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SOME GRAPH LABELINGS IN COMPETITION GRAPH OF CAYLEY DIGRAPHS

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ABSTRACT: In this paper we present an algorithm and prove the existence of graph labelings such as Z_3-magic, Cordial, total cordial, E-cordial, total E-cordial, Product cordial, total product cordial, Product E-cordial, total product E-cordial labelings for the Competition graph of the Cayley digraphs associated with the diheadral group D_n.

AMS SUBJECT CLASSIFICATION: 05C78.

KEYWORDS: Cayley digraph, Graph labeling, A-magic.

1. INTRODUCTION

The concept of graph labeling was introduced by Rosa in 1967 [7]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs serve as useful models for broad range of applications such as coding theory, X-ray, Crystallography, radar, astronomy, circuit design, communication networks and data base management and models for constraint programming over finite domain . Hence in the intervening years various labeling of graphs such as graceful labeling, harmonious labeling, magic labeling, antimagic labeling, bimagic labeling, prime labeling, cordial labeling, toal cordial labeling, k-graceful labeling and odd graceful labeling etc., have been studied in over 1100 papers [2].

Cahit has introduced cordial labeling [3]. In [4], it is proved that every tree is cordial; K_n is cordial if and only if $n \leq 3$, $K_{m, n}$ is cordial for all m and n. Friendship graph $C_3^{(t)}$ is cordial if and only if $t \equiv 2$ (mod 2) and all fans are cordial. In [1], Andaretal proved that the t-ply graph $P_t(u, v)$ is cordial except when it is Eulerian and the number of edges is congruent to 2 (mod 4). In [11], Youssef proved that every Skolem-graceful graph is cordial.

A new labeling called E-cordial was introduced by Yilmaz and Cahit in 1997 [10]. They proved the following graphs are E-cordial: trees with n vertices if and only if $n \neq 2 \pmod 4$; K_n if and only if $n \neq 2 \pmod 4$; $K_{m, n}$ if and only if $m + n \neq 2 \pmod 4$; C_n if and only if $n \neq 2 \pmod 4$; regular graphs of degree 1 on 2n vertices if and only if n is even; friendship graphs C_3^n for all n; fans F_n if and only if $n \neq 1 \pmod 4$; and wheels W_n if and only if $n \neq 1 \pmod 4$. They also observe that with $n \equiv 2 \pmod 4)$

vertices can not be E-cordial. More over the graph labelings on digraphs have been extensively studied in literatures [8, 9].

In 1878, Cayley constructed a graph with a generating set which is now popularly known as Cayley graphs. A directed graph or digraph is a finite set of points called vertices and a set of arrows called arcs connecting some vertices. The Cayley graphs and Cayley digraphs are excellent models for interconnection networks [2, 6]. Many well-known interconnection networks are Cayley digraphs. For example hypercube, butterfly, and cube-connected cycle's networks are Cayley graphs. The Cayley digraph of a group provides a method of visualizing the group and its properties. The properties such as commutativity and the multiplication table of a group can be recovered from a Cayley digraph.

The original concept of an A-magic graph is due to dedlack, who defined it to be a graph with real-valued edge labeling such that distinct edges have distinct non-negative labels which satisfies the condition that the sum of the labels of the edges incident to a particular vertex is the same for all vertices.

In this paper we prove the existence of graph labelings such as Z_3 magic, Cordial, total cordial, E-cordial, total E-cordial, Product cordial, total product cordial, Product E-cordial, total product E-cordial for the competition graph of the Cayley digraphs associated with the diheadral group D_n.

2. PRELIMINARIES

In this section we give the basic notation relevant to this paper. Let G = G (V, E) be a finite, simple and undirected graph with p vertices and q edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers called labels (usually the set of integers). In this paper we deal with the labeling with domain either the set of all vertices or the set of all edges or the set of all vertices and edges. We call these labelings as the vertex labeling or the edge labeling or the total labeling respectively.

Definition 2.1: A function f from the vertex set $V \to \{0, 1\}$ such that each edge uv assigning the label $|f(u) - f(v)|$ is said to be a cordial labeling if the number of vertices labeled 0 and the number of vertices labeled 1 differ at most by 1, and number of edges labeled 0 and the number of edges labeled 1 differ atmost by 1.

Definition 2.2: A function f from the vertex set $V \to \{0, 1\}$ such that each edge uv assign the label $f(u) \times f(v)$ is said to be a product cordial labeling if the number of vertices labeled 0 and the number of vertices labeled 1 differ atmost by 1, and number of edges labeled 0 and the number of edges labeled 1 differ atmost by 1.

Definition 2.3: A function f from the vertex set $V \to \{0, 1\}$ such that each edge uv assign the label $f(u) \times f(v)$ is said to be a total product cordial llabeling if the number of vertices and edges labeled with 0 and the number of vertices and edges labeled with 1 differ atmost by 1. A graph with total product cordial labeling is called total product cordial graph.

Definition 2.4: Let G be a graph with vertex set V and edge set E and let f be function from E to {0, 1}. Define f^* on V by $f^*(v) = \sum\{f(uv)/uv \in E\}$ (mod 2). The function f is called E-cordial labeling of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ atmost by 1. A graph that admits E-cordial labeling is called E-cordial.

Definition 2.5: A (p, q)-digraph G = (V, E) is defined by a set V of vertices such that $|V| = p$ and a set E of arcs or directed edges with $|E| = q$. The set E is a subset of elements (u, v) of $V \times V$. The out-degree (or in-degree of a vertex u of a digraph G is the number of arcs (u, v) (or (v, u)) of G) is denoted by $d^+(u)$ (or $d^-(u)$). A digraph is said to be regular if $d^+(u) = d^-(v)$ for every vertex u of G.

3. MAIN RESULT

In this section we present an algorithm and prove the existence of graph labeling such as Z_3-magic, Cordial, total cordial, E-cordial, total E-cordial, Product cordial, total product cordial, Product E-cordial, total Product E-cordial for the Competition graphs of Cayley digraphs associated with diheadral group D_n.

Definition 3.1: Let G (V, E) be a (p, q) digraph. It is said to admit E-cordial labeling if there exists a function f from E onto the set {0, 1} such that the induced map f^* on V is defined as $f^*(v_i) = \sum\{f(v_i, v_j)/v_i v_j \in E\}$ (mod 2) satisfing the property that the number of arcs labeled 0 and the number of arcs labeled 1 differ at most by 1.

Definition 3.2: Let G (V, E) be a (p, q) digraph. It is said to admit total E-cordial labeling if there exists a function f from E onto the set {0, 1} such that the induced map f^* on V is defined as $f^*(v_i) = \sum\{f(v_i v_j)/v_i v_j \in E\}$ (mod 2) satisfing the property that the number of vertices and arcs labeled with 0 and the number of vertices and arcs labeled with 1 differ at most by 1.

Definition 3.3: Let G (V, E) be a (p, q) digraph. It is said to admit product E-cordial labeling if there exists a function f from E onto the set {0, 1} such that the induced map f^* on V is defined as $f^*(v_i) = \prod\{f(v_i v_j)/v_i v_j \in E\}$ (mod 2) satisfying the property that if the number of vertices labeled 0 and the number of vertices labeled 1 differ atmost by 1, and number of arcs labeled 0 and the number of arcs labeled 1 differ at most by 1.

Definition 3.4: Let G (V, E) be a (p, q) digraph. It is said to admit total product E-cordial labeling if there exists a function f from E onto the set {0, 1} such that the induced map f^* on V is defined as $f^*(v_i) = \prod\{f(v_i v_j)/v_i v_j \in E\}$ (mod 2) satisfying the property that that the number of vertices and arcs labeled with 0 and the number of vertices and arcs labeled with 1 differ at most by 1.

Definition 3.5: Let G (V, E) be a (p, q) digraph. It is said to admit Z_3-magic labeling if there exists a function f from E onto the set {1, 2} such that the induced map f^* on V defined by $f^*(v_i) =\{\sum f(e) \pmod 3 = k$, a constant and $e = (v_i v_j) \in E\}$.

Defintion 3.6: Let G be a finite group and S be a generating subset of G. The Cayley digraph Cay (G, S) is the digraph whose vertices are the elements of G, and there is an edge from g to gs whenever $g \in G$ and $s \in S$. If $S = S^{-1}$ then there is an edge from g to gs if and only if there is an arc from gs to g.

Definition 3.7: Let $V = \{v_1, v_2, v_3, v_n\}$ and $E(E_a, E_b) = \{e_1, e_2, e_3, \ldots, e_{2n}\}$ be the vertex and arc sets of Cayley digraphs of diheadral group. The competition graph of Cay (D_n, (a, b)) denoted by ComCay (D_n, (a, b)) is a digraph consisting of same set of vertices and if for any path $v_i e_r v_j e_s v_k$ where $v_i, v_j, v_k \in V$ & $e_r, e_s \in E$, draw a new edge $v_i v_k$.

Definition 3.8: The structure of competition graph ComCay (D_n, (a, b)) is defined as follows. From the construction of competition graph using definition 3.7, the ComCay (D_n, (a, b)) has n vertices and 4n arcs. Let us denote the vertex set of ComCay (D_n, (a, b)) as $V = \{v_1, v_2, v_3, \ldots, v_n\}$. Denote the arc set of ComCay (D_n, (a, b)) as $E(E_{aa}, E_{ab}, E_{ba}, E_{bb}) = \{e_1, e_2, e_3, \ldots, e_{4n}\}$, where

E_{aa} = The set of all arcs obtained through (a, a),

E_{ab} = The set of all arcs obtained through (a, b),

E_{ba} = The set of all arcs obtained through (b, a),

E_{bb} = The set of all arcs obtained through (b, b).

Thus the arc set of the ComCay (D_n, (a, b)) is as follows:

(i) $v_i v_{i+2} \in E_{aa}$, where $1 \le i \le (n/2) - 2$ & $(n/2) + 1 \le i \le n - 2$

$v_i v_{i-2} \in E_{aa}$, where $(n/2) - 1 \le i \le (n/2)$ & $n - 1 \le i \le n$

(ii) $v_i v_{n-i}, v_{n/2} v_n, v_n v_{n/2} \in E_{ab}$, where $1 \le i \le (n/2) - 1$ & $(n/2) + 1 \le i \le n - 1$

(iii) $v_i v_{n-i+2}, v_{n-i+2} v_i, v_{(n/2)+1} v_1, v_1 v_{(n/2)+1} \in E_{ba}$, $2 \le i \le n/2$

(iv) $v_i v_j \in E_{bb}$, $i = j$ & $1 \le i \le n$.

Now we present an algorithm to get Z_3-magic, Cordial, E-cordial, Product cordial and Product E-cordial labeling for the competition graph ComCay $(D_n, (a, b))$.

Algorithm:

Input : The Diheadral group D_n with the generating set (a, b).

Step 1: Using definition 3.6, Construct Cayley digraph Cay $(D_n, (a, b))$

Step 2: Using definition 3.7, Construct competition graph ComCay $(D_n, (a, b))$.

Step 3: Denote the vertex set of ComCay $(D_n, (a, b))$ as $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and the the arc set as $E(E_{aa}, E_{ab}, E_{ba}, E_{bb}) = \{e_1, e_2, e_3, \ldots, e_{4n}\}$

where E_{aa} = The set of all arcs obtained through (a, a)

E_{ab} = The set of all arcs obtained through (a, b)

E_{ba} = The set of all arcs obtained through (b, a)

E_{bb} = The set of all arcs obtained through (b, b)

Step 4: (for Z_3-magic labeling)

Define f on E as follows:

$$f(v_i v_j) = \begin{cases} 2, & \text{where } v_i v_j \in E_{aa} \ \& \ E_{ba}; \\ 1, & \text{where } v_i v_j \in E_{ab} \ \& \ E_{bb}. \end{cases}$$

Step 5: (for cordial labeling)

Define f on V as follows:

$$f(v_i) = \begin{cases} 1, & 1 \le i \le n/2; \\ 0, & (n/2) + 1 \le i \le n, \quad v_i \in V. \end{cases}$$

Step 6: (for E-cordial labeling)

Define f on E as follows:

(i) For all $v_i v_{i+2}, v_i v_{i-2} \in E_{aa}$,

$$f(v_i v_{i+2}) = \begin{cases} 1, & 1 \le i \le (n/2) - 2; \\ 0, & (n/2) + 1 \le i \le n - 2. \end{cases}$$

$$f(v_i v_{i-2}) = \begin{cases} 1, & (n/2) - 1 \le i \le (n/2); \\ 0, & n - 1 \le i \le n. \end{cases}$$

(ii) For all $v_iv_j \in E_{ab}$, $f(v_i v_j) = 1$

(iii) For all $v_i v_j \in E_{ba}$, $f(v_i v_j) = 0$

(iv) For all $v_i v_j \in E_{bb}$ and $i = j$,

$$f(v_i v_j) = \begin{cases} 1, & i \equiv 1 \pmod 2; \\ 0, & i \equiv 0 \pmod 2). \end{cases}$$

Step 7: (for product cordial labeling)

Define f on V as follows: For all $1 \le i \le n$, $v_i \in V$

$$f(v_i v_j) = \begin{cases} 0, & i \equiv 1 \pmod 2; \\ 1, & i \equiv 0 \pmod 2). \end{cases}$$

Step 8: (for product E-cordial labeling)

Define f on E as follows:

(i) For all $v_iv_j \in E_{aa}$:

$$f(v_i v_{i+2}) = \begin{cases} 0, & 1 \le i \le (n/2) - 2; \\ 1, & (n/2) + 1 \le i \le n - 2. \end{cases}$$

$$f(v_i v_{i-2}) = \begin{cases} 0, & (n/2) - 1 \le i \le (n/2); \\ 1, & n - 1 \le i \le n. \end{cases}$$

(ii) For all $v_iv_j \in E_{ab}$:

$$f(v_i v_{n-i}) = \begin{cases} 1, & 1 \le i \le (n/2) - 1; \\ 0, & (n/2) + 1 \le i \le n - 1. \end{cases}$$

$f(v_{n/2} v_n) = 1$ & $f(v_n v_{n/2}) = 0$.

(iii) For all $v_iv_j \in E_{ba}$ & $2 \le i \le n/2$

$f(v_i v_{n-i+2}) = f(v_1 v_{(n/2)+1}) = 1$ & $f(v_{n-i+2} v_i) = f(v_{(n/2)+1} v_1) = 0$

(iv) For all $v_iv_j \in E_{bb}$ & $i = j$:

$$f(v_i v_j) = \begin{cases} 0, & 1 \le i \le (n/2); \\ 1, & (n/2) + 1 \le i \le n. \end{cases}$$

Output: Z_3-magic, Cordial, E-cordial, Product cordial and Product E-cordial labeling for the competition graph ComCay $(D_n, (a, b))$.

Theorem 3.1: The competition graph ComCay (D_n, (a, b)) admits Z_3-magic labeling.

Proof: From the construction of Competition graph ComCay (D_n, (a, b)), we have n vertices and 4n arcs. Denote the vertex set and arc set using step 3 of algorithm.

To prove ComCay (D_n, (a, b)) admits Z_3-magic labeling, we have to show that there exists a function f from E onto the set Z_3-{0} such that the induced map f^* on V defined by $f^*(v_i) = \{\sum f(e) \pmod 3 = k$, a constant/$e = (v_i v_j) \in E\}$. i.e., for any $v_i \in V$, the sum of the labels of the arcs incident at v_i is a constant. Consider an arbitrary vertex $v_i \in V$ of ComCay (D_n, (a, b)). Using step 4 of algorithm, define a map $f : E \to Z_3$-{0} such that

$$f(v_i v_j) = \begin{cases} 2, & \text{where} \quad v_i v_j \in E_{aa} \ \& \ E_{ba}; \\ 1, & \text{where} \quad v_i v_j \in E_{ab} \ \& \ E_{bb}. \end{cases}$$

Hence for the induced map $f^* : V \to Z_3$, for all $v_i \in V$, $f^*(v_i) = \sum f(v_i v_j) \pmod 3 = (2 + 2 + 1 + 1) \pmod 3 = 0 \pmod 3$, where $v_i v_j \in E_{aa}, E_{ba}, E_{ab}$ & E_{bb}. Thus $f^*(v_i) = 0 \pmod 3$ which is a constant for all i.

Hence Competition graph ComCay (D_n, (a, b)) admits Z_3-magic labeling.

Example 3.1: Competition graph of the Cayley digraph associated with the diheadral group D_8 and its Z_3-magic labeling is shown in Figure 1.

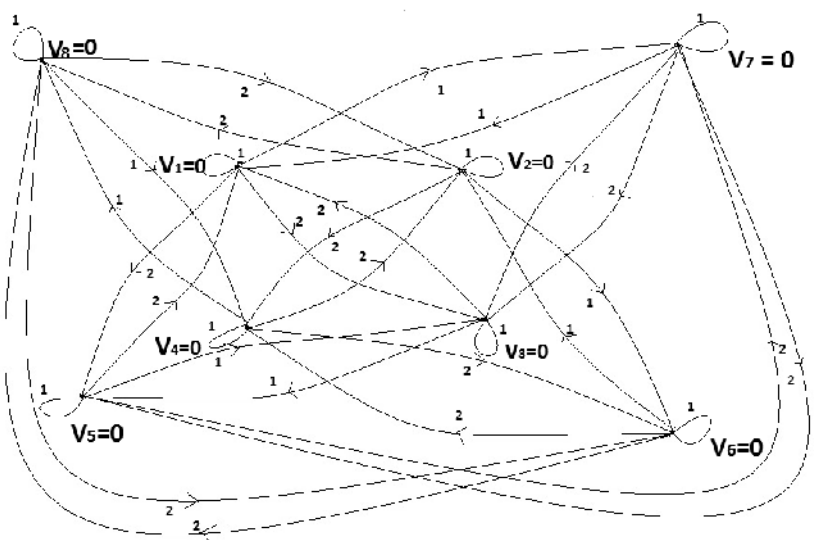

Figure 1: Z_3-Magic Labeling for ComCay (D_8, (a, b))

Theorem 3.2: The competition graph ComCay (D_n, (a, b)) admits Cordial labeling.

Proof: From the construction of Competition graph ComCay (D_n, (a, b)), we have n vertices and 4n arcs. Denote the vertex set and arc set using step 3 of algorithm. To prove ComCay (D_n, (a, b)) admits Cordial labeling, we have to show that there exists a function $f : V \to \{0, 1\}$ such that $f^*(v_i v_j) = \{(f(v_i) + f(v_j)) \pmod 2 / v_i v_j \in E(E_{aa}, E_{ba}, E_{ab}, E_{bb})\}$ which satisfies the property that the number of vertices labeled 0 and the number of vertices labeled 1differ atmost by 1 and number of arcs labeled 0 and the number of arcs labeled 1 differ by atmost 1. Consider the arbitrary vertex $v_i \in V$. Using step 5 of the Algorithm, we define a map $f : V \to \{0, 1\}$ as follows:

$$f(v_i) = \begin{cases} 1, & 1 \le i \le n/2; \\ 0, & (n/2)+1 \le i \le n, \quad v_i \in V. \end{cases}$$

Thus the number of vertices labeled 1 is n/2 and the number of vertices labeled 0 is n/2. Hence the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1. In order to get the labels for the arcs, define the induced map $f^* : E \to \{0, 1\}$ such that $f^*(v_i v_j) = (f(v_i) + f(v_j)) \pmod 2$, $v_i v_j \in E(E_{aa}, E_{ba}, E_{ab}, E_{bb})$

(i) $$f(v_i) = \begin{cases} 0 \pmod 2, & \text{for all} \quad v_i v_j \in E_{aa} \ \& \ E_{bb}; \\ 1 \pmod 2, & \text{for all} \quad v_i v_j \in E_{ab} \ \& \ E_{ba}. \end{cases}$$

That is, the number of arcs labeled 0 is n + n = 2n and the number of arcs labeled 1 is n + n = 2n. Thus the number of arcs labeled 0 and the number of arcs labeled 1 differ by at most 1. Hence ComCay (D_n, (a, b)) admits Cordial labeling.

Example 3.2: Competition graph of the Cayley digraph associated with the diheadral group D_8 and its Cordial labeling is shown in Figure 2.

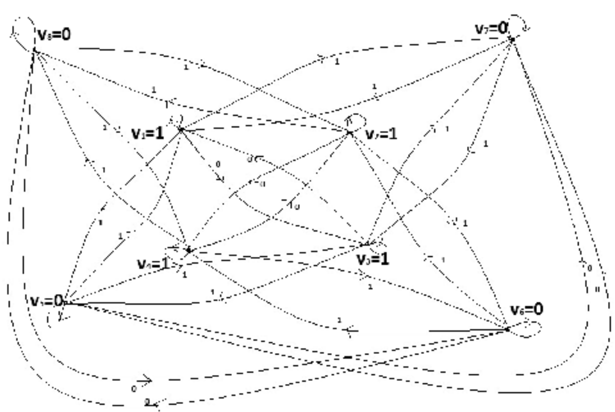

Figure2: Cordial Labeling for ComCay (D_8, (a, b))

Theorem 3.3: The competition graph ComCay (D_n, (a, b)) admits total cordial labeling.

Proof: To prove ComCay (D_n, (a, b)) admits total Cordial labeling, we have to show that there exists a function $f : V \to \{0, 1\}$ such that $f^*(v_i v_j) = \{(f(v_i) + f(v_j))$ (mod 2), $v_i v_j \in E(E_{aa}, E_{ba}, E_{ab}, E_{bb})\}$ which satisfies the property that that the number of zeroes on the vertices and arcs taken together differ by at most 1 with the number of one's on vertices and arcs taken together.

By the above theorem, using the map f on V and there by the induced map f^* on E, we have the number of arcs labeled 0 is 2n and the number of vertices labeled 0 is n/2. Also, the number of arcs labeled by 1 is 2n and the number of vertices labeled by 1 is n/2. Thus the total number of one's on vertices and arcs taken together is (n/2) + 2n = 5n/2 and the the total number of zeroes on vertices and arcs taken together is (n/2) + 2n = 5n/2. Thus the number of zeroes on the vertices and arcs taken together differ by at most 1 with the number of one's on vertices and arcs taken together. Hence, the competition graph ComCay (D_n, (a, b)) admits total cordial labeling.

Theorem 3.4: The competition graph ComCay (D_n, (a, b)) admits E-Cordial labeling.

Proof: From the construction of Competition graph ComCay (D_n, (a, b)), we have n vertices and 4n arcs. To prove Comcay (D_n, (a, b)) admits E-Cordial labeling, we have to show that there exists a function $f : E \to \{0, 1\}$ such that the induced function f^* on V is defined as $f^*(v_i) = \sum \{f(v_i v_j), v_i v_j \in E(E_{aa}, E_{ba}, E_{ab}, E_{bb})\}$ (mod 2) which satisfies the property that the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1 and the number of arcs labeled 0 and the number of arcs labeled 1 differ by at most 1. Consider the arbitrary vertex $v_i \in V$. Using step 6 of algorithm, we define a map $f : E \to \{0, 1\}$ as follows:

(i) For all $v_i v_{i+2}, v_i v_{i-2} \in E_{aa}$:

$$f(v_i v_{i+2}) = \begin{cases} 1, & 1 \le i \le (n/2) - 2; \\ 0, & (n/2) + 1 \le i \le n - 2. \end{cases}$$

$$f(v_i v_{i-2}) = \begin{cases} 1, & (n/2) - 1 \le i \le (n/2); \\ 0, & n - 1 \le i \le n. \end{cases}$$

(ii) For all $v_i v_j \in E_{ab}$, $f(v_i v_j) = 1$.

(iii) For all $v_i v_j \in E_{ba}$, $f(v_i v_j) = 0$.

(iv) For all $v_i v_j \in E_{bb}$ and $i = j$:

$$f(v_i v_j) = \begin{cases} 1, & i \equiv 1 \pmod 2; \\ 0, & i \equiv 0 \pmod 2. \end{cases}$$

From these definitions of the labeling functions, we have the total number of arcs labeled 0 is (n/2) + (n/2) + (n/2) + (n/2) = 2n and the total number of arcs labeled 1 is (n/2) + (n/2) + (n/2) + (n/2) = 2n. Thus the number of arcs labeled 0 and the number of arcs labeled 1 differ by at most 1. In order to get the labels for the vertices, define the induced map $f^* : V \to \{0, 1\}$ such that

(i) For all $1 \le i \le n/2$ & $v_i v_j \in E\,(E_{aa}, E_{ba}, E_{ab}, E_{bb})$

$$f(v_i) = \sum f(v_i v_i) = \begin{cases} 1+1+0+1=1, & i \equiv 1 \pmod 2; \\ 1+1+0+0=0, & i \equiv 0 \pmod 2. \end{cases}$$

(ii) For all $n/2 \le i \le n$ & $v_i v_j \in E\,(E_{aa}, E_{ba}, E_{ab}, E_{bb})$

$$f(v_i) = \sum f(v_i v_j) = \begin{cases} 0+1+0+1=0, & i \equiv 1 \pmod 2; \\ 0+1+0+0=1, & i \equiv 0 \pmod 2. \end{cases}$$

Under this map, the number of vertices labeled 1 is (n/4) + (n/4) = n/2 and the number of vertices labeled 0 is (n/4) + (n/4) = n/2. Thus the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1. Hence the competition graph ComCay (D_n, (a, b)) admits E-cordial labeling.

Example 3.3: Competition graph of the Cayley digraph associated with the diheadral group D_8 and its E-cordial labeling is shown in Figure 3.

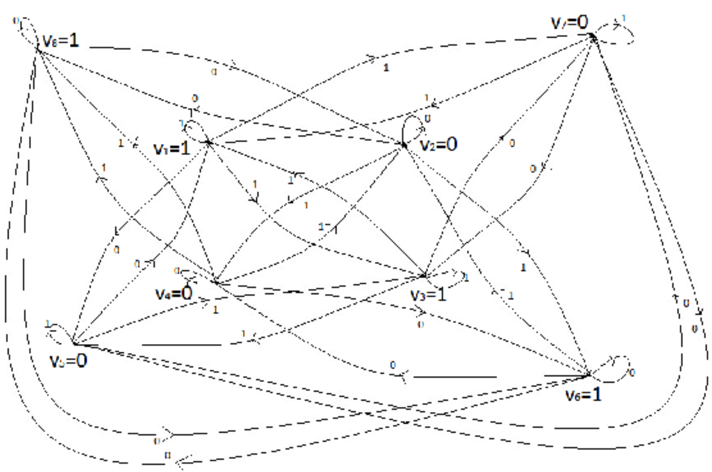

Figure 3: E-Cordial Lebeling for ComCay (D_n, (a, b))

Theorem 3.5: The competition graph ComCay (D_n, (a, b)) admits total E-cordial labeling.

Proof: To prove Comcay (D_n, (a, b)) admits total E-cordial labeling, we have to show that there exists a function $f : E \rightarrow \{0, 1\}$ such that the induced function f^* on V is defined as $f^*(v_i) = \sum \{f(v_i v_j), v_i v_j \in E(E_{aa}, E_{ba}, E_{ab}, E_{bb})\} \pmod 2$ which satisfies the property that the number of zeroes on the vertices and arcs taken together differ by at most 1 with the number of one's on vertices and arcs taken together.

By the above theorem, using the map f on E and there by the induced map f^* on V, we have the number of arcs labeled 0 is 2n and the number of vertices labeled 0 is n/2. Also, the number of arcs labeled by 1 is 2n and the number of vertices labeled by 1 is n/2.

Thus the total number of one's on vertices and arcs taken together is (n/2) + 2n = 5n/2 and the the total number of zeroes on vertices and arcs taken together is (n/2) + 2n = 5n/2.

Thus the number of zeroes on the vertices and arcs taken together differ by at most 1 with the number of one's on vertices and arcs taken together.

Hence The competition graph ComCay (D_n, (a, b)) admits total E-cordial labeling.

Theorem 3.6: The competition graph ComCay (D_n, (a, b)) admits Product Cordial labeling.

Proof: From the construction of Competition graph ComCay (D_n, (a, b)) using algorithm, we have n vertices and 4n arcs. To prove ComCay (D_n, (a, b)) admits Product Cordial labeling, we have to show that there exists a function $f : V \rightarrow \{0, 1\}$ such that $f^*(v_i v_j) = \{(f(v_i) \times f(v_j), v_i v_j \in E(E_{aa}, E_{ba}, E_{ab}, E_{bb})\}$ which satisfies the property that the number of vertices labeled 0 and the number of vertices labeled 1 differ atmost by 1 and number of arcs labeled 0 and the number of arcs labeled 1 differ by atmost 1. Consider the arbitrary vertex $v_i \in V$. Using step 7 of algorithm we define a map $f : V \rightarrow \{0, 1\}$ as follows. For all $1 \leq i \leq n$, $v_i \in V$

$$f(v_i) = \begin{cases} 0, & i \equiv 1 \pmod 2; \\ 1, & i \equiv 0 \pmod 2). \end{cases}$$

From this definition of the labeling functions, we have the total number of vertices labeled 0 is n/2 and the number of vertices labeled 1 is n/2. Hence the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1. In order to get the labels for the arcs, define the induced map $f^* : E \rightarrow \{0, 1\}$ such that $f^*(v_i v_j) = \{f(v_i) \times f(v_j) / v_i v_j \in E(E_{aa}, E_{ba}, E_{ab}, E_{bb})\}$.

Now for all $v_i v_j \in E_{aa}, E_{ab}, E_{ba}$ & E_{bb}

$$f^*(v_i v_j) = f(v_i) \times f(v_j) \begin{cases} 1, & i \;\&\; j \equiv 1 \pmod 2; \\ 0, & i \;\&\; j \equiv 0 \pmod 2). \end{cases}$$

Under this map the number of arcs labeled 0 is n + n = 2n and the number of arcs labeled 1 is n + n = 2n. Thus the number of arcs labeled 0 and the number of arcs labeled 1 differ by atmost 1.

Hence ComCay (D_n, (a, b)) admits Product Cordial labeling.

Example 3.4: Competition graph of the Cayley digraphs associated with the diheadral group D_8 and its Product cordial labeling is shown in Figure 4.

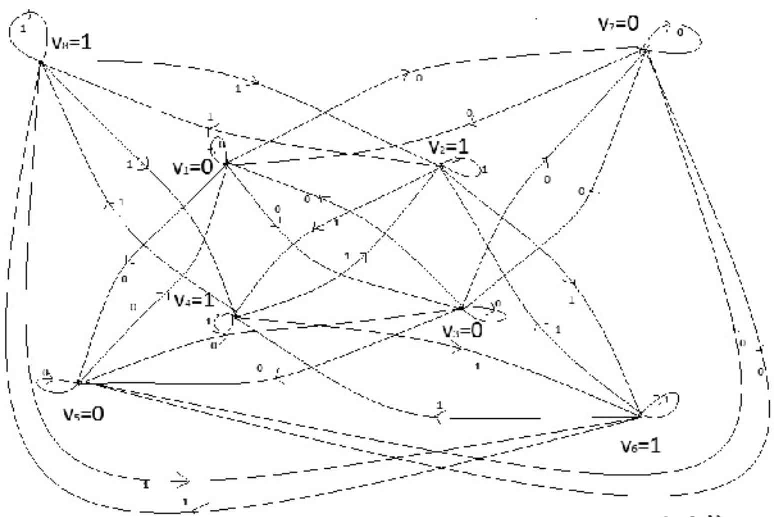

Figure 4: Product Cordial Lebeling for ComCay (D_8, (a, b))

Theorem 3.7: The competition graph ComCay (D_n, (a, b)) admits total product cordial labeling.

Proof: From the construction of Competition graph ComCay (D_n, (a, b) using algorithm, we have n vertices and 4n arcs.

To prove ComCay (D_n, (a, b)) admits total product cordial labeling, we have to show that there exists a function $f : V \to \{0, 1\}$ such that $f^*(v_i v_j) = \{(f(v_i) \times f(v_j)/v_i v_j \in E(E_{aa}, E_{ba}, E_{ab}, E_{bb})\}$ which satisfies the property that the number of zeroes on the vertices and arcs taken together differ by atmost 1 with the number of one's on vertices and arcs taken together.

By the above theorem, using the map f on E and there by the induced map f^* on E, we have the number of arcs labeled 0 is 2n and the number of vertices labeled 0 is n/2. Thus the total number of zeroes on vertices and arcs taken together is (n/2) + 2n = 5n/2. Also, the number of arcs labeled by 1 is 2n and the number of vertices labeled by 1 is n/2. Thus the total number of one's on vertices and arcs taken together is (n/2) + 2n = 5n/2. Thus the number of zeroes on the vertices and arcs taken together differ by atmost 1 with the number of one's on vertices and arcs taken together. Hence The competition graph ComCay (D_n, (a, b)) admits total product cordial labeling.

Theorem 3.8: The competition graph ComCay (D_n, (a, b)) admits Product E-cordial labeling.

Proof: From the construction of Competition graph ComCay (D_n, (a, b)) using algorithm, we have n vertices and 4n arcs.

To prove Comcay (D_n, (a, b)) admits Product E-Cordial labeling, we have to show that there exists a function $f : E \to \{0, 1\}$ such that the induced function f^* on V is defined as $f^*(v_i) = \prod \{f(v_i v_j)/v_i v_j \in E(E_{aa}, E_{ba}, E_{ab}, E_{bb})\}$ which satisfies the property that the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1 and the number of arcs labeled 0 and the number of arcs labeled 1 differ by atmost 1.

Consider the arbitrary vertex $v_i \in V$. Using step 8 of algorithm we define a map $f : V \to \{0, 1\}$ as follows:

(i) For all $v_i v_j \in E_{aa}$:

$$f(v_i v_{i+2}) = \begin{cases} 0, & 1 \le i \le (n/2) - 2; \\ 1, & (n/2) + 1 \le i \le n - 2. \end{cases}$$

$$f(v_i v_{i-2}) = \begin{cases} 0, & (n/2) - 1 \le i \le (n/2); \\ 1, & n - 1 \le i \le n. \end{cases}$$

(ii) For all $v_i v_j \in E_{ab}$:

$$f(v_i v_{n-i}) = \begin{cases} 1, & 1 \le i \le (n/2) - 1; \\ 0, & (n/2) + 1 \le i \le n - 1. \end{cases}$$

$$f(v_{n/2} v_n) = 1 \text{ \& } f(v_n v_{n/2}) = 0.$$

(iii) For all $v_i v_j \in E_{ba}$ & $2 \le i \le n/2$

$$f(v_i v_{n-i+2}) = f(v_1 v_{(n/2)+1}) = 1 \text{ \& } f(v_{n-i+2} v_i) = f(v_{(n/2)+1} v_1) = 0.$$

(iv) For all $v_iv_j \in E_{bb}$ & i = j :

$$f(v_i v_j) = \begin{cases} 0, & 1 \le i \le (n/2); \\ 1, & (n/2)+1 \le i \le n. \end{cases}$$

From these definitions of the labeling functions, we have the total number of arcs labeled 0 is n/2 + n/2 + n/2 + n/2 = 2n and the number of arcs labeled 1 is n/2 + n/2+ n/2 + n/2 = 2n. Hence the number of arcs labeled 0 and the number of arcs labeled1differ by atmost 1.

In order to get the labels for the vertices define the induced map $f : V \to \{0, 1\}$ such that $f^*(v_i) = \prod \{f(v_iv_j)/ v_iv_j \in E(E_{aa}, E_{ba}, E_{ab}, E_{bb})\}$. For all $v_iv_j \in E_{aa}, E_{ab}, E_{ba}$ & E_{bb}:

$$f^*(v_i) = \prod f(v_i v_j) = \begin{cases} 0\times0\times0\times0 = 0, & v_i \in V \ \& \ 1 \le i \le (n/2); \\ 1\times1\times1\times1 = 1, \quad \text{where} & v_i \in V \ \& \ (n/2)+1 \le i \le n. \end{cases}$$

Under this map the number of vertices labeled 0 is n/2 and the number of vertices labeled 1 is n/2. Thus the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1. Hence ComCay (D_n, (a, b)) admits Product E-Cordial labeling.

Example 3.5: Competition graph of the Cayley digraphs associated with the diheadral group D_8 and its Product E-cordial labeling is shown in Figure 5.

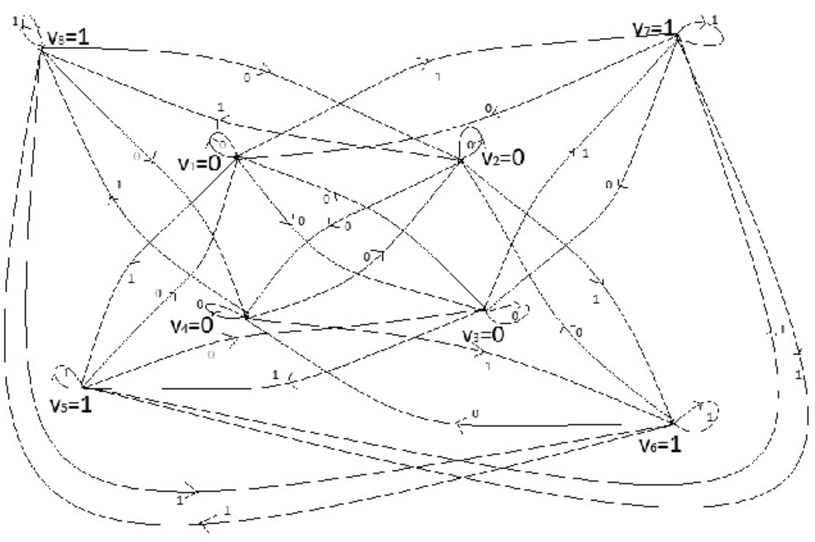

Figure 5: Product E-Cordial Lebeling for ComCay (D_8, (a, b))

Theorem 3.9: The competition graph ComCay (D_n, (a, b)) admits total product E-Cordial labeling.

Proof.: From the construction of Competition graph ComCay (D_n, (a, b), we have n vertices and 4n arcs.

To prove Comcay (D_n, (a, b)) admits total product E-Cordial labeling, we have to show that there exists a function $f : E \rightarrow \{0,1\}$ such that the induced function f^* on V is defined as $f^*(v_i) = \prod \{f(v_iv_j)/v_iv_j \in E(E_{aa}, E_{ba}, E_{ab}, E_{bb})\}$ which satisfies the property that the number of zeroes on the vertices and arcs taken together differ by atmost 1 with the number of one's on vertices and arcs taken together.

By the above theorem, using the map f on E and there by the induced map f^* on V, we have the number of arcs labeled 0 is 2n and the number of vertices labeled 0 is n/2. Thus the total number of zeroes on vertices and arcs taken together is (n/2) + 2n = 5n/2. Also, the number of arcs labeled by 1 is 2n and the number of vertices labeled by 1 is n/2. Thus the total number of one's on vertices and arcs taken together is (n/2) + 2n = 5n/2. Thus the number of zeroes on the vertices and arcs taken together differ by atmost 1 with the number of one's on vertices and arcs taken together. Hence The competition graph ComCay (D_n, (a, b)) admits total product E-cordial labeling.

4. CONCLUSION

In this paper we have presented an algorithm and proved that the competition graph of Cayley digraph associated with the diheadral graoup D_n denoted by ComCay (D_n, (a, b)) admits Z_3-magic, Cordial, total cordial, E-cordial, total E-cordial, Product cordial, total Product Cordial, Product E-Cordial and total Product E-cordial labelings.

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