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Stability Analysis for a Class of Nonlinear Fuzzy Systems with Delays

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The method of Takagi-Sugeno (T-S) fuzzy systems provides an effective representation of complex nonlinear systems by fuzzy sets and fuzzy reasoning. Most existing results of T-S method represent global nonlinear systems by connecting local linear systems. However, many complex nonlinear systems cannot be represented by linear systems. In this paper, by T-S method, a class of local nonlinear systems having nice dynamic properties is employed to represent some global complex nonlinear systems. The stability of these global complex nonlinear systems is studied based on these local nonlinear systems. Conditions for global exponential stability are derived. Examples are employed to illustrate the theory.

Key words: Fuzzy Systems, Control Systems, Uncertain Delays, Global Exponential Stability, Stabilization.

1. INTRODUCTION

The well known Takagi-Sugeno (T-S) model of fuzzy systems was first proposed in [1]. The T-S model gives an effective method to combine some simple local systems with their linguistic description to represent complex nonlinear dynamic systems. From the mathematical point of view, the essence of T-S model is to connect some simple local systems to form global complex nonlinear systems.

Stability of T-S model fuzzy systems is quite important for practical applications. It has been widely studied by many authors, see, for examples, [2-8].

T-S model of fuzzy systems with delays was first introduced in [9]. The stability between T-S model fuzzy systems with and without delays is essentially different. In recent years, some authors have paid their attention to control of nonlinear systems with delays by using T-S fuzzy models. There exist two kinds of delays, one is continuous, see, for examples, [3, 10-17] and the other is discrete, see, for examples, [5, 18, 19]. In [3], delays are assumed to be any uncertain bounded continuous functions. The delays are not required to be differentiable and it is also not necessary to know the bounds of the delays. The stability of the fuzzy systems can be determined by searching a common positive definite matrix P.

As we know, in most reported stability results of T-S model, simple linear systems are used to form global nonlinear fuzzy systems. However, there are many complex nonlinear fuzzy systems cannot be connected by using local linear systems. In this paper, unlike using local linear systems in previous study, a class of nonlinear systems with delays having nice dynamical properties [20] will be used as local systems to form some global complex nonlinear fuzzy systems by T-S method. We will study the stability of the global complex nonlinear fuzzy systems and derive some simple conditions to guarantee the exponential stability.

This paper is organized as follows. In Section 2, some preliminaries for delayed fuzzy control systems will be given. In Section 3, conditions for global exponential stability of fuzzy systems with delays will be proposed and proved. In Section 4, state feedback stabilization of delayed fuzzy control systems will be discussed. In Section 5, simulations will be given. This paper will be concluded in Section 6.

2. PRELIMINARIES

Let C_b^0 be the set of any nonnegative continuous and bounded functions defined on [0,+ ∞). For any $\tau_j^s(t) \in C_b^0 (s = 1, \ldots, n)$, consider a nonlinear time-delay system composed of n subsystems. The ith subsystem is described as follows:

$$\dot{x}_i(t) = -x_i(t) + \sum_{j=1}^{n} \left[a_{ij}^s f(x_j(t)) + b_{ij}^s f(x_j(t - \tau_s(t))) \right] + \sum_{j=1}^{m} c_{ij}^s u_j(t) \tag{1}$$

for $t \geq 0$ and $s = 1, \ldots, r$, $i = 1, \ldots, n$. Where $a_{ij}^s, b_{ij}^s, c_{ij}^s$ are constants.

The above local systems can be written in compact vector form

$$\dot{x}(t) = -x(t) + A_s f(x(t)) + B_s f(x(t - \tau_s(t))) + C_s u(t), \quad (s = 1, \ldots, r) \tag{2}$$

where $x(t) = (x_1(t), \ldots, x_n(t))^T$ is the state vector, $A_s = \left(a_{ij}^s\right)_{n\times n}$ and $B_s = \left(b_{ij}^s\right)_{n\times n}$ are constant matrices, $u(t)$ is the input vector, $\tau_s(t)$ is the time delay, r is the number of IF-THEN rules. For any $x \in R^n$, $f(x) = (f(x_1), \ldots, f(x_n))^T$, and the function f is defined as follows:

$$f(s) = \max\{0, s\},\ s \in R.$$

The function f is a piecewise linear function, which is continuous, unbounded and non-differentiable. Figure 1 shows this function. So the local system is nonlinear which is the main feature of this paper different from others.

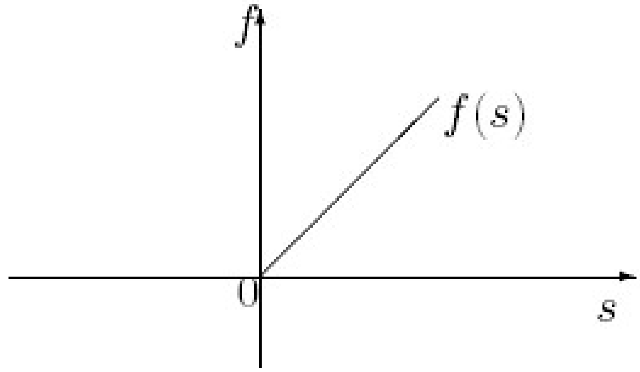

Fig. 1. The function f (s).

For each $s = 1, \ldots, r$, the T-S fuzzy time-delay model is composed of r plant rules that can be represented as follows.

Plant Rule s: IF $\alpha_1(t)$ is M_{1s} AND . . . AND $\alpha_p(t)$ is M_{ps} THEN

$$\dot{x}(t) = -x(t) + A_s f(x(t)) + B_s f(x(t - \tau_s(t))) + C_s u(t)$$

where $\alpha_1(t), \ldots, \alpha_p(t)$ are the premise variables and each M_{is} ($i = 1, \ldots, p$; $s = 1, \ldots r$) is the fuzzy set corresponding to $\alpha_i(t)$ and plant rule s. Let $M_{is}(\alpha_i(t))$ be the membership function of the fuzzy set M_{is} at the position $\alpha_i(t)$ and denote

$$w_s(\alpha(t)) = \prod_{i=1}^{p} M_{is}(\alpha_i(t)), (s = 1, \ldots, r).$$

Then, the resulting delayed fuzzy control system is inferred as the weighted average of the local model and has the form as

$$\dot{x}(t) = \frac{\sum_{s=1}^{r} w_s(\alpha(t))[-x(t) + A_s f(x(t)) + B_s f(x(t - \tau_s(t))) + C_s u(t)]}{\sum_{s=1}^{r} w_s(\alpha(t))}$$

$$= \sum_{s=1}^{r} h_s(\alpha(t))[-x(t) + A_x f(x(t) + B_s f(x(t - \tau_s(t))) + C_s u(t)] \qquad (3)$$

for $t \geq 0$, where

$$h_s(\alpha(t)) = \frac{w_s(\alpha(t))}{\sum_{i=1}^{r} w_i(\alpha(t))}$$

which satisfies

$$h_s(\mathrm{a}(t)) \geq 0, \text{ and } \sum_{s=1}^{r} h_s(\alpha(t)) = 1.$$

By using the membership functions, the local nonlinear delayed systems of (2) are smoothly connected to form a global nonlinear delayed fuzzy control system of (3).

For each system of (3), since the delays $\tau_s(t) \in C_b^0 (s = 1, \ldots, n)$, there must exit a constant $\tau > 0$ such that $0 \leq \tau_s(t) \leq \tau$ ($s = 1, \ldots, r$). For each solution, the initial value is assumed to be

$$x(t) = \phi(t), \ t \in [-\tau, 0]$$

where $\phi(t) = (\phi_1(t), \ldots, \phi_n(t))^T$ is a vector continuous function. We define

$$\| \phi \| = \sup_{-\tau \leq \theta \leq 0} \sqrt{\phi_1^2(\theta) + \ldots, + \phi_n^2(\theta)}.$$

In this paper, for a matrix S, we will use $S > 0$ and $S < 0$ to denote that S is a symmetric positive matrix or a symmetric negative matrix, respcetively.

Lemma 1 *[20] For above function* $f(s) = \max\{0, s\}$, $s \in R$.

$$sf(s) = f^2(s) = 2\int_0^s f(\theta) d\theta$$

for any $s \in R$.

Lemma 2 *[20] For above function f,*

$$f^T(x)x = x^T f(x) = f^T(x) f(x) = 2\sum_{i=1}^{n} \int_0^{x_i} f(\theta) d\theta$$

for any $x = (x_1, \ldots, x_n)^T \in R^n$.

Lemma 3 *[3] Let Q be any of a $n \times n$ matrix, we have for any constant $k > 0$ and any symmetric positive matrix $S > 0$ that*

$$2x^T Qy \le kx^T QS^{-1}Q^T x + \frac{1}{k} y^T Sy$$

for all $x, y \in R^n$.

D^+ is used to denote the *upper righthand Dini derivative* in this paper. For any continuous function $f : R \to R$, the *upper righthand Dini derivative* of $f(t)$ is defined as

$$D^+ f(t) = \lim_{\theta \to 0^+} \sup \frac{f(t+\theta) - f(t)}{\theta}.$$

It is easy to see that if $f(t)$ is locally Lipschitz then $|D^+ f(t)| < +\infty$.

3. STABILITY OF FREE FUZZY DELAYED SYSTEMS

First, we will give a class of fuzzy system with time delay

$$\dot{x}(t) = \sum_{s=1}^{r} h_s(\alpha(t))[-x(t) + A_s f(x(t)) + B_s f(x(t - \tau_s(t)))] \tag{4}$$

for $t \ge 0$. We can see that it is a global nonlinear fuzzy system and its nonlinear local delayed systems are represented as follows

$$\dot{x}(t) = -x(t) + A_s f(x(t)) + B_s f(x(t - \tau_s(t))), \ (s = 1, \ldots, r) \tag{5}$$

From [3], we can see that the fuzzy system (4) is globally exponentially stable, if there exist constants $\epsilon > 0$ and $\Pi \ge 1$ such that

$$||x(t)|| \le \Pi \, ||\phi|| e^{-\varepsilon t}$$

for all $t \ge 0$.

Theorem 1 *The free fuzzy system (4) is globally exponentially stable subject to any $\tau_s(t) \in C_b^0 (s = 1, \ldots, r)$ if there exists a diagonal matrix $D > 0$ and some constants $k_s > 0$ $(s = 1, \ldots, r)$ such that*

$$-D + DA_s + \frac{k_s}{2} DB_s D^{-1} B_s^T D + \frac{1}{2k_s} D < 0, (s = 1, \ldots, r) \tag{6}$$

Proof: For any $\tau_s(t) \in C_b^0 (s = 1, \ldots, r)$, there must exist a constant $\tau \geq 0$ such that $0 \leq \tau_s(t) \leq \tau$ ($s = 1, \ldots, r$) for all $t \geq 0$.

By (6), obviously, there exist a sufficient small constant $\epsilon > 0$ such that

$$\epsilon D - D + DA_s + \frac{k_s}{2} DB_s D^{-1} B_s^T D + \frac{e^{2\epsilon\tau}}{2k_s} D < 0$$

for $s = 1, \ldots, r$.

Using the above constant ϵ and the matrix D, we select a diffierentiable function

$$V(t) = e^{2\epsilon t} \sum_{i=1}^{n} d_i \int_0^{x_i t} f(s) ds$$

for all $t \geq 0$, denote $D = diag(d_i)(i = 1, \ldots, n)$. Then the derivative of the function $V(t)$ along the trajectories of (4) is

$$\dot{V}(t) = e^{2\epsilon t} \sum_{s=1}^{r} h_s(\alpha(t)) \Big[-f^T(x(t)) Dx(t) + f^T(x(t)) DA_s f(x)(t))$$

$$+ f^T(x(t)) DB_s f(x(t - \tau_s(t))) \Big] + 2\epsilon V(t)$$

for all $t \geq 0$. By the *Lemma 3* in last section, it follows that

$$\dot{V}(t) \leq e^{2\epsilon t} \sum_{s=1}^{r} h_s(\alpha(t)) \Big[-f^T(x(t)) Dx(t) + f^T(x(t)) DA_s f(x)(t))$$

$$+ \frac{1}{2} k_s f^T(x(t)) DB_s D^{-1} B_s^T Df(x(t))$$

$$+ \frac{1}{2k_s} f^T(x(t - \tau_s(t))) Df(x(t - \tau_s(t))) \Big] + 2\epsilon V(t)$$

for all $t \geq 0$. Using the *Lemma 1* and *Lemma 2* in last section, then

$$\dot{V}(t) \leq \sum_{s=1}^{r} h_s(\alpha(t)) \Big[e^{2\epsilon t} f^T(x(t)) (-D + \epsilon D + DA_s$$

$$+\frac{1}{2}k_2 DB_s D^{-1}B_s^T D\Big)x(t)+\frac{e^{2\epsilon t}}{k_s}V(t-\tau_s(t))\Big]$$

$$<e^{2\epsilon t}\sum_{s=1}^{r}\frac{h_s(\alpha(t))}{k_s}\Big[-\frac{1}{2}f^T(x(t))Dx(t)\,e^{2\epsilon t}$$

$$+V(t-\tau_s(t))]$$

$$=e^{2\epsilon t}\sum_{s=1}^{r}\frac{h_s(\alpha(t))}{k_s}[-V(t)+V(t-\tau_s(t))] \tag{7}$$

for all $t \geq 0$.

Let d_{max} and d_{min} to denote the largest and smallest ones of $d_i (i = 1, \ldots, n)$, respectively. Obviously, $d_{max} > 0$, $d_{min} > 0$. For any $c > 1$, by the *Lemma 1*, we can see that

$$V(t)=\frac{1}{2}e^{2\epsilon t}\sum_{i=1}^{n}d_i f^2(x_i(t))<\frac{1}{2}cd_{\max}\sum_{i=1}^{n}x_i^2(t)\leq\frac{1}{2}cd_{\max}\,\|\phi\|^2$$

for all $t \in [-\tau, 0]$. We will prove that $V(t)<\frac{1}{2}cd_{\max}\|\phi\|^2$ for all $t \geq 0$. If this is not true, there must exist a $t_1 > 0$ such that $V(t_1)=\frac{1}{2}cd_{\max}\|\phi\|^2$ and $V(t)<\frac{1}{2}cd_{\max}\|\phi\|^2$ for all all $t \in [-\tau, t_1)$. Hence, $\dot{V}(t_1)\geq 0.$ However, from (7) we have

$$\dot{V}(t_1)<e^{2\epsilon t}\sum_{s=1}^{r}\frac{h_s(\alpha(t_1))}{k_s}[-V(t_1)+V(t_1-\tau_s(t_1))]$$

$$\leq e^{2\epsilon t}\sum_{s=1}^{r}\frac{h_s(\alpha(t_1))}{k_s}\left(-\frac{1}{2}cd_{\max}\|\phi\|^2+\frac{1}{2}cd_{\max}\|\phi\|^2\right)$$

$$=0.$$

This leads to a contradiction and it proves that

$$V(t)<\frac{1}{2}cd_{\max}\|\phi\|^2$$

for all $t \geq 0$. By the definition of $V(t)$, we have

$$V(t) = \frac{1}{2} e^{2\epsilon t} \sum_{i=1}^{n} d_i f^2(x_i(t)) \geq \frac{1}{2} d_{\min} e^{2\epsilon t} f^2(x_i(t))$$

for all $t \geq 0$ and $i = 1, \ldots, n$. Hence,

$$|f(x_i(t))| \leq \sqrt{\frac{2V(t)}{d_{\min}}} e^{-\epsilon t} < \sqrt{\frac{c d_{\max}}{d_{\min}}} \|\phi\| e^{-\epsilon t}$$

By

$$\dot{x}_i(t) = -x_i(t) + \sum_{j=1}^{n} \left[a_{ij}^s f(x_j(t)) + b_{ij}^s f(x_j(t - \tau_s(t))) \right]$$

We have

$$D^+ |x_i(t)| \leq -|x_i(t)| + \sum_{j=1}^{n} \left[a_{ij}^s \| f(x_j(t))| + b_{ij}^s \| f(x_j - \tau_s(t)))| \right]$$

$$\leq -|x_i(t)| + \sum_{j=1}^{n} \left[|a_{ij}^s| \sqrt{\frac{c d_{\max}}{d_{\min}}} \|\phi\| e^{-\epsilon t} \right.$$

$$\left. + |b_{ij}^s| \sqrt{\frac{c d_{\max}}{d_{\min}}} \|\phi\| e^{-\epsilon(t - \tau_s(t))} \right]$$

$$\leq -|x_i(t)| + \sum_{j=1}^{n} (|a_{ij}^s| + |b_{ij}^s| e^{\epsilon t}) \sqrt{\frac{c d_{\max}}{d_{\min}}} \|\phi\| e^{-\epsilon t}$$

Thus, it follows that

$$|x_i(t)| \leq \left(1 - \frac{\sum_{j=1}^{n} (|a_{ij}^s| + |b_{ij}^s| e^{\epsilon t}) \sqrt{\frac{c d_{\max}}{d_{\min}}}}{1 - \epsilon} \right) \|\phi\| e^{-t}$$

$$+\frac{\sum_{j=1}^{n}(|a_{ij}^{s}|+|b_{ij}^{s}|e^{\epsilon t})\sqrt{\frac{cd_{\max}}{d_{\min}}}}{1-\epsilon}\|\phi\|e^{-\epsilon t}$$

$$=\|\phi\|\left[\left(1-\frac{\sum_{j=1}^{n}(|a_{ij}^{s}|+|b_{ij}^{s}|e^{\epsilon t})\sqrt{\frac{cd_{\max}}{d_{\min}}}}{1-\epsilon}\right)\right.$$

$$\left.+\frac{\sum_{j=1}^{n}(|a_{ij}^{s}|+|b_{ij}^{s}|e^{\epsilon t})\sqrt{\frac{cd_{\max}}{d_{\min}}}}{1-\epsilon}e^{-\epsilon t}\right]$$

for all $t \geq 0$ and $i = 1, \ldots, n$. The proof is completed.

From Theorem 1, we can get the condition to guarantee the exponential stability of the nonlinear time-delay fuzzy systems of (4). To check the inequalities of (6), it needs to find a common diagonal matrix $D > 0$. Generally, it is not easy to solve inequalities of (6) to find such a common diagonal matrix $D > 0$. By using Schur complement, we can have the following corollary which is equivalent to the matrix inequalities in (6) easily.

Corollary 1 *The fuzzy system (4) is globally exponentially stable subject to any* $\tau_s(t) \in C_b^0(s = 1, \ldots, r)$ *if there exists a common matrix* $D > 0$ *and constants* $k_s > 0(s = 1, \ldots, r)$ *such that the following LMI's hold*

$$\begin{bmatrix} -D + DA_s + \frac{1}{2k_s}D & DB_s \\ B_s^T D & -\frac{2}{k_s}D \end{bmatrix} < 0, (s = 1, \ldots,)$$

If we give a special D and select the parameters $k_s > 0(s = 1, \ldots, r)$ carefully, we can get the next corollary which gives some simple conditions for stability.

Corollary 2 *If*

$$(\sqrt{\lambda_{\max}(B_s B_s^T)}-1)I + A_s < 0, (s = 1, \ldots,)$$

where I is the n × n identity matrix, then the free fuzzy system (4) is globally exponentially stable subject to any $\tau_s(t) \in C_b^0 (s = 1, \ldots, r)$.

Proof: Let $D = I$, and choose

$$k_s \begin{cases} \frac{1}{\sqrt{\lambda_{\max}(B_s B_s^T)}}, & if \ \ \lambda_{\max}(B_s B_s^T) \neq 0 \\ \to +\infty, & otherwise \end{cases}$$

We can derive the above result from Theorem 1 directly. The proof is completed.

It is hard to check the above matrix inequalities if the dimensions of the matrices are much high. Next, we will derive some global exponential stability conditions which will be presented in some simple algebraic inequalities. They are very easy to check.

Theorem 2 *If*

$$-1 + a_{ii}^s + \sum_{j=1}^{n} \Big[|a_{ij}^s|(1-\delta_{ij}) + |b_{ij}^s| \Big] < 0$$

for all i = 1, . . . , *n and s* = 1, . . . , *r, where*

$$\delta_{ij} = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}$$

then, the free fuzzy system (4) is globally exponentially stable subject to any

$$\tau_s(t) \in C_b^0 (s = 1, \ldots, r).$$

Proof: For any delays $\tau_s(t) \in C_b^0 (s = 1, \ldots, n)$, the free fuzzy system of (4) can be rewritten as

$$\dot{x}_i(t) = -x_i(t) + \sum_{s=1}^{r} h_s(\alpha(t) \left[\sum_{j=1}^{n} \Big(a_{ij}^s f(x_j(t)) + b_{ij}^s f(x_j(t-\tau_s(t))) \Big) \right]$$

for all $t \geq 0$ and (i = 1, . . . , n). Then, it follows that

$$D^+ |x_i(t)| \le -|x_i(t)| + \sum_{s=1}^{r} h_s(\alpha(t))[a_{ii}^s |f(x_i(t))|$$

$$+\sum_{l=1}^{r}\Big(|a_{ij}^s|(1-\delta_{ij})|f(x_j(t))|$$

$$+|b_{ij}^s||f(x_j(t-\tau_s(t)))|\Big)\Big] \quad (8)$$

for all $t \ge 0$ and $(i = 1, \ldots, n)$.

By $\tau_s(t) \in C_b^0 (s = 1, \ldots, r)$, there exists a constant $\tau \ge 0$ such that $0 \le \tau_s(t) \le \tau$ $(s = 1, \ldots, r)$ for all $t \ge 0$. Since

$$-1 + a_{ii}^s + \sum_{j=1}^{n}\Big[|a_{ij}^s|(1-\delta_{ij}) + |b_{ij}^s|\Big] < 0$$

for $i = 1, \ldots, n$ and $s = 1, \ldots, r$, then there must exist a $\epsilon > 0$ such that

$$\epsilon - 1 + a_{ii}^s + \sum_{j=1}^{n}\Big[|a_{ij}^s|(1-\delta_{ij}) + e^{\epsilon t}|b_{ij}^s|\Big] < 0$$

for all $s = 1, \ldots, r$ and $i = 1, \ldots, n$. Denote

$$\eta_{is} = -\left[\epsilon - 1 + a_{ii}^s + \sum_{j=1}^{n}\Big[|a_{ij}^s|(1-\delta_{ij}) + e^{\epsilon t}|b_{ij}^s|\Big]\right]$$

for $s = 1, \ldots, r$ and $i = 1, \ldots, n$, and let

$$\sigma = \min_{1 \le i \le n, 1 \le s \le r} (\eta_{is}).$$

Obviously, $\sigma > 0$.

Define some functions

$$z_i(t) = |x_i(t)| e^{\epsilon t}, \ (i = 1, \ldots, n)$$

for all $t \ge -\tau$. Then, it follows from (8) that

$$D^{+}z_i(t) \le (-1+\epsilon)\,|\,x_i(t)\,|\,e^{\epsilon t} + \sum_{s=1}^{r} h_s(\alpha(t))[a_{ii}^{s}\,|\,f(x_i(t))\,|$$

$$+\sum_{j=1}^{n}\Big(|\,a_{ii}^{s}\,|(1-\delta_{ij})\,|\,f(x_j(t))\,|$$

$$+\,|\,b_{ij}^{s}\,||\,f(x_j(t-\tau_s(t)))\,|\Big)\Big]e^{\epsilon t}$$

for all $t \ge 0$ and $(i = 1, \ldots, n)$.

From the definition of function f, we can see that $|f(x_i(t))| \le |x_i(t)|$, $(i = 1, \ldots, n)$. So,

$$D^{+}z_i(t) \le (-1+\epsilon)\,|\,x_i(t)\,|\,e^{\epsilon t} + \sum_{s=1}^{r} h_s(\alpha(t))[a_{ii}^{s}\,|\,x_i(t)\,|$$

$$+\sum_{j=1}^{n}\Big(|\,a_{ij}^{s}\,|(1-\delta_{ij})\,|(x_j(t)\,|$$

$$+\,|\,b_{ij}^{s}\,||\,x_j(t-\tau_s(t))\,|\Big)\Big]e^{\epsilon t}$$

$$\le \;\big(-1+\epsilon\big)z_i(t) + \sum_{s=1}^{r} h_s(\alpha(t))[a_{ii}^{s} z_i(t)$$

$$+\sum_{j=1}^{n}\Big(|\,a_{ij}^{s}\,|(1-\delta_{ij})z_j(t)$$

$$+e^{\epsilon t}\,|\,b_{ij}^{s}\,|\,z_j(t-\tau_s(t))\Big)\Big]$$

$$\le \sum_{s=1}^{r} h_s(\alpha(t))\Big[-1+a_{ii}^{s}+\epsilon)z_i(t)$$

$$+\sum_{j=1}^{n}\Big(|\,a_{ij}^{s}\,|(1-\delta_{ij})z_j(t)$$

$$+e^{\epsilon t}\,|\,b_{ij}^{s}\,|\,z_j(t-\tau_s(t))\Big)\Big] \tag{9}$$

for all $t \geq 0$ and $(i = 1, \ldots, n)$. For any constant $c > 1$, it is easy to see that $z_i(t) = |\phi_i(t)|e^{\epsilon t} \leq \|\phi\| < c\|\phi\|$ $(i = 1, \ldots, n)$ for all $t \in [-\tau, 0]$. we will prove that $z_i(t) < c\|\phi\|$ $(i = 1, \ldots, n)$ for all $t \geq 0$. Otherwise, then there must exist some i and a time $t_1 > 0$ such that

$$z_i(t_1) = c\|\phi\|$$

and

$$z_j(t)\begin{cases} < c\|\phi\|, j = i, & for \ \ t \in [-\tau, t_1) \\ \leq c\|\phi\|, j \neq i, & for \ \ t \in [-\tau, t_1]. \end{cases}$$

Then, we have $D^+z_i(t_1) \geq 0$. But on the other hand, it follows from (9) that

$$D^+z_i(t_1) \leq \sum_{s=1}^{r} h_s(\alpha(t_1))[(-1 + a_{ii}^s + \epsilon)c\|\phi\|$$

$$+c\|\phi\|\sum_{j=1}^{n}\left(|a_{ij}^s|(1-\delta_{ij}) + e^{\epsilon\tau}|b_{ij}^s|\right)\Bigg]$$

$$= -c\|\phi\|\sum_{s=1}^{r} h_s(\alpha(t_1))\eta_{is}$$

$$\leq -\sigma c\|\phi\|$$

$$< 0.$$

This is a contradiction and it proves that

$$z_i(t) < c\|\phi\| \ (i = 1, \ldots, n)$$

for all $t \geq 0$. Letting $c \to 1$, we have $z_i(t) \leq \|\phi\|$ $(i = 1, \ldots, n)$ for all $t \geq 0$. Then, it follows that

$$|x(t)| \leq \|\phi\| e^{-\epsilon t}, \ (i = 1, \ldots, n)$$

for all $t \geq 0$. The proof is completed.

The above theorems provide some conditions to guarantee the exponential stability of the free fuzzy systems of (4) subject to any uncertain continuous bounded delays.

4. FUZZY FEEDBACK STABILIZATION OF DELAYED FUZZY SYSTEMS

In this section, we will study how to design a state feedback fuzzy controller to feedback control the stability of the fuzzy control system with uncertain delays

$$\dot{x}(t) = \sum_{s=1}^{r} h_s(\alpha(t))[-x(t) + A_s f(x(t)) + B_s f(x(t-\tau_s(t))) + C_s u(t)] \tag{10}$$

for $t \geq 0$.

We consider the following fuzzy control law for the delayed fuzzy system (10). For each $l = 1, \ldots, r$, we have

Regulator Rule l: IF $\alpha_1(t)$ is M_{1l} AND . . . AND $\alpha_p(t)$ is M_{pl} THEN

$$u(t) = -K_l f(x(t))$$

where each $K_l = \left(k_{ij}^l\right)$ is a $m \times n$ matrix.

The overall state feedback fuzzy controller can be inferred as

$$u(t) = -\frac{\sum_{l=1}^{r} w_l(\alpha(t)) K_l f(x(t))}{\sum_{l=1}^{r} w_l(\alpha(t))}$$

$$= -\sum_{l=1}^{r} h_l(\alpha(t)) K_l f(x(t)). \tag{11}$$

Using the above fuzzy feedback controller, from (10), we get the closed loop delayed fuzzy system

$$\dot{x}(t) = \sum_{s,l=1}^{r} h_s(\alpha(t)) h_l(\alpha(t))[-x(t) + (A_s - C_s K_l) f(x(t))$$

$$+ B_s f(x(t-\tau_s(t)))] \tag{12}$$

for $t \geq 0$.

If there exist matrices $K_l (l = 1, \ldots, r)$ such that the closed loop fuzzy system (12) is globally exponentially stable, then we say that the delayed fuzzy control system (10) can be fuzzy feedback globally exponentially stabilized by the fuzzy controller (11).

Obviously, the aim of the design of fuzzy feedback controller is to select the desired matrices $K_l (l = 1, \ldots, r)$ such that (12) is globally exponentially stable. In last Section, we have got some conditions of global exponential stability for free fuzzy systems. Since the closed loop delayed fuzzy system (12) can be looked as a free fuzzy system, we can directly use the results in last section to derive global exponential stability to (12) and then get criteria for the design of the feedback controller. Similar to the analysis of the last section, we have the following theorems which will provide some criteria for the selection of the matrices of $K_l (l = 1, \ldots, r)$.

Theorem 3 *If there exists a diagonal matrix $D > 0$ and some constants $\gamma_s > 0 (s = 1, \ldots, r)$ such that*

$$-D + D(A_s - C_s K_l) + \frac{\gamma_s}{2} DB_s D^{-1} B_s^T D + \frac{1}{2\gamma_s} D < 0 \tag{13}$$

or the LMI's

$$\begin{bmatrix} -D + D(A_s - C_s K_l) + \frac{1}{2\gamma_s} D & DB_s \\ B_s^T D & -\frac{2}{\gamma_s} D \end{bmatrix} < 0, (s = 1, \ldots,)$$

for $s, l = 1, \ldots, r$. Then the fuzzy system (10) can be globally exponentially stabilized by the fuzzy controller (11) subject to any $\tau_s(t) \in C_b^0 (s = 1, \ldots, r)$.

Corollary 3 *If*

$$(\sqrt{\lambda_{\max}(B_s B_s^T)} -)I + (A_s - C_s K_l) < 0$$

for $s, l = 1, \ldots, r$, where I is the $n \times n$ identity matrix. Then the fuzzy system (10) can be globally exponentially stabilized by the fuzzy controller (11) subject to any $\tau_s(t) \in C_b^0 (s = 1, \ldots, r)$.

Theorem 4 *Suppose that*

$$-1 + a_{ii}^s - \sum_{p=1}^{m} c_{ip}^s k_{pi}^l + \sum_{j=1}^{n} \left[\left| a_{ij}^s - \sum_{p=1}^{m} c_{ip}^s k_{pj}^l \right| (1 - \delta_{ij}) + | b_{ij}^s | \right] < 0$$

for all $i = 1, \ldots, n$ and $s, l = 1, \ldots, r$, where

$$\delta_{ij} = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}$$

then, the fuzzy system (10) can be globally exponentially stabilized by the fuzzy controller (11) subject to any $\tau_s(t) \in C_b^0 (s = 1, \ldots, r)$.

Since

$$\sum_{s,l=1}^{r} h_s(\alpha(t)h_l(\alpha(t)) = \sum_{s=1}^{r}\left(h_s(\alpha(t))\sum_{l=1}^{r} h_l(\alpha(t)) \right) = 1$$

the proofs of the above theorems can be derived by some slight modifications to the proofs of the theorems in last section. The details are omitted.

By solving the inequalities in the above theorems, the controllers can be obtained directly .

5. SIMULATIONS

In this section, we employ some examples to illustrate the above theory. Let us consider the following nonlinear system with delay

$$\begin{cases} \dot{x}_1(t) = -x_1(t) - f(x_1(t)).\left(1+\sin^2 x_2(t)\right) \\ \qquad\qquad + f(x_1(t-\tau(t))) + f(x_2(t-\tau(t))).\sin^2 x_2(t) \\ \dot{x}_2(t) = -x_2(t) - f(x_2(t)) + (f(x_1(t)) - f(x_2(t)).\cos^2 x_2(t) \\ \qquad\qquad + f(x_1(t-\tau(t))).\cos^2 x_2(t) \\ \qquad\qquad + f(x_2(t-\tau(t))).\sin^2 x_2(t) \end{cases} \tag{14}$$

for all $t \geq 0$.

Define some matrices

$$A_1 = \begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$$

and some functions

$$M_{11}(x_2(t)) = \sin^2 x_2(t),\ M_{22}(x_2(t)) = \cos^2 x_2(t).$$

We can interpret $M_{11}(x_2(t))$ and $M_{22}(x_2(t))$ as membership functions of some fuzzy sets M_{11} and M_{22}, respectively. Using these fuzzy sets, the above nonlinear system (14) can be presented by the following TS fuzzy model

Plant Rule 1: IF $x_2(t)$ is M_{11} THEN

$$\dot{x}(t) = -x(t) + A_1 f(x(t)) + B_1 f(x(t - \tau(t))). \tag{15}$$

Plant Rule 2: IF $x_2(t)$ is M_{22} THEN

$$\dot{x}(t) = -x(t) + A_2 f(x(t)) + B_2 f(x(t - \tau(t))). \tag{16}$$

Using the Theorem 2, it is easy to check that the nonlinear system (14) is globally exponentially stable subject to any $\tau(t) \in C_b^0$. The delay $\tau\ (t)$ could be any bounded continuous function, say, $\sin^2(t)$, $\cos^2(t)$, $1/(1 + |t|)$ (not differentiable), etc. Stability conditions for systems with uncertain delays are quite interesting, since in practical applications it is actually not easy to know the delays exactly.

Figure 2 shows the global exponential stability of the nonlinear system (14) with $\tau\,(t) = 1$.

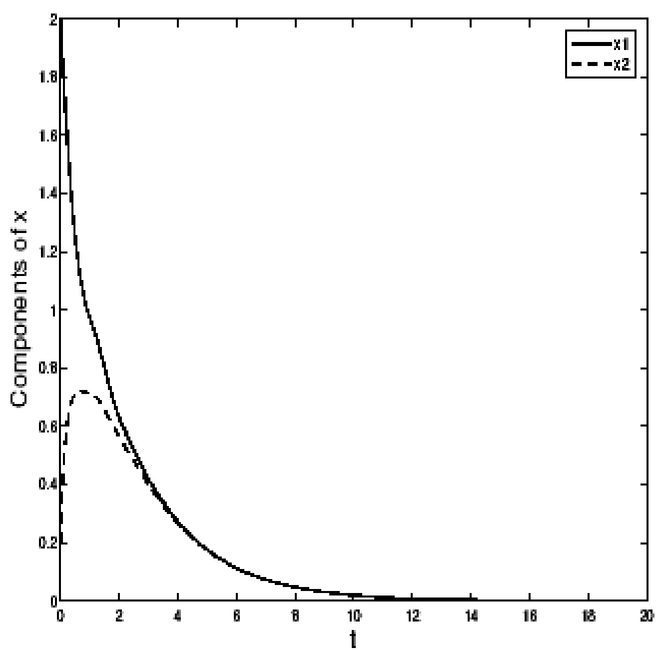

Fig. 2. GES of (14)with $\tau\,(t) = 1$

Figure 3 and Figure 4 show the global exponential stability of the local systems (15) and (16) with $\tau(t) = 1$.

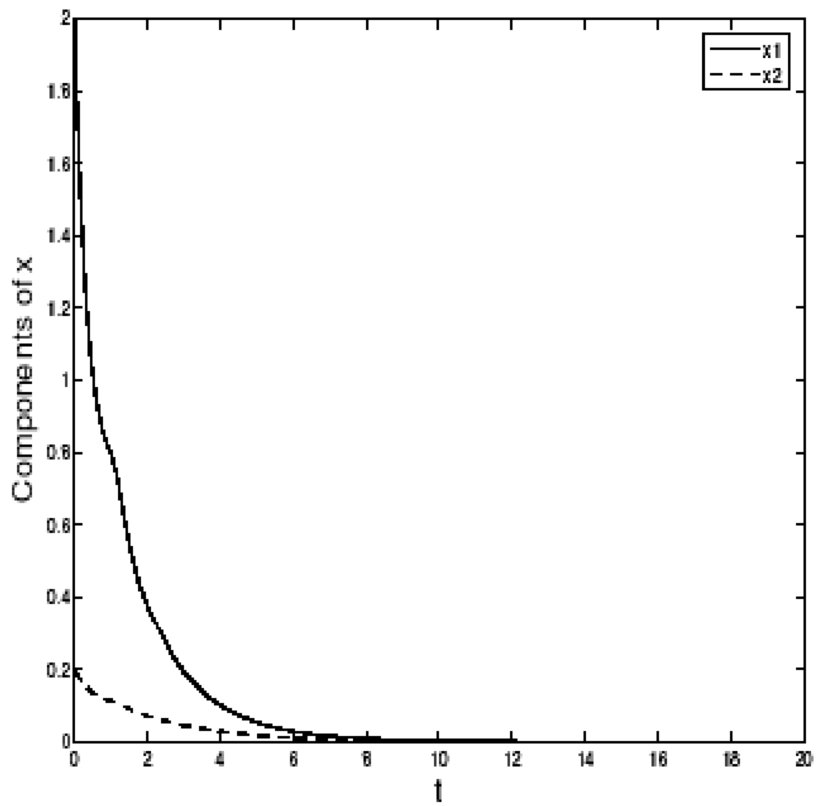

Fig. 3. GES of (15)with $\tau(t) = 1$

6. CONCLUSIONS

In this paper, the global exponential stability analysis for a class of fuzzy systems with time delays has been studied. First, we have discussed stabilization for delayed fuzzy control systems and some global exponential stability conditions for free delayed fuzzy systems have been proposed. Then we have given

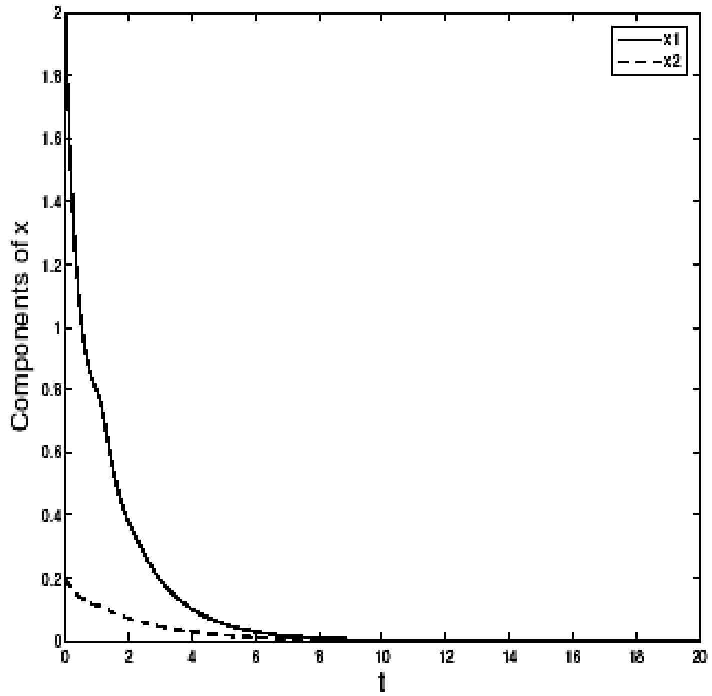

Fig. 4. GES of (16)with $\tau(t) = 1$

some criteria for feedback fuzzy controller design. Finally, an example has been used to illustrate the results. We believe that all of the results obtained in this paper can be extended to the fuzzy systems with multiple time delays or with time-varying delay only by changing another Lyapunov function.

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