	$\hat{\mathbf{m}}_{33}$	$\hat{\mathbf{m}}_{24}$
14	$\hat{\mathbf{m}}_{25}$	$\hat{\mathbf{m}}_{30}$	$\hat{\mathbf{m}}_{24}$	$\hat{\mathbf{m}}_{30}$	$\hat{\mathbf{m}}_{24}$	$\hat{\mathbf{m}}_{25}$
15	$\hat{\mathbf{m}}_{30}$	$\hat{\mathbf{m}}_{24}$	$\hat{\mathbf{m}}_{25}$	$\hat{\mathbf{m}}_{24}$	$\hat{\mathbf{m}}_{25}$	$\hat{\mathbf{m}}_{30}$
16	$\hat{\mathbf{m}}_{44}$	$\hat{\mathbf{m}}_{25}$	$\hat{\mathbf{m}}_{30}$	$\hat{\mathbf{m}}_{25}$	$\hat{\mathbf{m}}_{30}$	$\hat{\mathbf{m}}_{33}$
17	$\hat{\mathbf{m}}_{43}$	$\hat{\mathbf{m}}_{43}$	$\hat{\mathbf{m}}_{43}$	$\hat{\mathbf{m}}_{43}$	$\hat{\mathbf{m}}_{43}$	$\hat{\mathbf{m}}_{43}$
18	$\hat{\mathbf{m}}_{17}$	$\hat{\mathbf{m}}_{44}$	$\hat{\mathbf{m}}_{44}$	$\hat{\mathbf{m}}_{44}$	$\hat{\mathbf{m}}_{44}$	$\hat{\mathbf{m}}_{44}$
19	$\hat{\mathbf{m}}_{50}$	$\hat{\mathbf{m}}_{50}$	$\hat{\mathbf{m}}_{50}$	$\hat{\mathbf{m}}_{28}$	$\hat{\mathbf{m}}_{28}$	$\hat{\mathbf{m}}_{28}$
20	$\hat{\mathbf{m}}_{28}$	$\hat{\mathbf{m}}_{28}$	$\hat{\mathbf{m}}_{28}$	$\hat{\mathbf{m}}_{50}$	$\hat{\mathbf{m}}_{50}$	$\hat{\mathbf{m}}_{50}$
21	$\hat{\mathbf{m}}_{8}$	$\hat{\mathbf{m}}_{8}$	$\hat{\mathbf{m}}_{8}$	$\hat{\mathbf{m}}_{8}$	$\hat{\mathbf{m}}_{8}$	$\hat{\mathbf{m}}_{8}$
22	$\hat{\mathbf{m}}_{10}$	$\hat{\mathbf{m}}_{10}$	$\hat{\mathbf{m}}_{10}$	$\hat{\mathbf{m}}_{10}$	$\hat{\mathbf{m}}_{10}$	$\hat{\mathbf{m}}_{10}$
23	$\hat{\mathbf{m}}_{23}$	$\hat{\mathbf{m}}_{23}$	$\hat{\mathbf{m}}_{23}$	$\hat{\mathbf{m}}_{23}$	$\hat{\mathbf{m}}_{23}$	$\hat{\mathbf{m}}_{23}$

24	$\hat{m}_3$	$\hat{m}_1$	$\hat{m}_1$	$\hat{m}_1$	$\hat{m}_1$	$\hat{m}_1$
25	$\hat{m}_5$	$\hat{m}_3$	$\hat{m}_{20}$	$\hat{m}_{20}$	$\hat{m}_{20}$	$\hat{m}_{20}$
26	$\hat{m}_1$	$\hat{m}_5$	$\hat{m}_3$	$\hat{m}_3$	$\hat{m}_3$	$\hat{m}_3$
27	$\hat{m}_{48}$	$\hat{m}_{20}$	$\hat{m}_5$	$\hat{m}_5$	$\hat{m}_5$	$\hat{m}_5$
28	$\hat{m}_{20}$	$\hat{m}_{48}$	$\hat{m}_{48}$	$\hat{m}_{48}$	$\hat{m}_{48}$	$\hat{m}_{48}$
29	$\hat{m}_{26}$	$\hat{m}_{26}$	$\hat{m}_{18}$	$\hat{m}_{18}$	$\hat{m}_{18}$	$\hat{m}_{18}$
30	$\hat{m}_{27}$	$\hat{m}_{27}$	$\hat{m}_{21}$	$\hat{m}_{21}$	$\hat{m}_{21}$	$\hat{m}_{21}$
31	$\hat{m}_7$	$\hat{m}_{18}$	$\hat{m}_{26}$	$\hat{m}_{26}$	$\hat{m}_{26}$	$\hat{m}_{26}$
32	$\hat{m}_{41}$	$\hat{m}_{21}$	$\hat{m}_{27}$	$\hat{m}_{27}$	$\hat{m}_{27}$	$\hat{m}_{27}$
33	$\hat{m}_{18}$	$\hat{m}_7$	$\hat{m}_7$	$\hat{m}_7$	$\hat{m}_7$	$\hat{m}_7$
34	$\hat{m}_{21}$	$\hat{m}_{41}$	$\hat{m}_{41}$	$\hat{m}_{41}$	$\hat{m}_{41}$	$\hat{m}_{41}$
35	$\hat{m}_{42}$	$\hat{m}_{42}$	$\hat{m}_{42}$	$\hat{m}_{42}$	$\hat{m}_{42}$	$\hat{m}_{42}$
36	$\hat{m}_{46}$	$\hat{m}_{46}$	$\hat{m}_{46}$	$\hat{m}_{46}$	$\hat{m}_{35}$	$\hat{m}_{35}$
37	$\hat{m}_{47}$	$\hat{m}_{47}$	$\hat{m}_{47}$	$\hat{m}_{47}$	$\hat{m}_{46}$	$\hat{m}_{46}$
38	$\hat{m}_{34}$	$\hat{m}_{35}$	$\hat{m}_{35}$	$\hat{m}_{35}$	$\hat{m}_{47}$	$\hat{m}_{47}$
39	$\hat{m}_{35}$	$\hat{m}_{34}$	$\hat{m}_{34}$	$\hat{m}_{34}$	$\hat{m}_{34}$	$\hat{m}_{34}$
40	$\hat{m}_{19}$	$\hat{m}_{19}$	$\hat{m}_{19}$	$\hat{m}_{19}$	$\hat{m}_{19}$	$\hat{m}_{40}$
41	$\hat{m}_{40}$	$\hat{m}_{40}$	$\hat{m}_{40}$	$\hat{m}_{40}$	$\hat{m}_{40}$	$\hat{m}_{51}$
42	$\hat{m}_{51}$	$\hat{m}_{51}$	$\hat{m}_{51}$	$\hat{m}_{51}$	$\hat{m}_{51}$	$\hat{m}_{19}$
43	$\hat{m}_{31}$	$\hat{m}_9$	$\hat{m}_9$	$\hat{m}_9$	$\hat{m}_9$	$\hat{m}_9$
44	$\hat{m}_9$	$\hat{m}_{31}$	$\hat{m}_{31}$	$\hat{m}_{31}$	$\hat{m}_{31}$	$\hat{m}_{19}$
45	$\hat{m}_{32}$	$\hat{m}_{49}$	$\hat{m}_{49}$	$\hat{m}_{49}$	$\hat{m}_{49}$	$\hat{m}_{32}$
46	$\hat{m}_{49}$	$\hat{m}_{32}$	$\hat{m}_{32}$	$\hat{m}_{32}$	$\hat{m}_{32}$	$\hat{m}_{32}$
47	$\hat{m}_{39}$	$\hat{m}_{39}$	$\hat{m}_{39}$	$\hat{m}_{39}$	$\hat{m}_{39}$	$\hat{m}_{39}$
48	$\hat{m}_6$	$\hat{m}_6$	$\hat{m}_6$	$\hat{m}_6$	$\hat{m}_6$	$\hat{m}_6$
49	$\hat{m}_{12}$	$\hat{m}_{13}$	$\hat{m}_{13}$	$\hat{m}_{13}$	$\hat{m}_{13}$	$\hat{m}_{13}$
50	$\hat{m}_{13}$	$\hat{m}_{12}$	$\hat{m}_{12}$	$\hat{m}_{12}$	$\hat{m}_{12}$	$\hat{m}_{12}$
51	$\hat{m}_{45}$	$\hat{m}_{45}$	$\hat{m}_{45}$	$\hat{m}_{45}$	$\hat{m}_{45}$	$\hat{m}_{11}$
52	$\hat{m}_{11}$	$\hat{m}_{11}$	$\hat{m}_{11}$	$\hat{m}_{11}$	$\hat{m}_{11}$	$\hat{m}_{45}$

From the table of mean square error and relative efficiency it is observed that estimator $\widehat{m}_{52}$ performs well in term of MSE and RE. This estimator is recently developed by Noor-ul-Amin (2021) and called memory base ratio estimator. Second most efficient estimator from our study is $\widehat{m}_{37}$ for all sample size. This estimator was developed by Subrammani and Prabavathy (2014). In this estimator author make use of median of study and auxiliary variable. These three estimators are efficient for all sample sizes under study.

For large sample size fourth efficient estimator is$\widehat{m}_{16}$, this estimator was developed by Chakrabarty (1979) which is the combination of classical ratio estimator and simple mean. Fifth ranking is obtained by $\widehat{m}_{15}$ for sample size 30. For sample size 40, 50 and 60 efficient estimator is found to be$\widehat{m}_{38}$, this estimator was developed by Enang, Akpan and Ekpenyong (2014). This estimator utilized by using the correlation coefficient and regression-cum ratio estimator.

From the table of mean square error and relative efficiency it is observed that estimator $\widehat{m}_{52}$ performs well in term of MSE and RE. This estimator is recently developed by Noor-ul-Amin (2021) and called memory base ratio estimator. Second most efficient estimator from our study is $\widehat{m}_{37}$ for all sample size. This estimator was developed by Subrammani and Prabavathy (2014). In this estimator author introduced median of study and auxiliary variable in addition of population mean of auxiliary variable. The third most efficient estimator for this case is $\widehat{m}_{36}$, which is the another estimator developed by Subrammani and Prabavathy (2014). In this estimator author make use of median of study and auxiliary variable. These three estimators are efficient for all sample sizes under study.

Fourth efficient ratio estimator in ranking is $\widehat{m}_{29}$, for sample size 30, 40 and 50, while this estimator is not efficient for sample size 100 and 200. For large sample size fourth efficient estimator is $\widehat{m}_{16}$, this estimator was developed by Chakrabarty (1979) which is the combination of classical ratio estimator and simple mean. Fifth ranking is obtained by $\widehat{m}_{15}$ for sample size 30. For sample size 40, 50 and 60 efficient estimator is found to be $\widehat{m}_{38}$, this estimator was developed by Enang, Akpan and Ekpenyong (2014). This estimator utilized by using the correlation coefficient and regression-cum ratio estimator. For sample size n=100 $\widehat{m}_{29}$, and for n=200 $\widehat{m}_{22}$, got fifth position for large sample size. Estimator $\widehat{m}_{26}$, developed by Subramani and Kumarpandiyan (2012) in which he make use of quartile deviation in addition of population mean of auxiliary variable. Estimators $\widehat{m}_{2}$,$\widehat{m}_{4}$,$\widehat{m}_{14}$,$\widehat{m}_{15}$,$\widehat{m}_{22}$ and $\widehat{m}_{29}$, estimators also comes in the orbit of top most 10 ranking of efficient estimators. Theses estimators make use of coefficient of variation, measure of kurtosis and correlation coefficient of auxiliary variable. So, we can conclude that make use of thesis measures in the construction is helpful for all sample sizes.

Ratio estimator that is worst in performance is $\widehat{m}_{11}$, which is developed by Goodman and Hartley (1958). This estimator makes use of Sample size and population size in its construction along with the correlation coefficient. The performance of this estimator is worst in performance for all sample sizes. The second worst estimator in performance w.r.t. MSE and RE is found to be $\widehat{m}_{45}$. This estimator is developed by Kumar, Bharti, Yadav and Kumar (2019). This estimator also make use of median of auxiliary variable along with the covariances in term of mean and median.

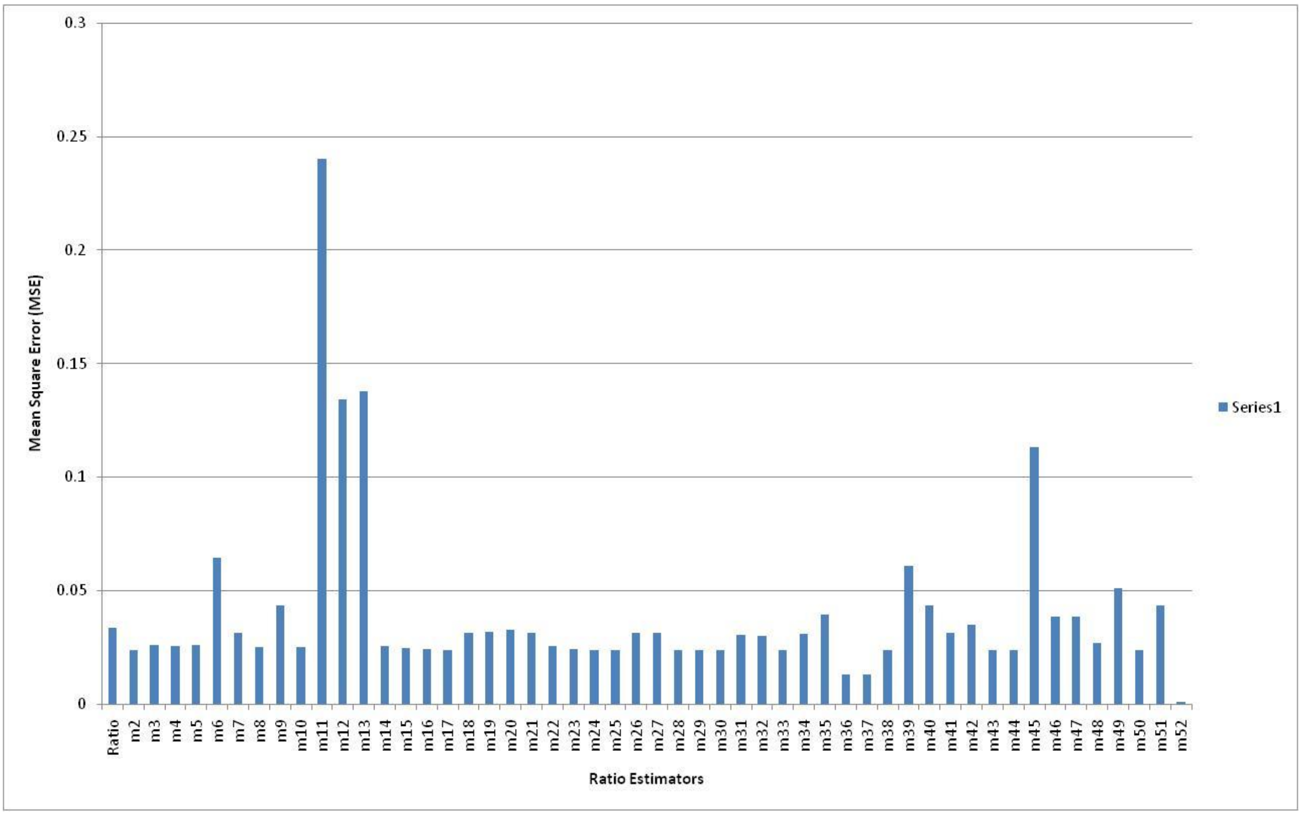

Figure 1:Graphical Representation of Ratio Estimators using term of MSE

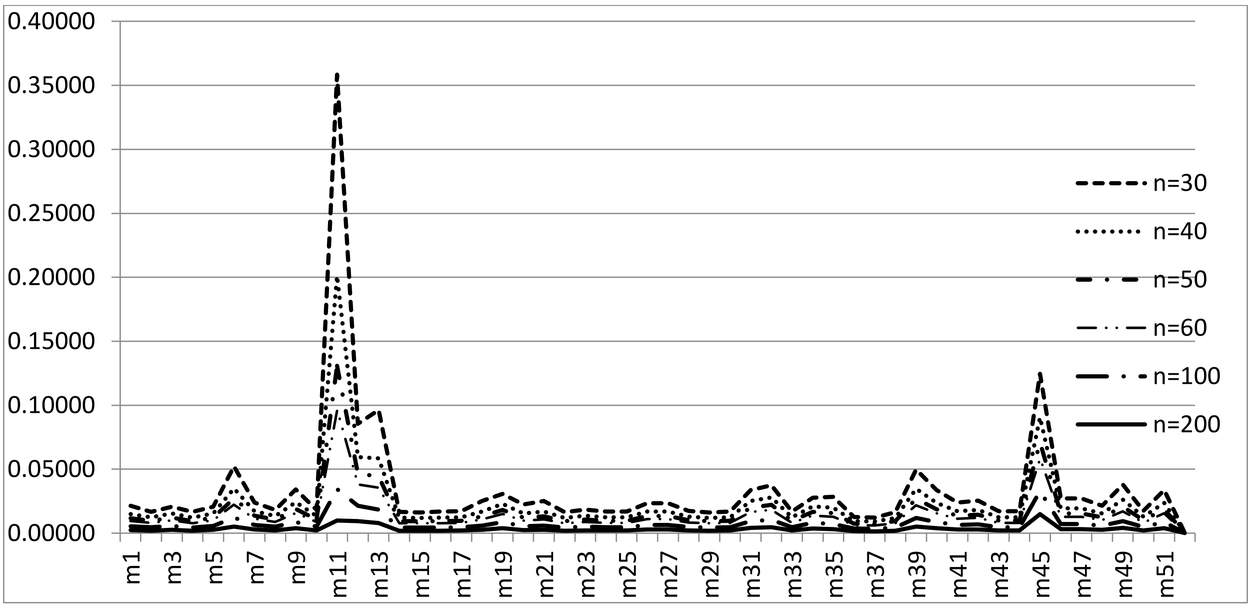

Figure 2:Graphical Representation of Ratio Estimators using term of MSE for different sample sizes

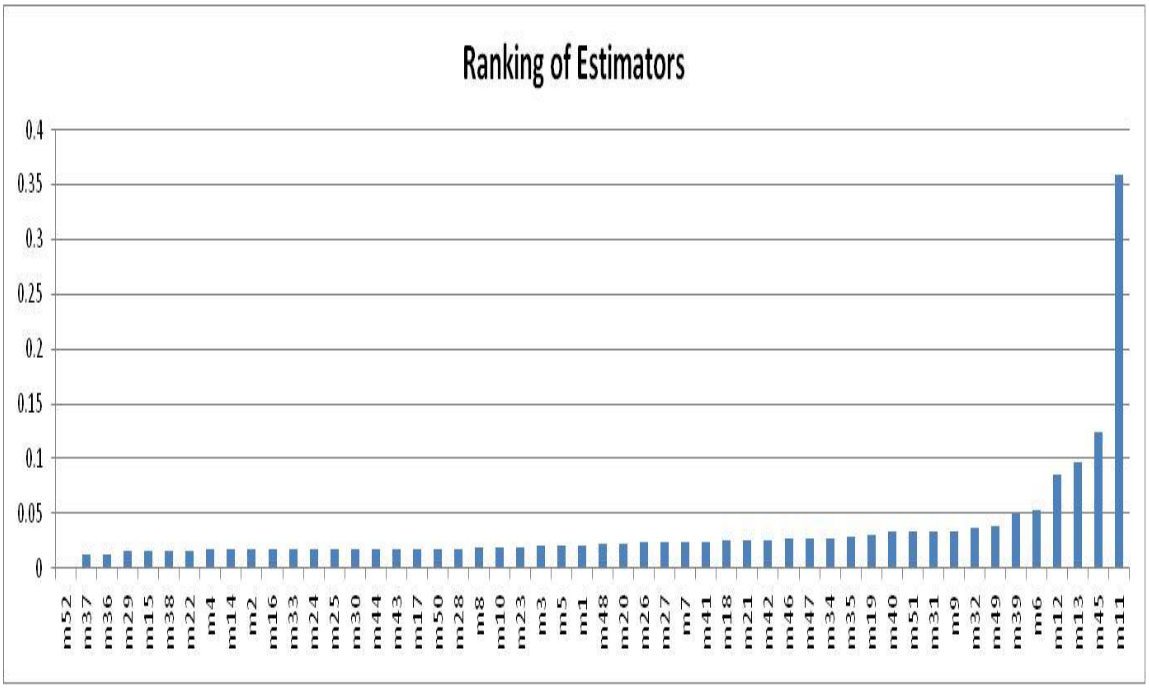

Figure 3:Relative Position of Ratio Estimators in term of MSE

CONCLUSION

In this article we made an attempt to compare all the existing ratio estimators in simple random sampling. There are about fifty-two ratio estimator in simple random sampling which have been proposed by different authors time to time. For the comparison purpose we use simulation study using three different cases. These cases are discussed to see the effect of average and dispersion in the estimation of average using proposed ratio estimators. For this purpose, simulation study carried out and we conclude that approximately fifty-two ratio estimators in simple random sampling. Simulation study shows that estimator $\hat{m}_{52}$ performs well in term of MSE and RE. This estimator is recently developed by Noor-ul-Amin (2021) and called memory base ratio estimator. Second most efficient estimator from our study is $\hat{m}_{37}$ for all sample size. This estimator was developed by Subrammani and Prabavathy (2014). In this estimator author introduced median of study and auxiliary variable in addition of population mean of auxiliary variable. The third most efficient estimator for this case is $\hat{m}_{36}$, which is the another estimator developed by Subrammani and Prabavathy (2014). In this estimator author make use of median of study and auxiliary variable. These three estimators are efficient for all sample sizes under study.

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