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Parameter Excited Chaos Synchronizations of Integral and Fractional Order Nano Resonator Systems

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In this paper, the chaos synchronizations of two uncoupled integral and fractional order chaotic nano resonator systems are obtained by replacing their corresponding parameters by the same function of chaotic state variables of a third chaotic system. It is named parameter excited chaos synchronization which can be successfully obtained for very low total fraction order 0.2. Numerical simulations are illustrated by phase portraits, Poincaré maps and state error plots.

1. INTRODUCTION

Chaos synchronization [1-9] is a very important topic in the nonlinear [10-12] science and it has been developed extensively. Recently many scientists in various fields have been attracted to investigate chaos synchronization due to its application in a variety of fields including secure communications, chemical, physical, and biological systems, neural networks and so on. So various synchronization schemes, such as variable structure control [13], parameters adaptive control [14-21], observer based control [22, 23], active control [24-30], nonlinear control [31, 32], anti-control [33-39] and so on, have been successfully applied to the chaos synchronization.

Fractional calculus is a 300-year-old mathematical topic [40-43]. Although it has a long history, for many years it was not used in physics and engineering. However, during the last 10 years or so, fractional calculus starts to attract increasing attention of physicists and engineers from an application point of view [44, 45]. It was found that many systems in interdisciplinary fields can be elegantly described with the help of fractional derivatives. Many systems are known to display fractional-order dynamics, such as viscoelastic systems [46], dielectric polarization [47], electrode-electrolyte polarization [48], electromagnetic waves [49], quantitative finance [50], and quantum evolution of complex systems [51].

More recently, many investigations are devoted to control [52–56] and dynamics [57–68] of fractional order dynamical systems. For example, in [57], it is shown that the fractional order Chua's circuit with an appropriate cubic nonlinearity and with order as low as 2.7 can produce a chaotic attractor. In [58], it is shown that nonautonomous fractional order Duffing systems of order less than 2 can still behave in a chaotic manner. In [59], chaotic behaviors of the fractional order "jerk" model is studied, in which a chaotic attractor can be generated with the system order as low as 2.1, and in [60] control of this fractional order chaotic systems is investigated. In [61, 62], the fractional order Wien bridge oscillator is studied, where it is shown that limit cycle can be generated for any fractional order, with a proper value of the amplifier gain. In [63, 64], bifurcation and chaotic dynamics of the fractional order cellular neural networks are studied. In [65], chaos and hyperchaos in fractional order Rössler equations are discussed, in which it is shown that chaos can exist in the fractional-order Rössler equation with order as low as 2.4, and hyperchaos can also exist in the fractional order Rössler hyperchaos equation with order as low as 3.8. Chaotic behaviors in the fractional order Chen system is studied [66, 67] and a chaos control approach is also presented for this fractional order system [66].

Mechanical resonance is widely applied in high-precision oscillators for a multitude of time-keeping and frequency reference applications. The extraordinary small size and high level of integration that can be achieved with nano resonators appear to open exceptional possibilities for creating miniature-scale precision oscillators to be used in e.g. mobile communication and navigation devices.

Nano resonator system [69] studied in this paper is a modified form of nonlinear damped Mathieu system. In this paper, the chaos synchronizations of two uncoupled integral and fractional order chaotic nano resonator systems are obtained by replacing their corresponding parameters by the same function of chaotic state variables of a third chaotic system. It is named parameter excited chaos synchronization which can be successfully obtained for very low total fraction order 0.2. Numerical simulations are illustrated by phase portraits, Poincaré maps and state error plots. Investigations of chaos synchronization of fractional order systems can also be found in [70, 71].

This paper is organized as follows. In Section 2, a method for the approximation of the fractional derivative is given. In Section 3, numerical simulations for integral and fractional order nano resonator systems are given for order 1 ~ 0.1. In Section 4, conclusions are drawn.

2. METHOD FOR THE APPROXIMATION OF A FRACTIONAL DERIVATIVE

The idea of fractional integrals and derivatives has been known since the development of the regular calculus, with the first reference probably being associated with Leibniz in 1695 [72].

Two commonly used definitions for the general fractional differintegral are the Grunwald definition and the Riemann-Liouville definition. The latter is given here

$$\frac{d^q f(t)}{dt^q} = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{q-n+1}} d\tau \tag{1}$$

where $n-1 \le q < n$ and $\Gamma(\cdot)$ is an Euler's gamma function.

The Laplace transformation of the Riemann-Liouville fractional derivative (1) is

$$L\left\{\frac{d^q f(t)}{dt^q}\right\} = s^q L\{f(t)\} - \sum_{k=0}^{n-1} s^k \left[\frac{d^{q-1-k} f(t)}{dt^{q-1-k}}\right]_{t=0}, \text{ for } n-1 \le q < n \tag{2}$$

By considering the initial conditions to be zero, this formula reduces to the more expected and comforting form

$$L\left\{\frac{d^q f(t)}{dt^q}\right\} = s^q L\{f(t)\} \tag{3}$$

and the fractional integral of order q can be described as $F(s) = \frac{1}{s^q}$ in the frequency domain.

The standard definitions of the fractional differintegral do not allow direct implementation of the operator in time domain simulations of complicated systems with fractional elements. Using the standard integer order operators to approximate the fractional operators is an effective method to analyze such systems.

The approximation approach taken here is to approximate the system behavior in the frequency domain [73]. By utilizing frequency domain techniques based in Bode diagrams, one can obtain a linear approximation of a fractional order integrator. Thus an approximation of any desired accuracy over any frequency band can be achieved.

Table I of Ref. [57] gives approximations for $\frac{1}{s^q}$ with $q = 0.1 \sim 0.9$ in steps of 0.1 with errors of approximately 2 dB from $\omega = 10^{-2}$ to 10^2 rad/s. These approximations will be used in the following numerical simulations.

3. NANO RESONATOR SYSTEM, MODIFIED NONLINEAR DAMPED MATHIEU SYSTEM WITH ITS FRACTIONAL ORDER FORM

Mechanical resonance is widely applied in high-precision oscillators for a multitude of time-keeping and frequency reference applications. In all such cases, the high-precision resonating element consists of an off-chip passive component, such as a quartz crystal. Major drawback of these off-chip resonator technologies is that they are bulky and must interface with transistor chips at the boards, posing a bottleneck against the ultimate miniaturization of e.g. wireless devices. The extraordinary small size and high level of integration that can be achieved with nano resonators appear to open exceptional possibilities for creating miniature-scale precision oscillators to be used in e.g. mobile communication and navigation devices.

Nano resonator system studied in this paper is a modified form of nonlinear damped Mathieu system which is obtained when the nano Mathieu oscillator has nonlinear time-dependent spring constant [69]. The nonlinear damped Mathieu system is a nonautonomous system with two states *x* and *y*:

$$\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = -(a + b\sin \omega_1 t)x - (a + b\sin \omega_1 t)x^3 - cy + d\sin \omega_2 t \end{cases} \tag{4}$$

where *a*, *b*, *c*, *d* are constant parameters, and ω_1, ω_2 are circular frequencies. The phase portraits, Poincaré maps, bifurcation diagram and the Lyapunov exponent for system (4) are showed in Fig. 1 of [74] where $a = 0.2$, $b = 0.2$, $c = 0.4$, $\omega_1 = \omega_2 = \omega = 1$. Let $\omega_1 = \omega_2 = \omega$, and replace $\sin \omega t$ by *z* which is the periodic time function solution of the nonlinear oscillator

$$\begin{cases} \dfrac{dz}{dt} = w \\ \dfrac{dw}{dt} = -ez - fz^3 \end{cases} \tag{5}$$

where *e*, *f* are constant parameters. Then we have the modified nonlinear damped Mathieu system:

$$\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = -(a+bz)x - (a+bz)x^3 - cy + dz \\ \dfrac{dz}{dt} = w \\ \dfrac{dw}{dt} = -ez - fz^3 \end{cases} \tag{6}$$

It becomes an autonomous system with four states where a, b, c, d, e and f are constant parameters of the system. System (4) consists of two parts:

$$\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = -(a+bz)x - (a+bz)x^3 - cy + dz \end{cases} \tag{7}$$

and

$$\begin{cases} \dfrac{dz}{dt} = w \\ \dfrac{dw}{dt} = -ez - fz^3 \end{cases} \tag{8}$$

Eq. (8) affords the periodic time function solution to system (7) as an excitation which induces the chaos in system (7). As a result, Eq. (7) can be considered as a nonautonomous system with two states, while Eq. (7) and Eq. (8) together can be considered as an autonomous system with four states. Our main interest devotes to Eq. (6), while Eq. (8) remains an integral order system. The phase portraits, Poincaré maps, bifurcation diagram and the Lyapunov exponent for (6) are showed in Fig. 2 of [74]. The corresponding modified nonlinear fractional order damped Mathieu system, the fractional order nano resonator system, is:

$$\begin{cases} \dfrac{d^{\alpha} x}{dt^{\alpha}} = y \\ \dfrac{d^{\beta} y}{dt^{\beta}} = -(a+bz)x - (a+bz)x^{3} - cy + dz \\ \dfrac{dz}{dt} = w \\ \dfrac{dw}{dt} = -ez - fz \end{cases} \tag{9}$$

where α and β are the fractional orders.

4. NUMERICAL SIMULATIONS FOR THE SYNCHRONIZATIONS OF INTEGRAL AND FRACTIONAL ORDER CHAOTIC NANO RESONATOR SYSTEMS

It is well known that a chaotic system is very sensitive to its initial conditions, means that the behaviors of two same chaotic systems which have distinct initial conditions are totally different. In this paper, these two chaotic fractional order nano resonator systems

$$\begin{cases} \dfrac{d^{\alpha} x_1}{dt^{\alpha}} = y_1 \\ \dfrac{d^{\beta} y_1}{dt^{\beta}} = -(a+bz_1)x_1 - (a+bz_1){x_1}^{3} - cy_1 + dz_1 \\ \dfrac{dz_1}{dt} = w_1 \\ \dfrac{dw_1}{dt} = -ez_1 - fz_1 \end{cases} \tag{10}$$

and

$$\begin{cases} \dfrac{d^{\alpha} x_2}{dt^{\alpha}} = y_2 \\ \dfrac{d^{\beta} y_2}{dt^{\beta}} = -(a+bz_1)x_1 - (a+bz_1){x_1}^{3} - cy_2 + dz_2 \\ \dfrac{dz_2}{dt} = w_2 \\ \dfrac{dw_2}{dt} = -ez_2 - fz_2 \end{cases} \tag{11}$$

where α and β are the fractional orders, are synchronized by replacing corresponding parameters by the same function of chaotic states of chaotic nano resonator system

$$\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = -(a+bz)x - (a+bz)x^3 - cy + dz \\ \dfrac{dz}{dt} = w \\ \dfrac{dw}{dt} = -ez - fz^3 \end{cases} \tag{12}$$

where a = 0.2, b = 0.2, c = 0.4, d = 50, e = 1 and f = 0.3 are constant parameters of the system. Define the error states as $e_1 = x_1 - x_2$ and $e_2 = y_1 - y_2$ for system (10) and (11). The synchronization scheme is to replace the corresponding parameters d, c, b or a in system (10) and (11) by the same function of chaotic states of system (12) such that $\|e(t)\| \to 0$ as $t \to \infty$. In following simulations, for various derivative orders α and β, we replace the system parameter d in system (10) and (11) by sin x, and by sin y, where x and y are state variables in system (12); replace c in system (10) and (11) by $\sin x + \sin y$, where x and y are state variables in system (12); replace b in system (10) and (11) by 10 ($\sin^2 x + \sin^2 y$), where x and y are state variables in system (12) and replace a in system (10) and (11) by 10 ($\sin^3 x + \sin^3 y$), where x and y are state variables in system (12). Simulations are performed under $\alpha = \beta = 0.1 \sim 1$ in steps of 0.1. In our numerical simulations, six parameters $a = 0.2$, $b = 0.2$, $c = 0.4$, $d = 50$, $e = 1$ and $f = 0.3$ of system (12) are fixed. The initial states of system (12) are $x(0) = 3$, $y(0) = 4$, $z(0) = 1$ and $w(0) = 0$. The numerical simulations are carried out by MATLAB.

Case 1: The parameters $a = 0.2$, $b = 0.2$, $c = 0.4$, $e = 1$ and $f = 0.3$ of system (10) and (11) are fixed. The initial states of system (10) and (11) are $x_1(0) = 0.003$, $y_1(0) = 0.004$, $x_2(0) = 0.004$, $y_2(0) = 0.003$, $z(0) = 1$ and $w(0) = 0$. The parameter

d of system (10) and (11) is replaced by the same sin x, where x is the state variable of system (12). All synchronizations for $\alpha = \beta = 0.1 \sim 1$ are successfully obtained. For saving space, only results for $\alpha = \beta = 0.1$, 0.4, 0.7 and 1 are shown in Fig. 1-8.

Case 2: The parameters $a = 0.2$, $b = 0.2$, $c = 0.4$, $e = 1$ and $f = 0.3$ of system (10) and (11) are fixed. The initial states of system (10) and (11) are $x_1(0) = 0.003$, $y_1(0) = 0.004$, $x_2(0) = 0.004$, $y_2(0) = 0.003$, $z(0) = 1$ and $w(0) = 0$. The parameter d of system (10) and (11) is replaced by the same sin y, where y is the state variable of system (12). All synchronizations for $\alpha = \beta = 0.1 \sim 1$ are successfully obtained. For saving space, only results for $\alpha = \beta = 0.1$, 0.4, 0.7 and 1 are shown in Fig. 9-16.

Case 3: The parameters $a = 0.2$, $b = 0.2$, $c = 0.4$, $e = 1$ and $f = 0.3$ of system (10) and (11) are fixed. The initial states of system (10) and (11) are $x_1(0) = 0.003$, $y_1(0) = 0.004$, $x_2(0) = 0.004$, $y_2(0) = 0.003$, $z(0) = 1$ and $w(0) = 0$. The parameter c of system (10) and (11) is replaced by the same $\sin x + \sin y$, where x and y are the state variables of system (12). All synchronizations for $\alpha = \beta = 0.1 \sim 0.9$ are successfully obtained. For saving space, only results for $\alpha = \beta = 0.1$, 0.4, 0.7 and 0.9 are shown in Fig. 17-24.

Case 4: The parameters $a = 0.2$, $b = 0.2$, $c = 0.4$, $e = 1$ and $f = 0.3$ of system (10) and (11) are fixed. The initial states of system (10) and (11) are $x_1(0) = 0.003$, $y_1(0) = 0.004$, $x_2(0) = 0.004$, $y_2(0) = 0.003$, $z(0) = 1$ and $w(0) = 0$. The parameter b of system (10) and (11) is replaced by the same 10 ($\sin^2 x + \sin^2 y$), where x and y are the state variables of system (12). All synchronizations for $\alpha = \beta = 0.1 \sim 0.9$ are successfully obtained. For saving space, only results for $\alpha = \beta = 0.1$, 0.4, 0.7 and 0.9 are shown in Fig. 25-32.

Case 5: The parameters $a = 0.2$, $b = 0.2$, $c = 0.4$, $e = 1$ and $f = 0.3$ of system (10) and (11) are fixed. The initial states of system (10) and (11) are $x_1(0) = 0.003$, $y_1(0) = 0.004$, $x_2(0) = 0.004$, $y_2(0) = 0.003$, $z(0) = 1$ and $w(0) = 0$. The parameter a of system (10) and (11) is replaced by the same 10 ($\sin^3 x + \sin^3 y$), where x and y are the state variables of system (12). All synchronizations for $\alpha = \beta = 0.1 \sim 0.9$ are successfully obtained. For saving space, only results for $\alpha = \beta = 0.1$, 0.4, 0.7 and 0.9 are shown in Fig. 33-40.

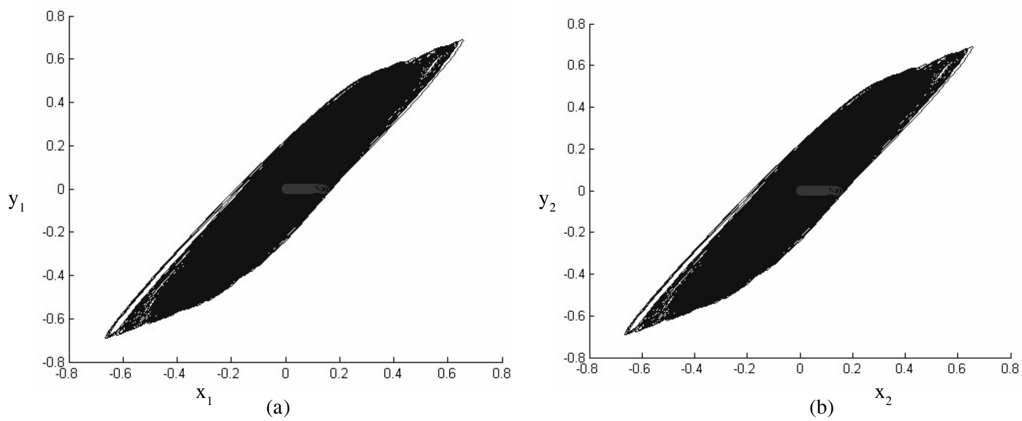


Figure 1: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.1$ for Case 1.

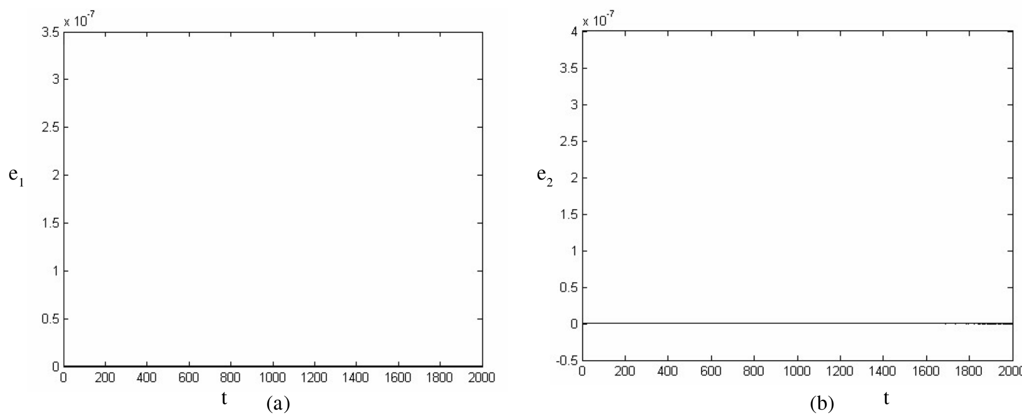


Figure 2: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.1$ for Case 1.

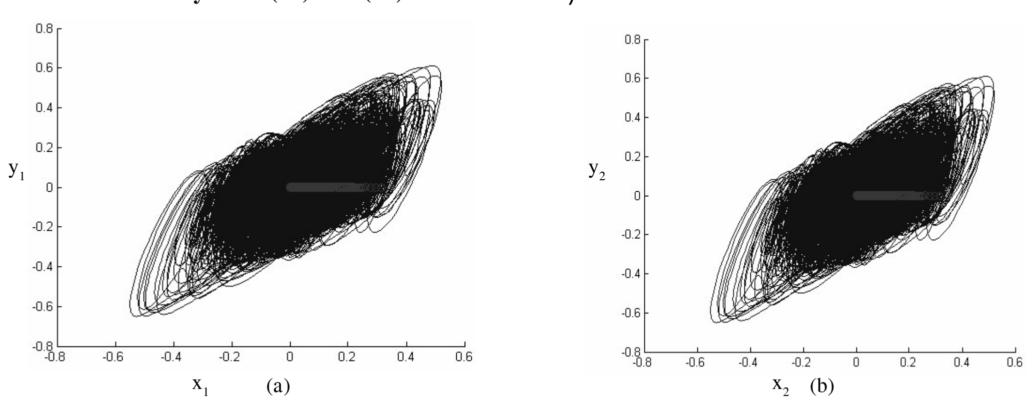


Figure 3: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.4$ for Case 1.

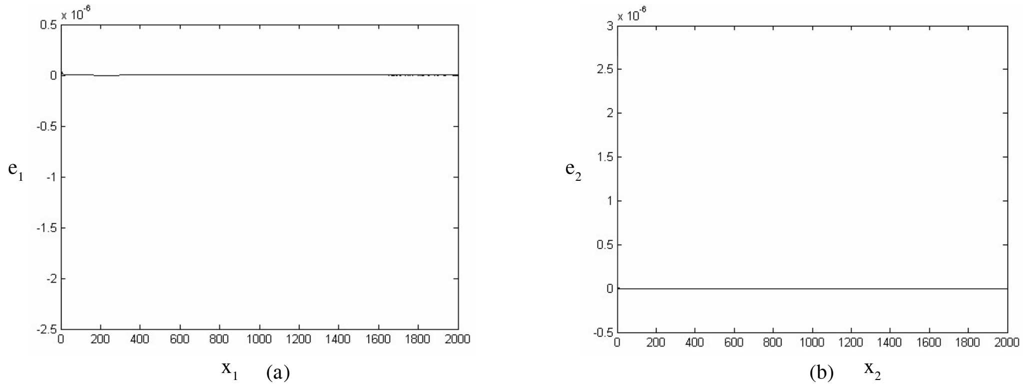


Figure 4: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.4$ for Case 1.

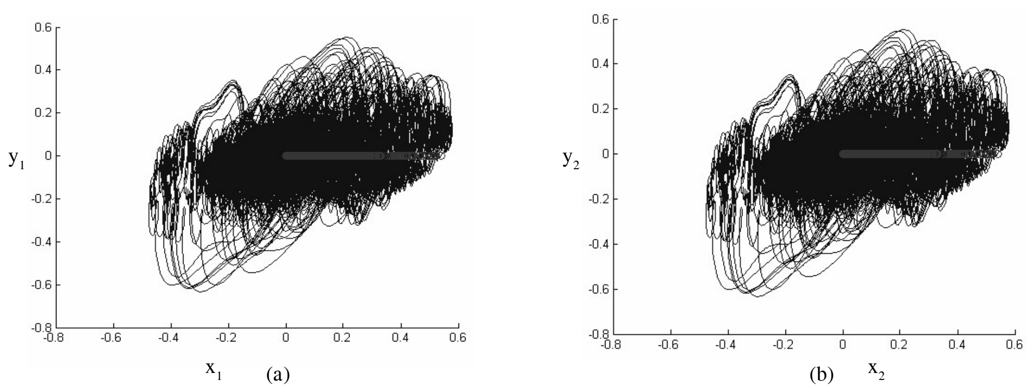


Figure 5: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.7$ for Case 1.

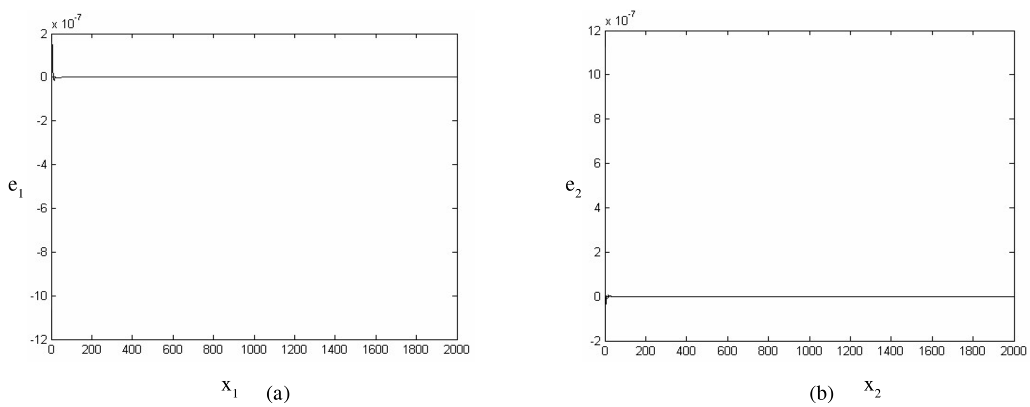


Figure 6: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.7$ for Case 1.

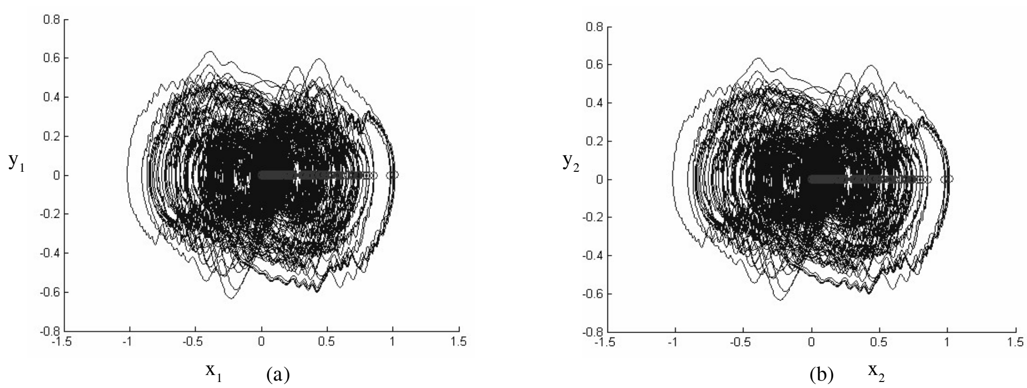


Figure 7: The phase portraits of the synchronized nano resonator integral order systems (10) and (11) with order $\alpha = \beta = 1$ for Case 1.

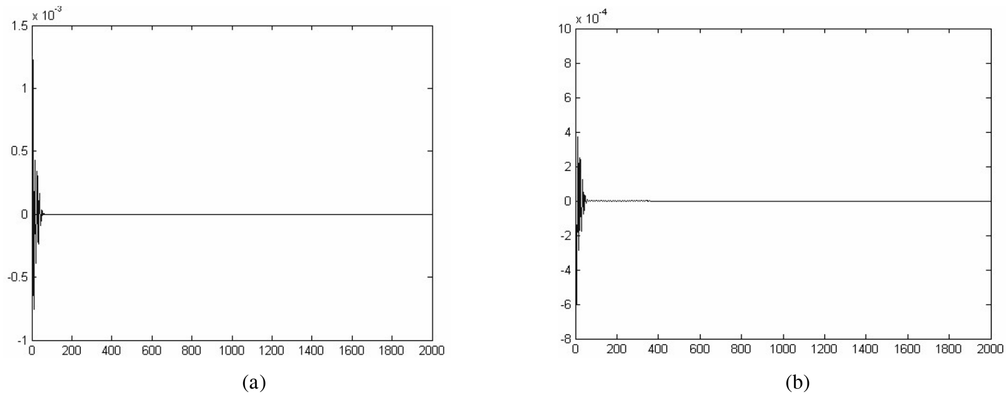


Figure 8: The time histories of the errors of the states of the synchronized nano resonator integral order systems (10) and (11) with order $\alpha = \beta = 1$ for Case 1.

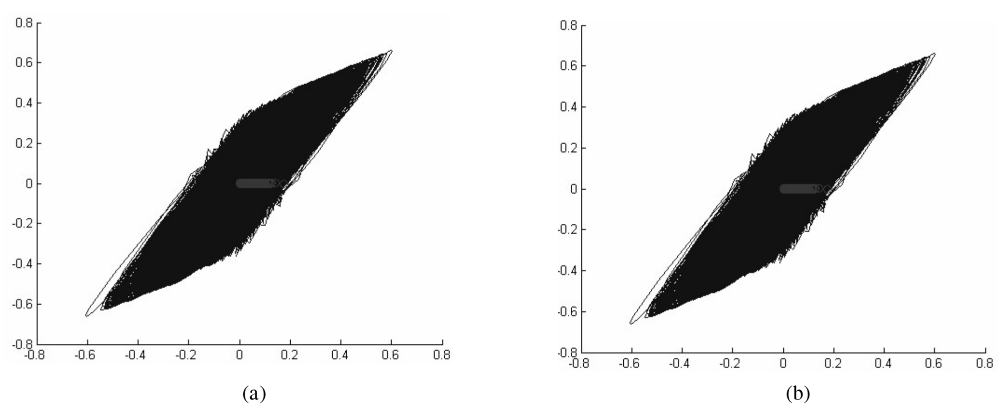
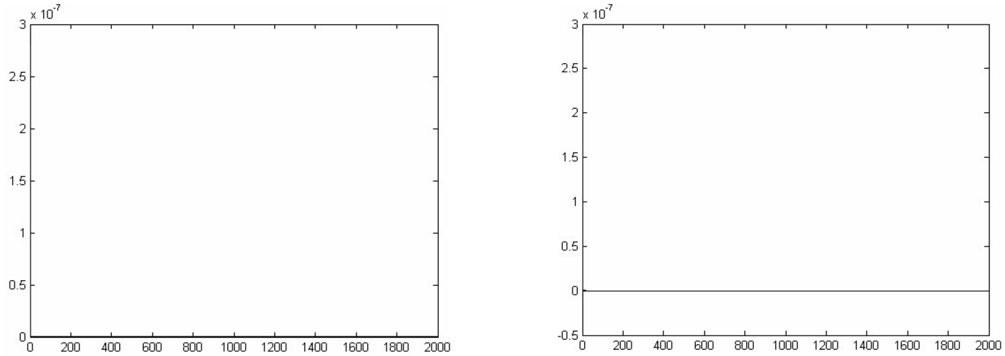
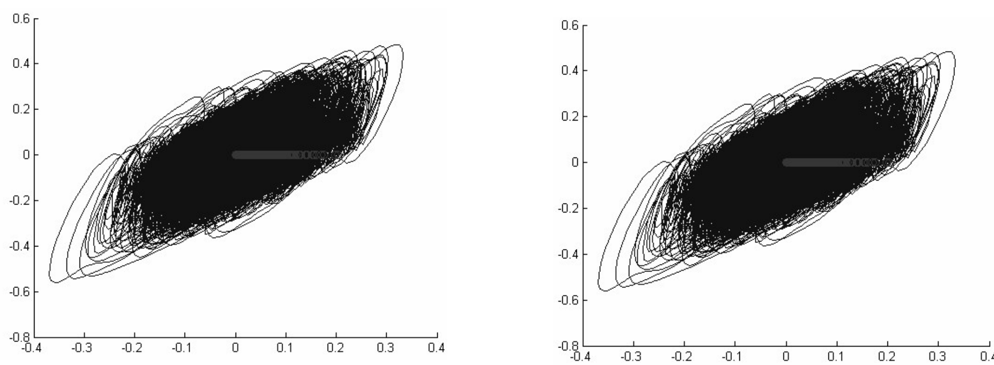


Figure 9: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.1$ for Case 2.



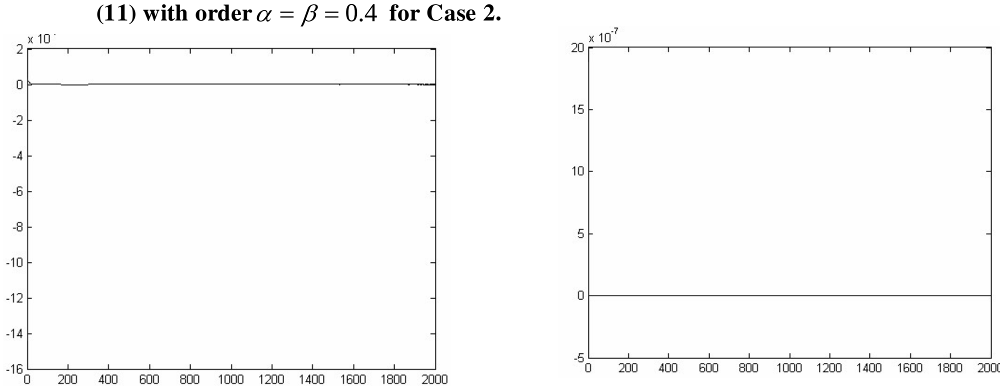
(a) (b)

Figure 10: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.1$ for Case 2.



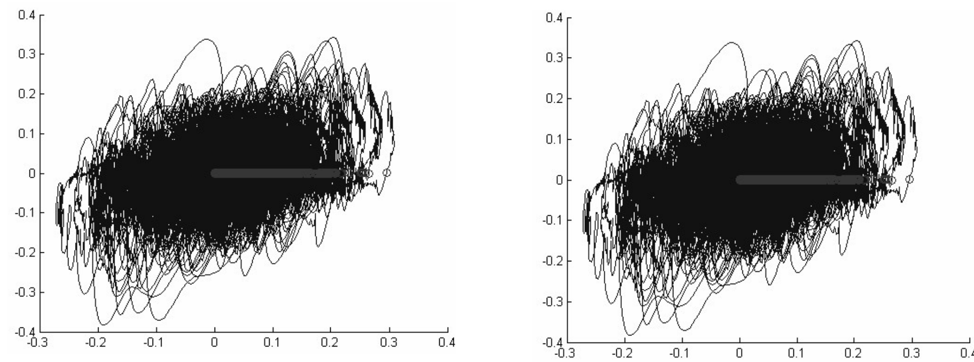
(a) (b)

Figure 11: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.4$ for Case 2.



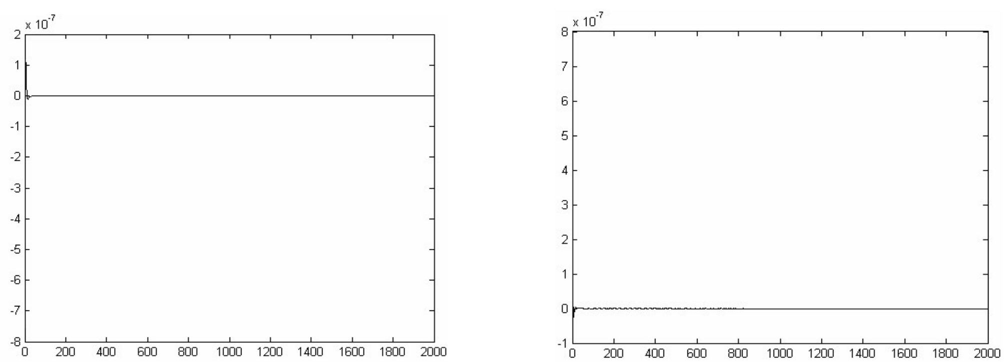
(a) (b)

Figure 12: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.4$ for Case 2.



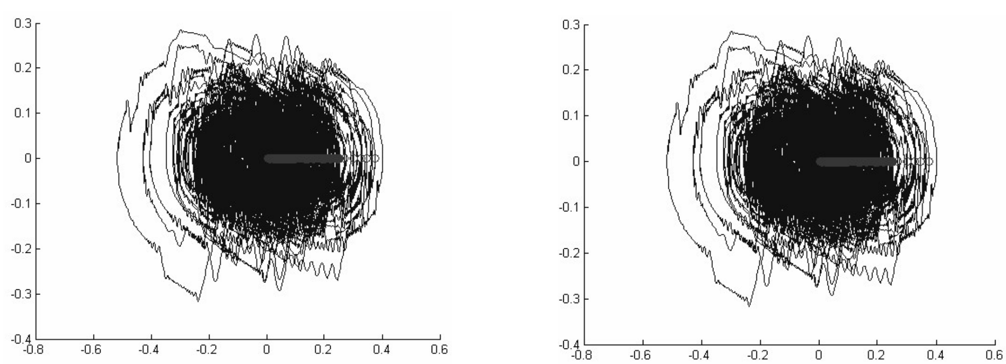
(a) (b)

Figure 13: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.7$ for Case 2.



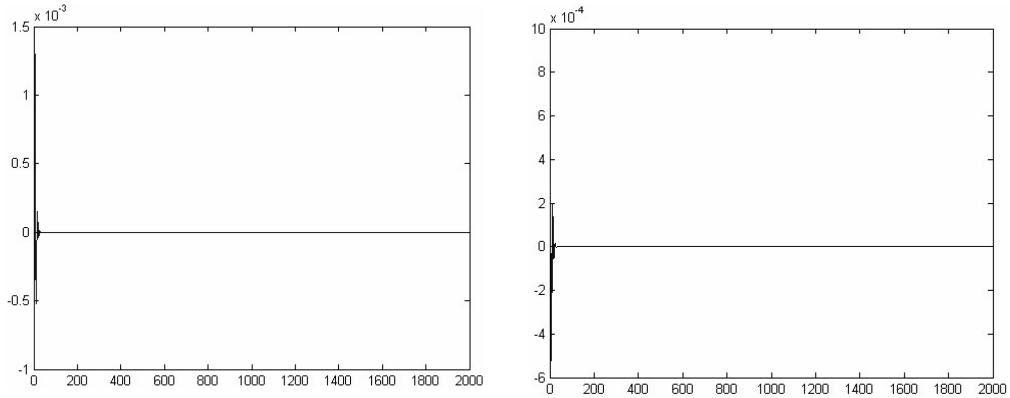
(a) (b)

Figure 14: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.7$ for Case 2.



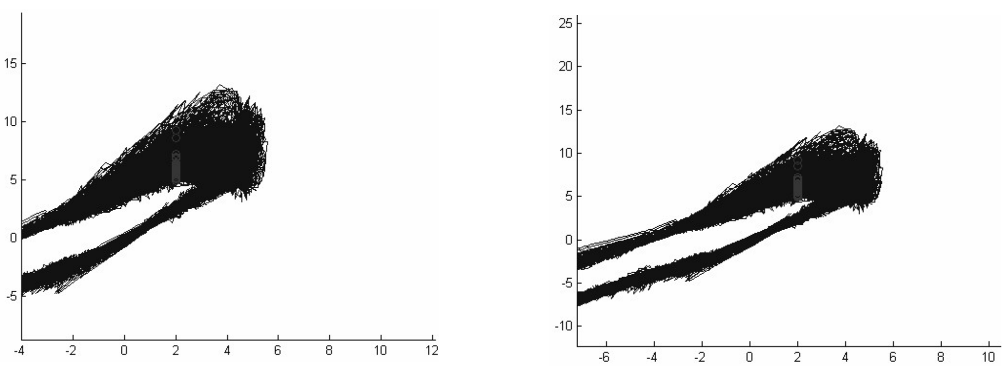
(a) (b)

Figure 15: The phase portraits of the synchronized nano resonator integral order systems (10) and (11) with order $\alpha = \beta = 1$ for Case 2.



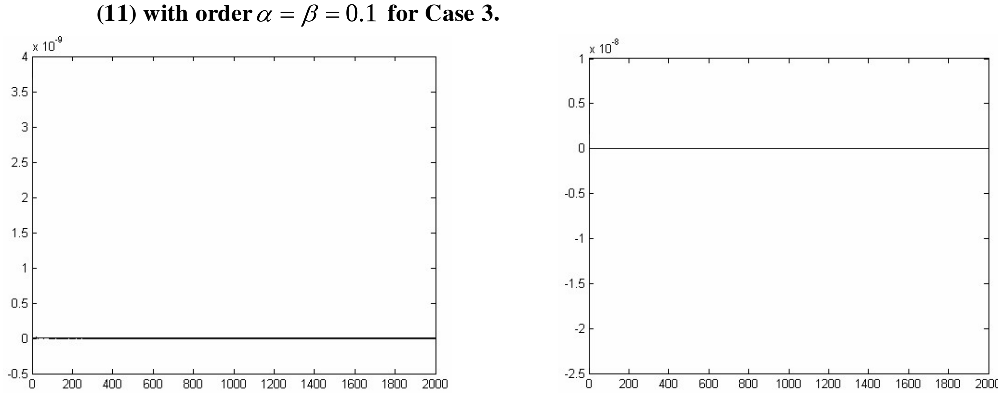
(a) (b)

Figure 16: The time histories of the errors of the states of the synchronized nano resonator integral order systems (10) and (11) with order $\alpha = \beta = 1$ for Case 2.



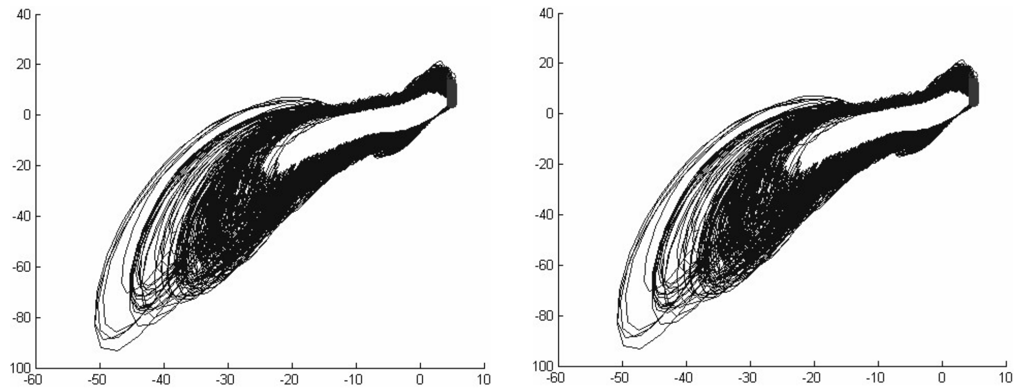
(a) (b)

Figure 17: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.1$ for Case 3.



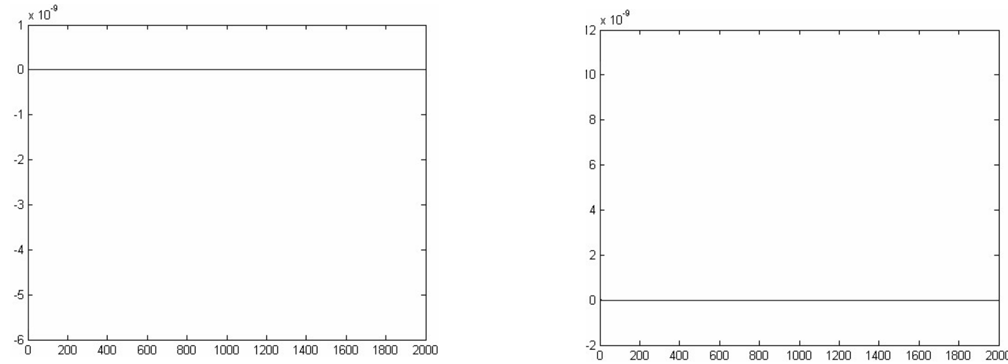
(a) (b)

Figure 18: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.1$ for Case 3.



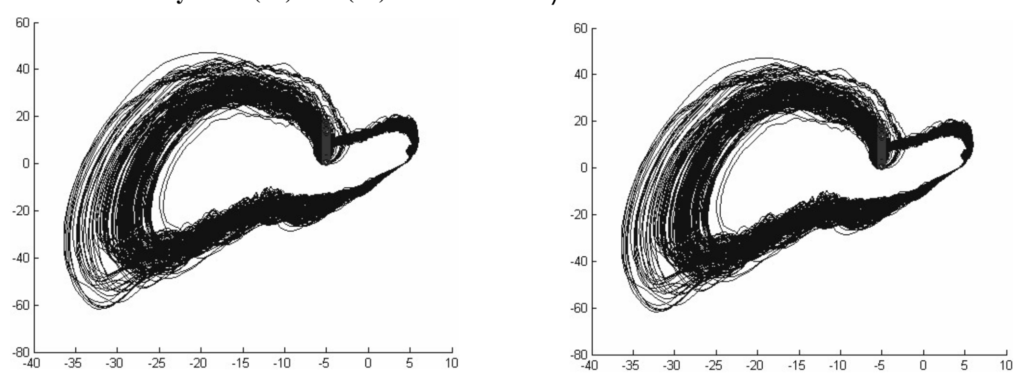
(a) (b)

Figure 19: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.4$ for Case 3.



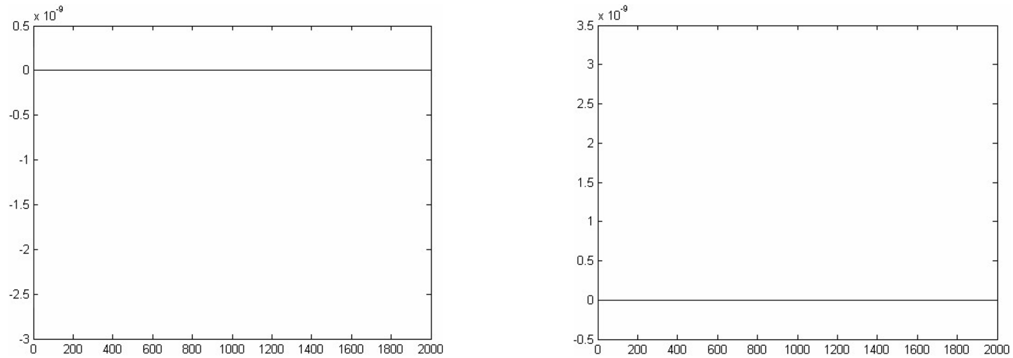
(a) (b)

Figure 20: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.4$ for Case 3.



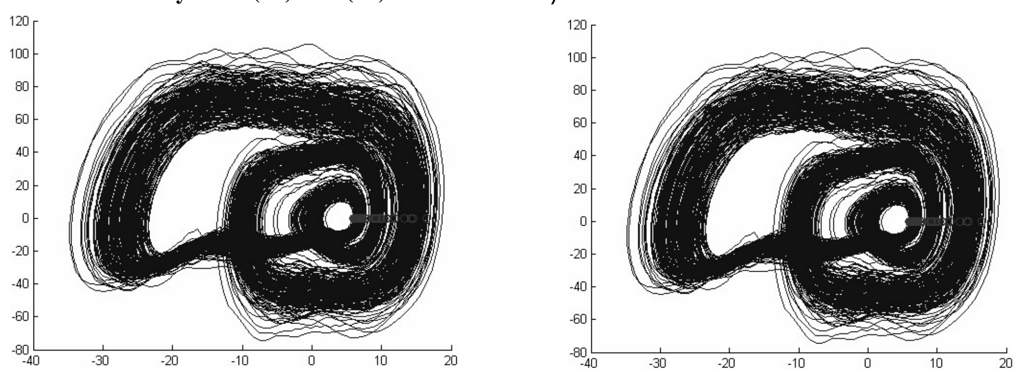
(a) (b)

Figure 21: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.7$ for Case 3.



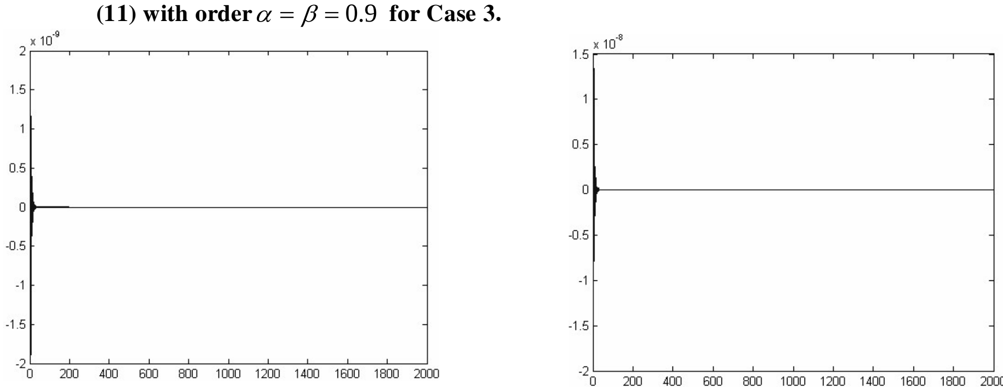
(a) (b)

Figure 22: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.7$ for Case 3.



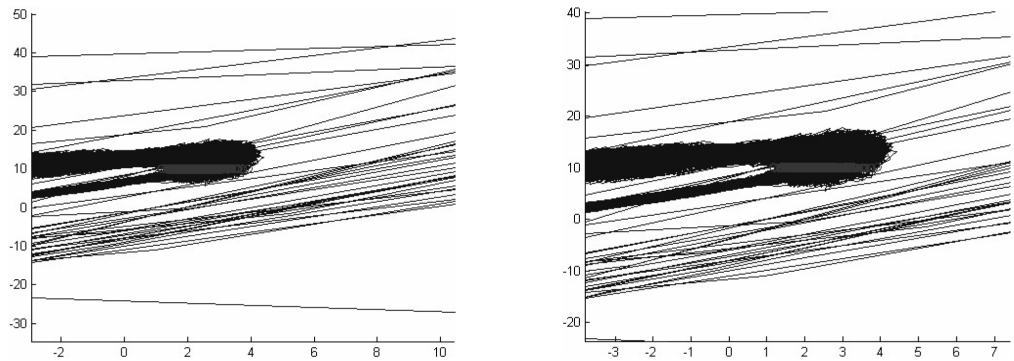
(a) (b)

Figure 23: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.9$ for Case 3.



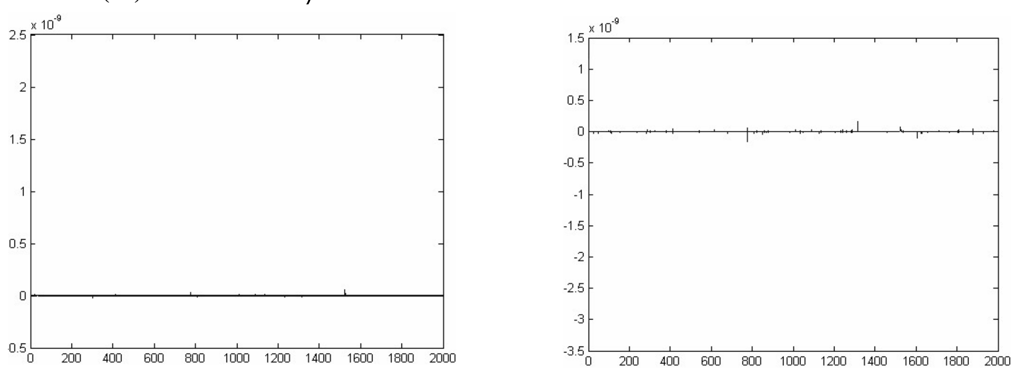
(a) (b)

Figure 24: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.9$ for Case 3.



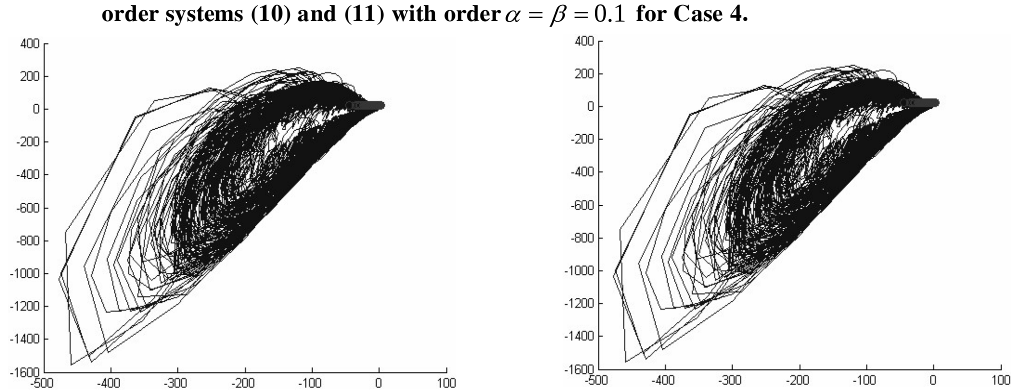
(a) (b)

Figure 25: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.1$ for Case 4.



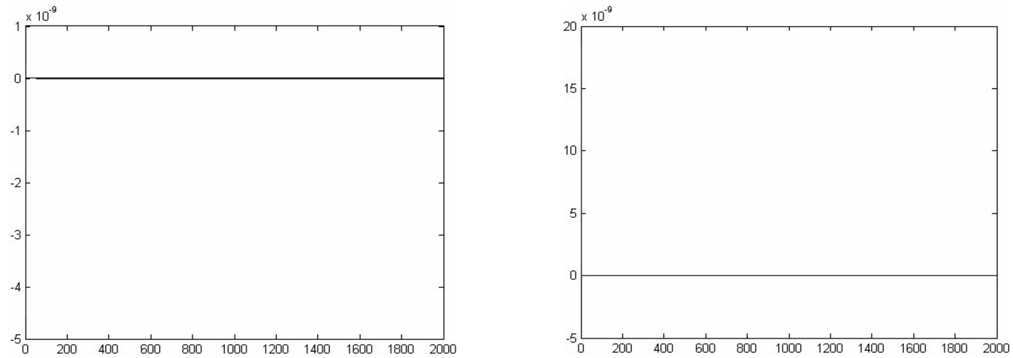
(a) (b)

Figure 26: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.1$ for Case 4.



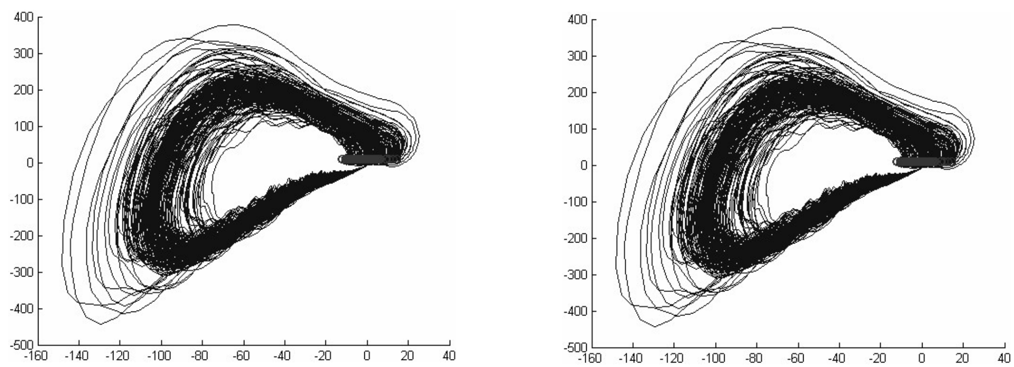
(a) (b)

Figure 27: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.4$ for Case 4.



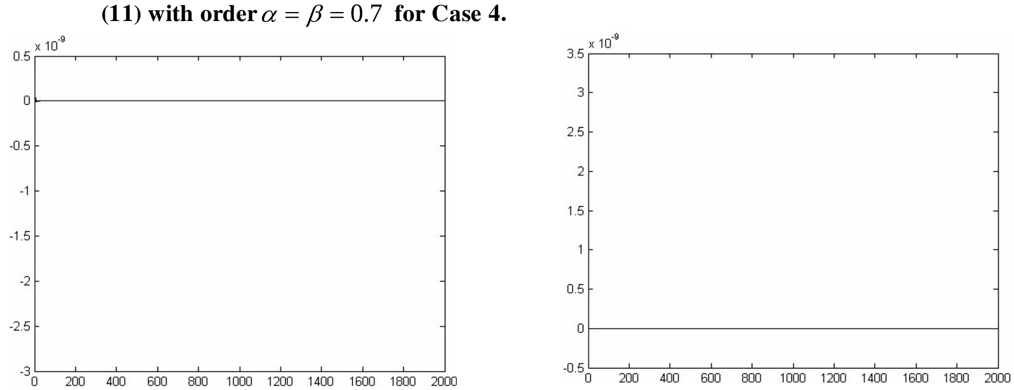
(a) (b)

Figure 28: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.4$ for Case 4.



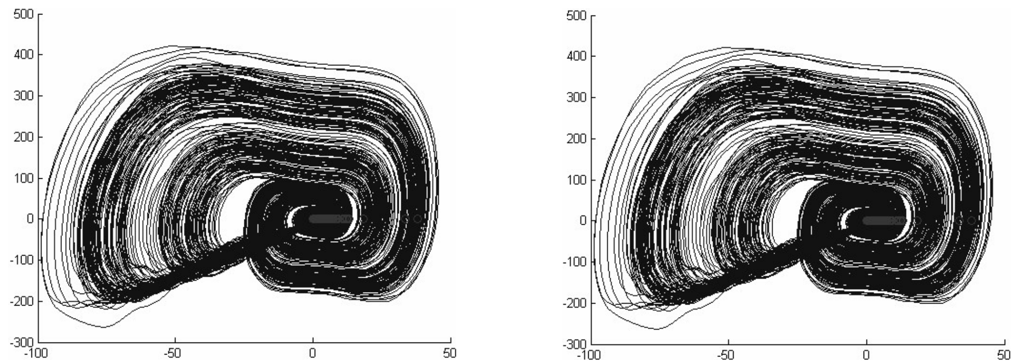
(a) (b)

Figure 29: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.7$ for Case 4.



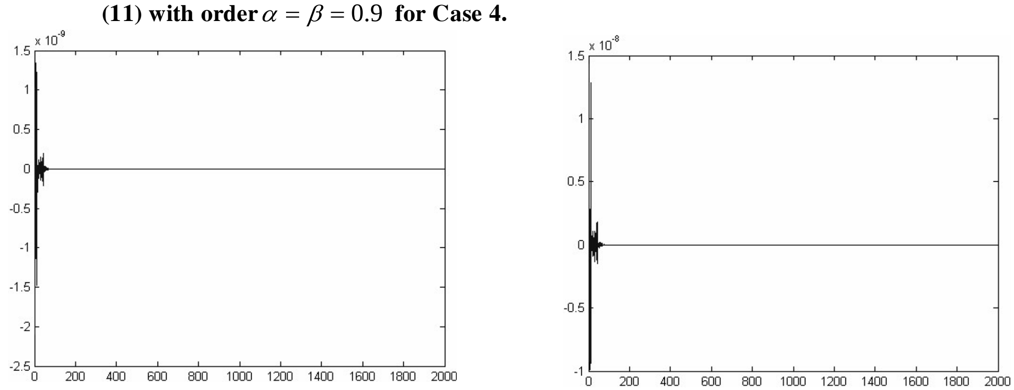
(a) (b)

Figure 30: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.7$ for Case 4.



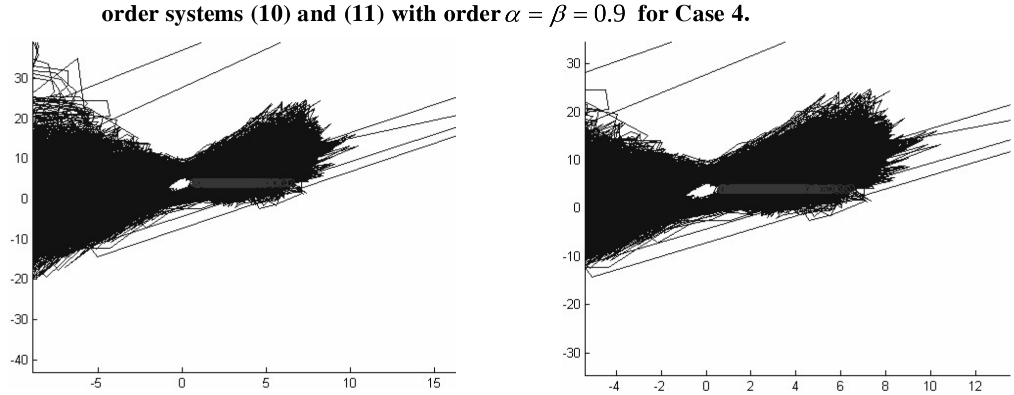
(a) (b)

Figure 31: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.9$ for Case 4.



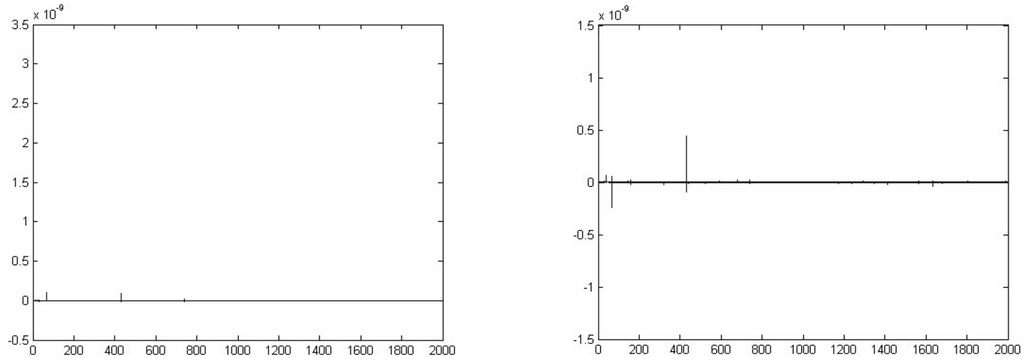
(a) (b)

Figure 32: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.9$ for Case 4.



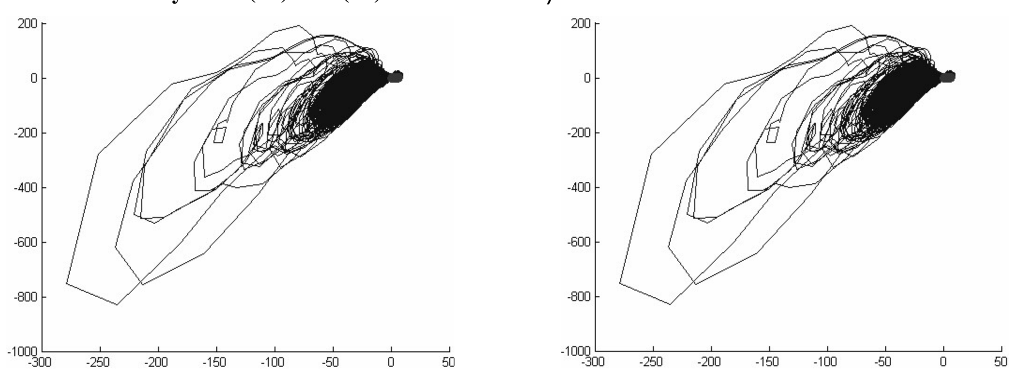
(a) (b)

Figure 33: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.1$ for Case 5.



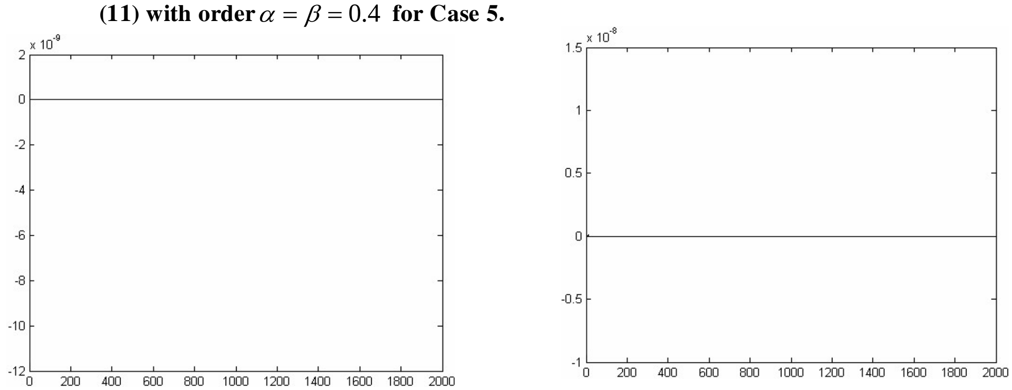
(a) (b)

Figure 34: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.1$ for Case 5.



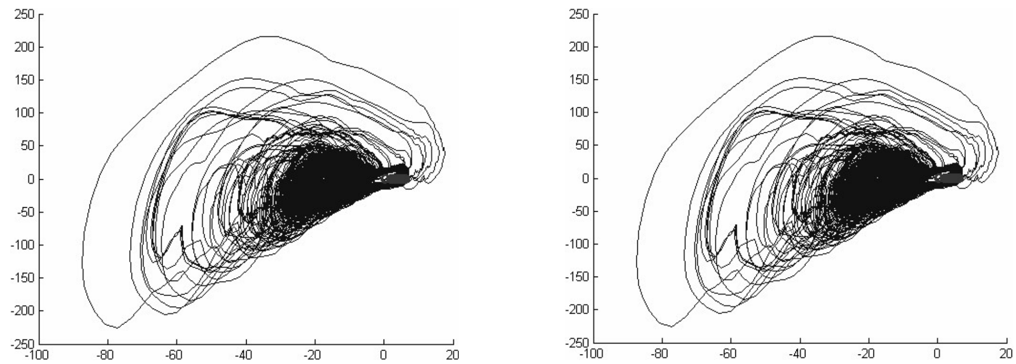
(a) (b)

Figure 35: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.4$ for Case 5.



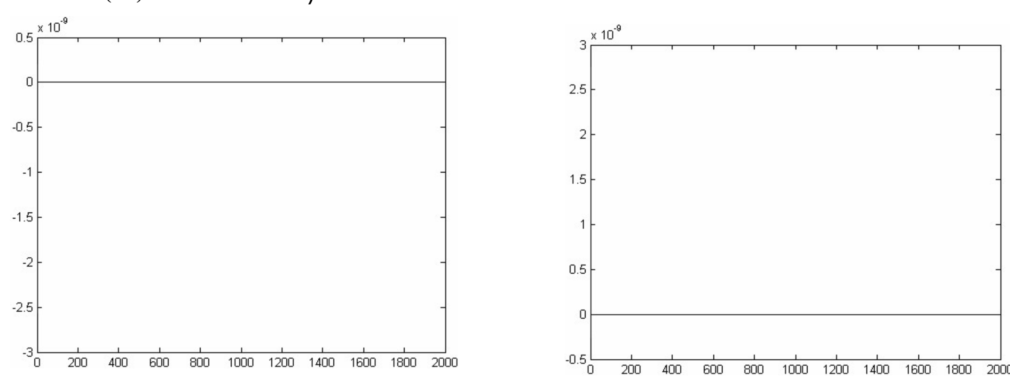
(a) (b)

Figure 36: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.4$ for Case 5.



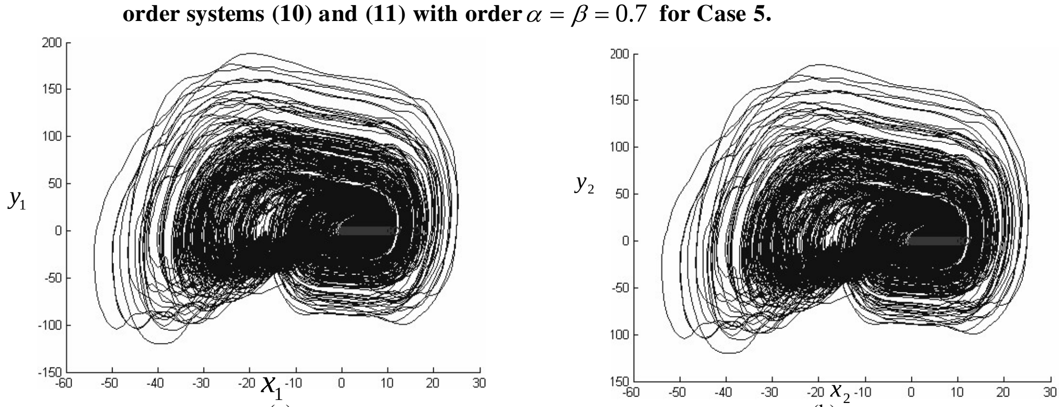
(a) (b)

Figure 37: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.7$ for Case 5.



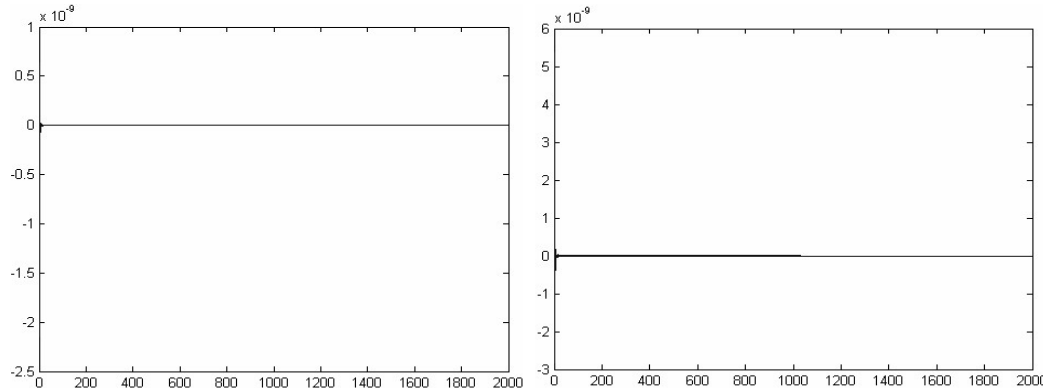
(a) (b)

Figure 38: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.7$ for Case 5.



(a) (b)

Figure 39: The phase portraits of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.9$ for Case 5.



(a) (b)

Figure 40: The time histories of the errors of the states of the synchronized nano resonator fractional order systems (10) and (11) with order $\alpha = \beta = 0.9$ for Case 5.

4. CONCLUSIONS

In this paper, parameter excited chaos synchronizations of uncoupled integral and fractional order nano resonator systems are studied by means of phase portraits, Poincaré maps and the state error plots. It is found that this approach is very effective even for very low total fractional order 0.2. An interesting phenomenon is found that the lower the total fractional order is, the faster the synchronization is accomplished.

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