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ON MEAN LABELING: QUADRILATERAL SNAKE ATTACHMENT OF PATH: $P_M(QS_N)$ AND CYCLE: $C_M(QS_N)$

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ABSTRACT: The notation QS_n is Quadrilateral Snake with 'n' number of C_4 attached in a series connection as defined below:

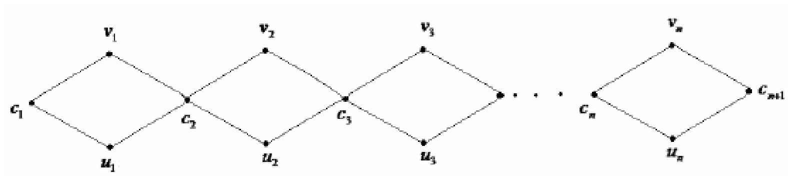

Figure 1.1 Quadrilateral Snake QS_n

u_i is adjacent to c_k and c_{k-1}, $1 < i < n$, $1 < k < n$

v_j is adjacent to c_k and c_{k-1}, $1 < j < n$, $1 < k < n$

c_k's are not adjacent.

In this paper,we proved that the graphs $P_m(QS_n)$ are Mean graph for every $m > 2$ and $n > 1$. The graphs $C_m(QS_n)$ are Mean graph for every $m > 3$, $n > 1$.

AMS SUBJECT CLASSIFICATION: 05C78.

KEYWORDS: Graph labeling, Mean labeling, Path, Cycle, Quadrilateral Snake graph.

1. INTRODUCTION

In the literature of graph labeling, it is interesting to observe that many mathematicians have constructed a larger graceful graph from certain standard graphs by using various graph operations. Join and product operations are used extensively among the graphs such as paths, cycles, stars, complete graphs, complete bipartite graphs, complement of complete graphs and graceful trees etc., to get larger graceful or harmonious graph etc., (refer Acharya and Gill (1981) [7], Balakrishnan and Kumar (1994), Bu (1994), Frucht and Gallian (1988), Grace (1983), Jungreis and Reid (1992)). Sethuraman and Kishore (1999)) are adjoined at one common edge and the resultant graphs are proved to be graceful. For an exhaustive survey of these topics one may refer to the excellent survey paper of Gallian (2010). [2]

S. Somasundaram and R. Ponraj [1] have introduced the notion of Mean labelings of graphs.

Definition 1: A graph G with p vertices and q edges is called a Mean labeling if there is an injective function ‘f’ from the vertices of G to {0, 1, 2, 3, ..., q} such that when each edge uv is labeled with $\frac{f(u)+f(v)}{2}$. If f(u) + f(v) is even and $\left(\frac{f(u)+f(v)+1}{2}\right)$. If f(u) + f(v) is odd then the resulting edge labels are distinct. If the Graph G satisfies mean labeling then it is Mean graph.

Definition 2: The graph G = $P_m(QS_n)$ is defined as isomorphic Quadrilateral snake attached with each vertex of path P_m, n is the number of C_4 attached in the Quadrilateral snake.

The graph G is as shown in the following diagram:

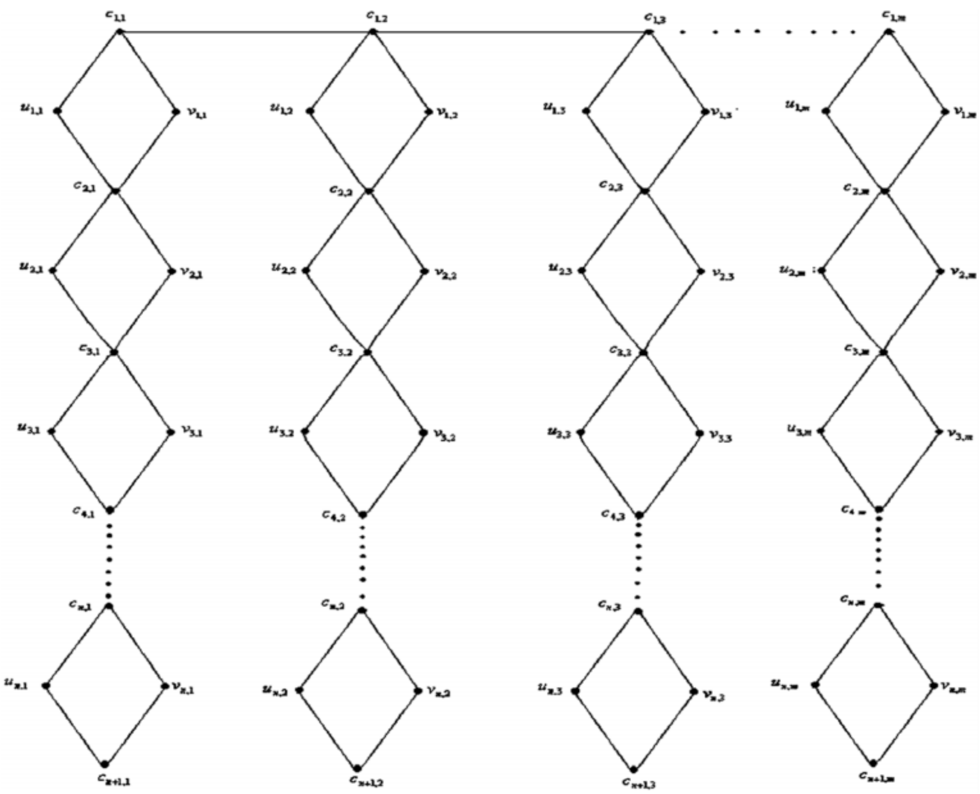

Figure 1.2: $P_m(QS_n)$

Definition 3: The graph G = $C_m(QS_n)$ is defined as isomorphic Quadrilateral Snake attached with each vertex of Cycle C_m. ‘n’ is the number of C_4 attached in the Quadrilateral Snake. The graph G is as shown in the following diagram

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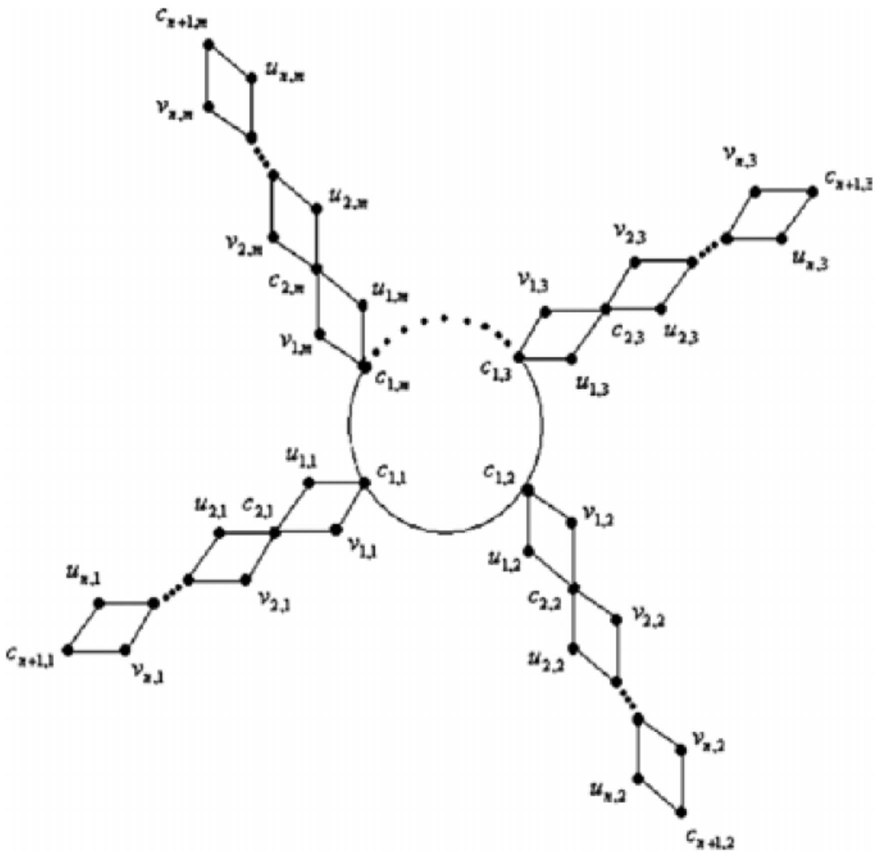

Figure 1.3: $C_m(QS_n)$

Theorem 1: The graph $P_m(QS_n)$ is Mean graph for every $m > 2$ and $n > 1$.

Proof: The graph $G = P_m(QS_n)$ has $m(3n + 1)$ vertices and $m(4n + 1) - 1$edges.

The Mean labelings for vertices of G is defined by

$$c_{i,j} = \begin{cases} 4mn+m-2i+1-(4n+1)(j-1), & j \text{ is odd}, \ i=1,\dots,n+1, \ j=1,\dots,m-1 \\ 4mn+m-4i+3-(4n+1)(j-1), & j \text{ is odd}, \ i=1,\dots,n+1, \ j=m \end{cases}$$

$$c_{i,j} = \{4mn+m+2i+j(4n+1)-1, \quad j \text{ is even}, \ i=1,\dots,n+1, \ j=1,\dots,m$$

$$u_{i,j} = \begin{cases} 4mn+m-2i-(4n+1)(j-1), & j \text{ is odd}, \ i=1,\dots,n, \ j=1,\dots,m-1 \\ 4mn+m-4i+2-(4n+1)(j-1), & j \text{ is odd}, \ i=1,\dots,n, \ j=m \end{cases}$$

$$u_{i,j} = \{4mn+m+4n-j(4n+1)+2i-2, \quad j \text{ is even}, \ i=1,\dots,n, \ j=1,\dots,m$$

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$$v_{i,j} = \begin{cases} 4mn + m - 2i + 2 - j(4n+1), & j \text{ is odd}, \ i = 1, ..., n, \ j = 1, ..., m-1 \\ 4mn + m - 4i + 1 - (4n+1)(j-1), & j \text{ is odd}, \ i = 1, ..., n, \ j = m \end{cases}$$

$$v_{i,j} = \{4mn + m + 2i - 2 - j(4n+1), \quad j \text{ is even}, \ i = 1, ..., n, \ j = 1, ..., m$$

from the above assignment the labeling of vertices and edges are distinct.

Hence the graph G = $P_m(QS_n)$ is Mean graph.

The following is an illustration of the labeling is given in the proof of 1

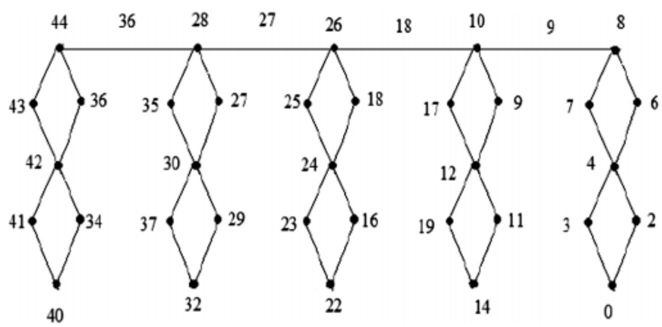

Figure 1.4: Mean Labeling of $P_5(QS_2)$

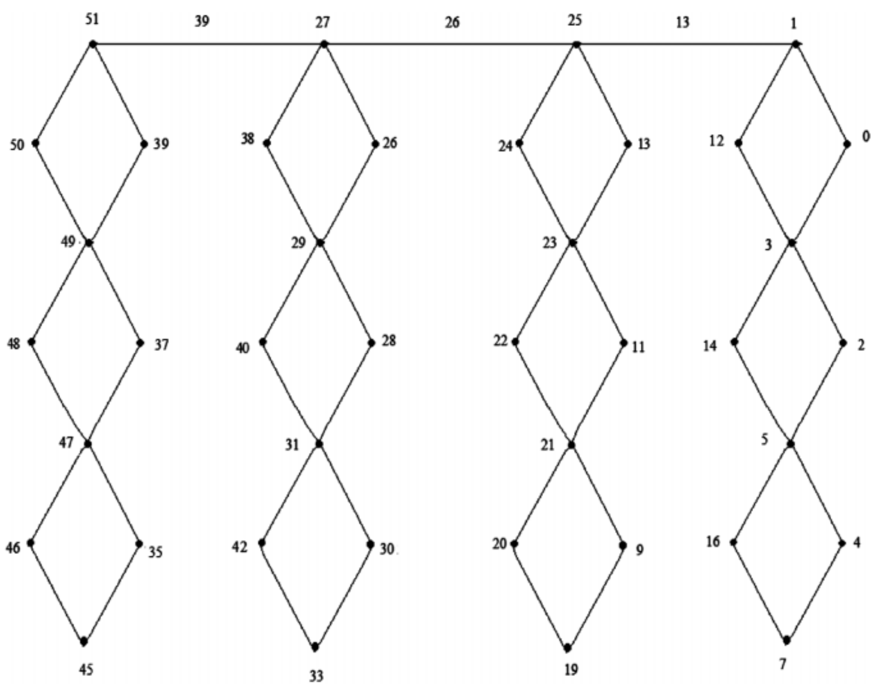

Figure 1.5: Mean Labeling of $P_4(QS_3)$

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Theorem 2: The graph $C_m(QS_n)$ is Mean graph for every $m > 3$, $n > 1$.

Proof: The graph $G = C_m(QS_n)$ has $m(1 + 3n)$ vertices and $m(1 + 4n)$ edges.

The Mean labelings for vertices of G is defined by

$$c_{i,j} = \begin{cases} & i=1,\dots,n+1 \\ m+4mn-(4n+1)(j-1)-2(i-1), \ j \text{ is odd}, & j=1,\dots,\frac{m-1}{2}, \ m \text{ is odd} \\ & j=1,\dots,\frac{m}{2}, \ m \text{ is even} \end{cases}$$

$$c_{i,j} = \begin{cases} & i=1,\dots,n+1 \\ m+4mn-2i+1-(4n+1)(j-1), \ j \text{ is odd}, & j=\frac{m+3}{2},\dots,m-2, \ m \text{ is odd} \\ & j=\frac{m+4}{2},\dots,m-1, \ m \text{ is even} \end{cases}$$

$$c_{i,j} = \begin{cases} & i=1,\dots,n+1 \\ m+4mn-4i+3-(4n+1)(j-1), \ j \text{ is odd}, & j=m, \ m \text{ is odd} \end{cases}$$

$$c_{i,j} = \begin{cases} & i=1,\dots,n+1 \\ m+4mn-8n+2i-3-(4n+1)(j-2), \ j \text{ is even}, & 2,\dots,\frac{m+1}{2}, \ m \text{ is odd} \\ & 2,\dots,\frac{m+2}{2}, \ m \text{ is even} \end{cases}$$

$$c_{i,j} = \begin{cases} & i=1,\dots,n+1 \\ m+4mn-8n+2i-4-(4n+1)(j-2), \ j \text{ is even}, & j=\frac{m+5}{2},\dots,m-1, \ m \text{ is odd} \\ & j=\frac{m+6}{2},\dots,m, \ m \text{ is even} \end{cases}$$

$$u_{i,j} = \begin{cases} & i=1,\dots,n \\ m+4mn-2i+1-(4n+1)(j-1), \ j \text{ is odd}, & j=1,\dots,\frac{m-1}{2}, \ m \text{ is odd} \\ & j=1,\dots,\frac{m}{2}, \ m \text{ is even} \end{cases}$$

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$$u_{i,j} = \begin{cases} & i = 1,\dots,n \\ m + 4mn - 2i - (4n+1)(j-1),\ j \text{ is odd}, & j = \frac{m+3}{2},\dots,m-2,\ m \text{ is odd} \\ & j = \frac{m+4}{2},\dots,m-1,\ m \text{ is even} \end{cases}$$

$$u_{i,j} = \begin{cases} & i = 1,\dots,n \\ 4mn + m - 4i + 2 - (4n+1)(j-1),\ j \text{ is odd}, & j = m,\ m \text{ is odd} \end{cases}$$

$$u_{i,j} = \begin{cases} & i = 1,\dots,n \\ m + 4mn + 4n + 2i + 1 - j(4n+1),\ j \text{ is even}, & j = 2,\dots,\frac{m-1}{2},\ m \text{ is odd} \\ & j = 2,\dots,\frac{m}{2},\ m \text{ is even} \end{cases}$$

$$u_{i,j} = \begin{cases} & i = 1,\dots,n \\ m + 4mn - 4n + 2i - 4 - (8n+1)\left(\frac{j-2}{2}\right),\ j \text{ is even}, & j = \frac{m+1}{2},\dots,m-1,\ m \text{ is odd} \\ & j = \frac{m+2}{2},\dots,m,\ m \text{ is even} \end{cases}$$

$$v_{i,j} = \begin{cases} & i = 1,\dots,n \\ m + 4mn - 2(i-1) - 4n - (4n+1)(j-1),\ j \text{ is odd}, & j = 1,\dots,\frac{m-1}{2},\ m \text{ is odd} \\ & j = 1,\dots,\frac{m}{2},\ m \text{ is even} \end{cases}$$

$$v_{i,j} = \begin{cases} & i = 1,\dots,n \\ m + 4mn - 2i + 2 - j(4n+1),\ j \text{ is odd}, & j = \frac{m+3}{2},\dots,m-2,\ m \text{ is odd} \\ & j = \frac{m+4}{2},\dots,m-1,\ m \text{ is even} \end{cases}$$

$$v_{i,j} = \begin{cases} & i = 1,\dots,n \\ 4mn + m - 4i + 1 - (4n+1)(j-1),\ j \text{ is odd}, & j = m,\ m \text{ is odd} \end{cases}$$

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$$v_{i,j} = \begin{cases} & i = 1, \ldots, n \\ m + 4mn + 2i - j(4n+1), \quad j \text{ is even}, & j = 2, \ldots, \frac{m-1}{2}, \text{ m is odd} \\ & j = 2, \ldots, \frac{m}{2}, \text{ m is even} \end{cases}$$

$$v_{i,j} = \begin{cases} & i = 1, \ldots, n \\ m + 4mn - 8n + 2i - 5 - (8n+1)\left(\frac{j-2}{2}\right), \ j \text{ is even}, & j = \frac{m+1}{2}, \ldots, m-1, \ m \text{ is odd} \\ & j = \frac{m+2}{2}, \ldots, m, \ m \text{ is even} \end{cases}$$

From the above assignment the labeling of vertices and edges are distinct.

Hence the graph G = $C_m(QS_n)$ is Mean graph.

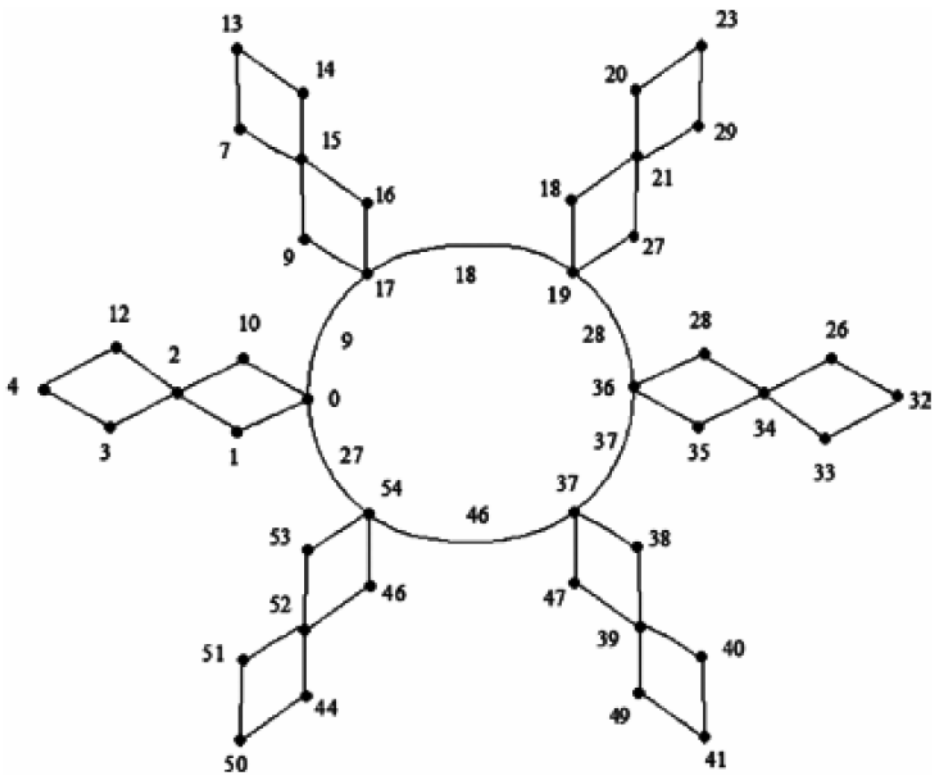

Figure 1.6: Mean Labeling of $C_6(QS_2)$

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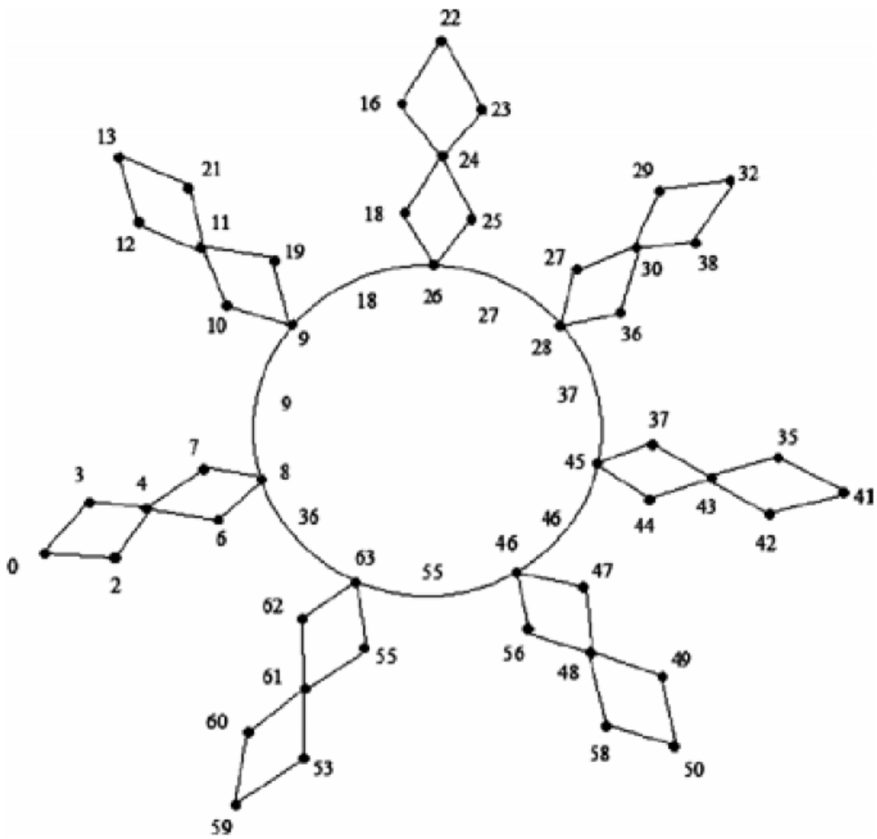

Figure 1.7: Mean Labeling of $C_7(QS_2)$

2. CONCLUSION

In this Paper ,we have given Mean labelings for the graphs $P_m(QS_n)$, for every $m > 2$, $n > 1$ and The graphs $C_m(QS_n)$ for every $m > 3$, $n > 1$ are Mean graph.

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