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Parameter Identification of Chaos System Based on Unknown Parameter Observer

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Parameter identification of chaos system is an important problem in the field of chaos control and synchronization. In this paper, the method of parameter identification based on unknown parameter observer is studied in detail. The key to observer design is to choose an appropriate gain function and construct an auxiliary function. The right side of the differential equation of chaos system is divided into the linear part and nonlinear part. The techniques to construct observer are discussed respectively under different cases, i.e., the diagonal element, non-diagonal element of coefficient matrix of the linear part in chaos system's differential equation, and the coefficient of a certain nonlinear term are unknown parameters respectively. Some general approaches to choosing an appropriate gain function and constructing an auxiliary function are proposed and the design methods of observer are provided under corresponding cases. Illustrative observers are designed for two chaotic systems. Simulations results demonstrate the effectiveness and feasibility of the designed observers.

1. INTRODUCTION

Since Lorenz found the first canonical chaotic attractor in 1963 [1], chaos control attracts more and more attention [2] and many different techniques have been proposed, including OGY method [3], linear state space feedback [4], differential geometric approach [5], sampled-date feedback method [6], inverse optimal control [7], and adaptive control [8]. Especially, after the pioneering work of Pecora and Carrol in 1990 [9], synchronization of chaotic systems has attracted much attention due to its potential application in secure communication, chemical and biological systems, information science, biotic science and so on [2]. Great efforts have been devoted to achieving chaos synchronization in the last few years and a wide variety of approaches have been proposed, such as linear and nonlinear feedback synchronization methods [10, 11], time delay feedback synchronization approaches [12], adaptive synchronization methods [13], impulsive synchronization methods [14] and so on.

In the fields of chaos control and synchronization, how to achieve the uncertain parameters of chaotic system is an important problem. In recent years, a number of different techniques have been proposed to estimate parameters of chaotic system, including synchronization method based on the theory of Lyapunove function [15, 16], Bayesian approach [17], least square method [18], random optimization method [19], among many others [20, 21].

Guan et al. [22] introduced the method based on unknown parameter observer to estimate the parameter of the third equation in Lorenz system. Wu et al. [23] used this method to identify the parameter of the second equation in Lü system. This approach is very convenient in practical application because it uses the state vector of chaos system directly and avoids some complex theories in other methods. However, only a special case was discussed and no general methods to choose gain function and design auxiliary function was given in [22, 23]. In general cases, there are two difficult problems: one is how to choose an appropriate gain function; another is how to construct an auxiliary function that can eliminate the derivative of the state variable in the preliminary observer. In this paper, some observers are designed to identify the unknown parameters of chaos systems in usual cases, and the general methods to choose the gain function and design the auxiliary function are proposed.

The layout of the paper is as follows: the problem is formulated in Section 2. Section 3, 4 and 5 are devoted to identify the uncertain parameters of chaos systems and discuss the design of the unknown parameter observer in some different cases. Some examples and numerical simulations are given to demonstrate the effectiveness and feasibility of the designed observers in Section 6. Section 7 is the conclusions.

2. PROBLEM FORMULATION

Consider chaos system

$$\dot{x} = Ax + F(x), \tag{1}$$

where $x \in \mathbf{R}^n$ is a state vector, $A \in [a_{ij}]_{n\times n}$ is the coefficient matrix of the linear part in system (1), $F(x) = [f_i(x)]_{n\times 1}$ is the nonlinear part of system (1).

As some unknown parameter in A and $F(x)$, our objective is to design observers to identify these unknown parameters.

3. IDENTIFICATION OF THE UNKNOWN PARAMETER a_{ii}

In this section, we discuss how to design an observer to identify the unknown parameter a_{ii} of the system (1).

*Lemma: If $x \in$ **R** is the state vector of a chaos system, and $f(x) \in [\mathbf{R}^n \to \mathbf{R}]$ is chaotic, then the system $\dot{y} = -k\,|\,f(x)\,|\,y \;\; (k > 0, y \in \boldsymbol{R})$ is asymptotically stable.*

Proof: From the differential equation of the system, we get $y = y_0 e^{-\int_0^t k|f(x)|dt}$. Since $f(x)$ is chaotic, $f(x)$ don't converge zero when $t \to \infty$. According to Barbalat theorem, $\int_0^\infty k\,|\,f(x)\,|\,dt \to \infty$ correspondingly, $y(\infty) \to 0$, i.e., the system $\dot{y} = -k\,|\,f(x)\,|\,y$ is asymptotically stable. This completes the proof.

Theorem 1: *Assume system (1) satisfies the conditions as follows: a) a_{ii} is the unique unknown parameter in the i-th equation of system (1), b) a_{ii} is a constant. If the state vector x of system (1) can be obtained, then we can design an observer to identify the unknown parameter.*

Proof: Since parameter a_{ii} is a constant, we have

$$\dot{a}_{ii} = 0. \tag{2}$$

Because the unknown parameter a_{ii} can act as a state variable, system (1) can be augmented by Eq. (2). According to the *i*-th equation of system (1), we have

$$a_{ii} x_i = \dot{x}_i - \sum_{j=1, j\neq i}^{n} a_{ij} x_j - f_i(x). \tag{3}$$

Then we design the observer as follows

$$\dot{\hat{a}}_{ii} = -L(x_i) x_i \hat{a}_{ii} + L(x_i)[\dot{x}_i - \sum_{j=1, j\neq i}^{n} a_{ij} x_j - f_i(x)], \tag{4}$$

where $L(x_i)$ is a gain function. Let

$$e(t) = \hat{a}_{ii} - a_{ii}, \tag{5}$$

then according to Eqs. (2), (3) and (4), we obtain

$$\dot{e}(t) = \dot{\hat{a}} - \dot{a}_{ii} = -L(x_i) x_i e(t). \tag{6}$$

If we choose such a gain function $L(x_i)$ that makes the system $\dot{e}(t) + L(x_i) x_i e(t) = 0$ asymptotical stable for all x_i, then $\hat{a}_{ii}(t)$ converges to a_{ii} when $t \to \infty$.

In the practical situation, x_i is hard to be observed and the observer (4) is not applicable directly. Hence, we introduce an auxiliary variable

$$p_i = \hat{a}_{ii} + \Phi(x_i), \tag{7}$$

where $\Phi(x_i)$ is a design auxiliary function that satifies

$$\frac{d\Phi(x_i)}{dx_i} = -L(x_i). \tag{8}$$

According to Eqs. (7), (4) and (8), we have

$$\begin{aligned}\dot{p}_i &= \dot{\hat{a}}_{ii} + \dot{\Phi}(x_i) \\ &= -L(x_i)x_i\hat{a}_{ii} + L(x_i)[\dot{x}_i - \sum_{j=1, j\neq i}^{n} a_{ij}x_j - f_i(x)] + \frac{d\Phi(x_i)}{dx_i}\dot{x}_i \\ &= -L(x_i)x_i p_i + L(x_i)[x_i\Phi(x_i) - \sum_{j=1, j\neq i}^{n} a_{ij}x_j - f_i(x)]\end{aligned} \tag{9}$$

The next work is to choose an appropriate $L(x_i)$ or $\Phi(x_i)$ that makes system (6) asymptotical stable for all x_i . In the following, we will discuss it under different cases respectively.

3.1 $x_i > 0$ (or $x_i < 0$) for all Time

For simplicity, we discuss only the case of $x_i > 0$. In this case, there are a few choices for us. Two simple choices are as follows

a. We choose

$$L(x_i) = k \ (k > 0) \tag{10}$$

According to the lemma, the error system $\dot{e}(t) = -L(x_i)x_i e(t) = -kx_i e(t)$ is exponentially asymptotical stability for all x_i, and $\hat{a}_{ii} \to a_{ii}$ *aii* when $t \to \infty$.

From Eq. (8), we have $\Phi(x_i) = -kx_i$. Thus, the observer is designed

$$\begin{cases} \dot{p}_i = -kx_i p_i - k^2 x_i^2 - k\left[\sum_{j=1, j\neq i}^{n} a_{ij}x_j + f_i(x)\right], \\ \hat{a}_{ii} = p_i + kx_i \end{cases} \tag{11}$$

b. We choose

$$L(x_i) = \frac{k}{x_i}\ (k > 0), \tag{12}$$

then,the system $\dot{e}(t) = -L(x_i)x_i e(t) = -ke(t)$ is exponentially asymptotical stable. From Eq. (8), we have $\Phi(x_i) = -k \ln x_i$. Thus, the observer is designed as follows

$$\begin{cases} \dot{p}_i = -kp_i - k^2 \ln x_i - \frac{k}{x_i}\left[\sum_{j=1, j\neq i}^{n} a_{ij}x_j + f_i(x)\right], \\ \hat{a}_{ii} = p_i + k \ln x_i. \end{cases} \tag{13}$$

Theoretically, all forms like $L(x_i) = kx_i^n$ are feasible in this case.

3.2 The set $[x_i \mid x_i(t) = 0, t \in R^+]$ is a Denumerable Set

In this case, the sign of x_i is alternative. According to the lemma, we can choose $L(x_i) = kx_i^{2m-1}, (k > 0, m = 1, 2, \ldots)$ that makes $\dot{e}(t) = -L(x_i)x_i e(t) = -kx_i^{2m} e(t)$, ($k > 0$, $m = 1,2, \ldots$) asymptotical stability. The simplest choice is $L(x_i) = kx_i$. Here, $\Phi(x_i) = -\frac{1}{2}kx_i^2$ and the observer is designed as

$$\begin{cases} \dot{p}_i = -kx_i^2 p_i - \frac{1}{2}k^2 x_i^4 - kx_i\left[\sum_{j=1, j\neq i}^{n} a_{ij}x_j + f_i(x)\right], \\ \hat{a}_{ii} = p_i + \frac{1}{2}kx_i^2. \end{cases} \tag{14}$$

From the above deduction, we can see that the observer of the unknown parameter a_{ii} only relies on the i-th equation and is independent of others in chaos system (1). So, as long as a_{ii} is the unique unknown parameter in the i-th equation of chaos system (1), it can be identified by means of above observer. If all a_{ii}, ($i = 1,2, \ldots n$) are unknown and the rest parameters are certain, then the observers of a_{ii}, ($i = 1,2, \ldots n$) can be designed to identify them at the same time. The proof of the theorem is completed.

4. IDENTIFICATION OF THE UNKNOWN PARAMETER A_{IK}

In this section, we discuss how to design an observer to identify the unknown parameter a_{ik} of the system (1) in two different cases.

4.1 a_{ik} is the Unique Unknown Parameter in the i-th Equation of Chaos System (1)

Theorem 2: *Assume that system (1) satisfies the conditions as follows: a) the i-th equation has no unknown parameter except a_{ik} and aik is a constant, b) all parameters of the k-th equation of (1) are certain or only a_{kk} is unknown. If the state vector of system (1) can be obtained, then we can design an observer to identify the unknown parameter a_{ik}.*

Proof: As a_{ik} is a constant, we get

$$\dot{a}_{ik} = 0. \tag{15}$$

Because the unknown parameter a_{ik} can act as a state variable, system (1) can be augmented by Eq. (15). According to the *i*-th equation of system (1), we have

$$a_{ik} x_k = \dot{x}_i - \sum_{j=1, j\neq k}^{n} a_{ij} x_j - f_i(x). \tag{16}$$

Then we design the observer as follows

$$\dot{\hat{a}}_{ik} = -L(x_k) x_k \hat{a}_{ik} + L(x_k)[\dot{x}_i - \sum_{j=1, j\neq k}^{n} a_{ij} x_j - f_i(x)] \tag{17}$$

where $L(x_k)$ is a gain function. Let

$$e(t) = \hat{a}_{ik} - a_{ik}, \tag{18}$$

then according to Eqs. (16), (17) and (18), we obtain

$$\dot{e}(t) = \dot{\hat{a}}_{ik} - \dot{a}_{ik} = -L(x_k) x_k e(t). \tag{19}$$

If we choose an appropriate gain function $L(x_k)$ that makes the error system $\dot{e}(t) + L(x_k) x_k e(t) = 0$ asymptotically stable for all x_k, then $\hat{a}_{ik}(t)$ converges to a_{ik} when $t \to \infty$.

In the practical situation, $\dot{x}_i$ is hard to be observed and the observer (17) is not applicable directly. Hence, we introduce an auxiliary variable

$$p_{ik} = \hat{a}_{ik} + \Phi(x_i, x_k), \tag{20}$$

where $\Phi(x_i, x_k)$ is a design auxiliary function that satisfies

$$\frac{\partial \Phi(x_i, x_k)}{\partial x_i} = -L(x_k). \tag{21}$$

According to Eqs. (20), (17) and (21), we get

$$\begin{aligned}
\dot{p}_{ik} &= \dot{\hat{a}}_{ik} + \dot{\Phi}(x_i, x_k) \\
&= -L(x_k)x_k\hat{a}_{ik} + L(x_k)[\dot{x}_i - \sum_{j=1, j\neq k}^{n} a_{ij}x_j - f_i(x)] \\
&\quad + \frac{\partial \Phi(x_i, x_k)}{\partial x_i}\dot{x}_i + \frac{\partial \Phi(x_i, x_k)}{\partial x_k}\dot{x}_k \\
&= -L(x_k)x_k p_{ik} + L(x_k)[x_k\Phi(x_i, x_k) - \sum_{j=1, j\neq k}^{n} a_{ij}x_j - f_i(x)] \\
&\quad + \frac{\partial \Phi(x_i, x_k)}{\partial x_k}\dot{x}_k
\end{aligned} \tag{22}$$

Based on the condition b), $\dot{x}_k$ can be obtained from the k-th equation of system (1), i.e., $\dot{x}_k = \sum_{j=1}^{n} a_{kj}x_j + f_k(x)$.

Remark: IF a_{kk} is unknown, it can be identified using the method in Section 3.

So, Eq. (22) becomes

$$\begin{aligned}
\dot{p}_{ik} = -L(x_k)x_k p_{ik} + L(x_k)[x_k\Phi(x_i, x_k) - \sum_{j=1, j\neq k}^{n} a_{ij}x_j - f_i(x)] \\
+ \frac{\partial \Phi(x_i, x_k)}{\partial x_k}[\sum_{j=1}^{n} a_{kj}x_j + f_k(x)].
\end{aligned} \tag{23}$$

According to the lemma, we can choose $L(x_k) = kx_k^{2m-1}(k > 0, m = 1,2,\ldots)$ and $\Phi(x_i, x_k) = -kx_k^{2m-1}x_i$ that make the error system (19) asymptotically stable. Thus, the observer consisted of (23) and (20) is applicable to identify the unknown parameter a_{ik}. If the sign of x_k is alternative, the simplest choice is $L(x_k) = kx_k$ and $\Phi(x_i, x_k) = -kx_ix_k$. If $x_k \geq 0$ (or $x_k \leq 0$) for all time, the ordinary choice is $L(x_k) = k$ (or $-k$)($k > 0$) and $\Phi(x_i, x_k) = -kx_i$ (or kx_i). This completes the proof.

4.2 Parameters $|a_{ik_1}|=|a_{ik_2}|=\cdots=|a_{ik_m}|=|a_{ii}|$ are Unknown and the Others are Certain in *i*-th Equation of System (1)

Theorem 3: *Assume that system (1) satisfies the conditions as follows: a) the i-th equation have unknown parameters $|a_{ik_1}|=|a_{ik_2}|=\cdots=|a_{ik_m}|=|a_{ii}|=a$ constant, and the others are certain; b) all parameters of the k_l-th ($l = 1, 2, \ldots, m$) equation are certain or only $a_{k_lk_l}$ are unknown. If the state vector of system (1) can be obtained, then we can design an observer to identify the unknown parameter.*

Proof: Similar to Section 3, according to the assume, we have

$$\dot{a}_{ii} = 0. \tag{24}$$

From the *i*-th equation of system (1), we get

$$a_{ii}(x_i + y) = \dot{x}_i - \sum_{j=1, j\neq i, k_l}^{n} a_{ij}x_j - f_i(x), \tag{25}$$

where y is the algebraic sum of $x_{k_1}, x_{k_2}, \ldots, x_{km}$. Then we design the observer as follows

$$\dot{\hat{a}}_{ii} = -L(x_i, y)(x_i + y)\hat{a}_{ii} + L(x_i, y)[\dot{x}_i - \sum_{j=1, j\neq i, k_l}^{n} a_{ij}x_j - f_i(x)] \tag{26}$$

where $L(x_i, y)$ is a gain function. Let $e(t) = \hat{a}_{ii} - a_{ii}$, and we have

$$\dot{e}(t) = \dot{\hat{a}}_{ii} - \dot{a}_{ii} = -L(x_i, y)(x_i + y)e(t). \tag{27}$$

If we choose an appropriate gain function $L(x_i, y)$ that makes the error system (27) asymptotically stable for all $x_i, x_{k_1}, x_{k_2}, \ldots, x_{k_m}$ then $\hat{a}_{ii}(t)$ converges to a_{ii} when $t \to \infty$. However, $\dot{x}_i$ is hard to be observed in the practical situation, and the observer (26) is not applicable directly. To eliminate $\dot{x}_i$, we introduce an auxiliary variable

$$p_i = \hat{a}_{ii} + \Phi(x_i, y), \tag{28}$$

where $\Phi(x_i, y)$ is a design auxiliary function that satisfies

$$\frac{\partial \Phi(x_i, y)}{\partial x_i} = -L(x_i, y). \tag{29}$$

From Eqs. (28), (29) and (26), we get

$$\begin{aligned}
\dot{p}_i &= \hat{a}_{ii} + \dot{\Phi}(x_i, y) \\
&= -L(x_i, y)(x_i + y)\hat{a}_{ii} + L(x_i, y)[\dot{x}_i - \sum_{j=1, j\neq i, k_l}^{n} a_{ij}x_j - f_i(x)] \\
&+ \frac{\partial \Phi(x_i, y)}{\partial x_i}\dot{x}_i + \sum_{l=1}^{m} \frac{\partial \Phi(x_i, y)}{\partial x_{k_l}}\dot{x}_{k_l} \\
&= -L(x_i, y)(x_i + y)p_i + L(x_i, y)[(x_i + y)\Phi(x_i, y) - \sum_{j=1, j\neq i, k_l}^{n} a_{ij}x_j - f_i(x)] \\
&+ \sum_{l=1}^{m} \frac{\partial \Phi(x_i, y)}{\partial x_{k_l}}\dot{x}_{k_l}.
\end{aligned} \tag{30}$$

Based on the condition b), $\dot{x}_{k_l}$ can be obtained from the k_l-th equation of system (1), i.e., $\dot{x}_{k_l} = \sum_{j=1}^{n} a_{k_l j}x_j + f_{k_l}(x)$.

Remark: IF $a_{k_l k_l}$ is unknown, it can be identified using the method in Section 3.

So, Eq. (30) becomes

$$\begin{aligned}
\dot{p}_i &= -L(x_i, y)(x_i + y)p_i + L(x_i, y)[(x_i + y)\Phi(x_i, y) - \sum_{j=1, j\neq i, k_l}^{n} a_{ij}x_j - f_i(x)] \\
&+ \sum_{l=1}^{m} \frac{\partial \Phi(x_i, y)}{\partial x_{k_l}}[\sum_{j=1}^{n} a_{k_l j}x_j + f_{k_l}(x)].
\end{aligned} \tag{31}$$

According to the lemma, if we choose $L(x_i, y) = k(x_i + y)^{2l-1}(k > 0, l = 1, 2, \ldots)$ and $\Phi(x_i, y) = -\frac{1}{2l}k(x_i + y)^{2l}$, then the error system (27) is asymptotical stable. The observer consisted of (31) and (28) is applicable to identify the unknown parameters $a_{ik_1}, a_{ik_2}, \ldots, a_{ik_m}$ and a_{ii}. The proof of the theorem is completed.

5. IDENTIFICATION OF THE UNKNOWN PARAMETER IN NONLINEAR PART

If there is unique unknown parameter b_i in the nonlinear part of the i-th equation of system (1), the i-th equation can be expressed as

$$\dot{x}_i = \sum_{j=1}^{n} a_{ij}x_j + b_i f_{i_1}(x) + f_{i_2}(x), \tag{32}$$

where $f_{i_1}(x) = \prod_{j=1}^{n} x_j^{n_j}$, $f_{i_2}(x)$ is the rest nonlinear part.

Theorem 4: *Assume that system (1) satisfies the conditions as follows: a) b_i is the unique unknown parameter of the i-th equation and constant, b) all parameters of the j-th ($j \neq i$, $n_j \neq 0$) equation in system (1) are certain or only a_{jj} is unknown. If the state vector of system (1) can be obtained, then we can design an observer to identify the unknown parameter b_i.*

Proof: Because b_i is a constant, we have

$$\dot{b}_i = 0. \tag{33}$$

From Eq. (32), we get $b_i f_{i_1}(x) = \dot{x}_i - [\sum_{j=1}^{n} a_{ij}x_j + f_{i_2}(x)],$. Then the observer is designed as

$$\dot{\hat{b}}_i = -L[f_{i_1}(x)]f_{i_1}(x)\hat{b}_i + L[f_{i_1}(x)][\dot{x}_i - \sum_{j=1}^{n} a_{ij}x_j - f_{i_2}(x)], \tag{34}$$

where $L[f_{i_1}(x)]$ is the gain function. Let $e(t) = \hat{b}_i - b_i$, and we have

$$\dot{e}(t) = -L[f_{i_1}(x)]f_{i_1}(x)e(t). \tag{35}$$

If we choose an appropriate gain function $L[f_{i_1}(x)]$ that makes the error system (35) asymptotically stable for all x, then $\hat{b}_i(t)$ converges to b_i when $t \to \infty$. However, $\dot{x}_i$ is hard to be observed in the practical situation, and the observer (34) is not applicable directly. To eliminate $\dot{x}_i$, we introduce an auxiliary variable

$$p_i = \hat{b}_i + \Phi(x), \tag{36}$$

where $\Phi(x)$ is an auxiliary function that satisfies

$$\frac{\partial \Phi(x)}{\partial x_i} = -L[f_{i_1}(x)]. \tag{37}$$

From Eqs. (36), (34) and (37), we can get

$$\begin{aligned} p_i &= \hat{b}_i + \dot{\Phi}(x) \\ &= -L[f_{i_1}(x)]f_{i_1}(x)\hat{b}_i + L[f_{i_1}(x)] \\ &\times[\dot{x}_1 - \sum_{j=1}^{n} a_{ij}x_j - f_{i_2}(x)] + \sum_{j=1}^{n} \frac{\partial \Phi(x)}{\partial x_j}\dot{x}_j \\ &= -L[f_{i_1}(x)]f_{i_1}(x)p_i + L[f_{i_1}(x)][f_{i_1}(x)\Phi(x) - \sum_{j=1}^{n} a_{ij}x_j - f_{i_2}(x)] \\ &+ \sum_{j=1, j\neq i}^{n} \frac{\partial \Phi(x)}{\partial x_j}\dot{x}_j. \end{aligned} \tag{38}$$

According to the condition b), $\dot{x}_j(j \neq i)$ can be obtained from the j-th equation of system (1), i.e., $\dot{x}_j = \sum_{k=1}^{n} a_{jk}x_k + f_j(x)$.

Remark: IF a_{jj} is unknown, it can be identified by means of the method in Section 3.

So, Eq. (38) becomes

$$\dot{p}_i = -L[f_{i_1}(x)]f_{i_1}(x)p_i + L[f_{i_1}(x)][f_{i_1}(x)\Phi(x) - \sum_{j=1}^{n} a_{ij}x_j - f_{i_2}(x)]$$

$$+\sum_{j=1,j\neq i}^{n}\frac{\partial\Phi(x)}{\partial x_j}[\sum_{k=1}^{n}a_{jk}x_k+f_j(x)]. \tag{39}$$

Based on the lemma, we can choose $L[f_{i_1}(x)]=[f_{i_1}(x)]^{2m-1}(m=1,2,\dots)$ that makes the error system $\dot{e}(t)+L[f_{i_1}(x)]f_{i_1}(x)e(t)=0$ asymptotically stable for all x, and we can get $\Phi(x)=\frac{1}{n_i(2m-1)+1}x_i^{n_i(2m-1)+1}\Pi_{j=1,j\neq i}^{n}x_j^{n_j(2m-1)}$ from Eq. (36). Especially, if $f_{i_1}(x)\geq 0$ or $f_{i_1}(x)\leq 0$ for all time, we can choose $L[f_{i_1}(x)]=k$ (or $-k$)($k>0$), consequently, $\Phi(x)=-kx_i$ (or kx_i). Thus, the observer consisted of (39) and (36) is applicable to identify the unknown parameters b_i. This completes the proof.

6. SIMULATIONS

In this section, some examples and numerical simulations are presented to demonstrate and verify the performance of the observer. In all simulations, the differential equations are solved through using ode45 in Matlab 7.04.

Examples 1: Parameter identification of Lorenz system

Lorenz system is described as

$$\begin{cases}\dot{x}_1=a(x_2-x_1)\\ \dot{x}_2=bx_1-x_2-x_1x_3\\ \dot{x}_3=x_1x_2-cx_3\end{cases}.$$

When $a=10$, $b=28$, $c=\frac{8}{3}$, Lorenz system is chaotic.

a. The parameter c is uncertain and the rests are known.

Using the method in Section 3, the observer is designed as follows

$$\begin{cases}\dot{p}=-kx_3^2p+\frac{1}{2}k^2x_3^2+kx_1x_2x_3,\\ \hat{c}=p-\frac{1}{2}kx_3^2.\end{cases}$$

Fig.1 (a) shows its simulation result.

Because $x_3>0$ for all time in Lorenz system, the observer can also be designed as

$$\begin{cases} \dot{p} = -kx_3 p + k^2 x_3^2 + kx_1 x_2, \\ \hat{c} = p - kx_3. \end{cases}$$

Fig. 1 (b) shows its simulation result.

b. The parameter *b* is unknown and the rests are certain.

Adopting the method in Section 4.1, we can get the observe as follows

$$\begin{cases} \dot{p} = -kx_1^2 p - k^2 x_1^3 x_2 + kx_1(x_2 + x_1 x_3) - kx_2[a(x_2 - x_1)] \\ \hat{b} = p + kx_1 x_2. \end{cases}$$

Fig.1 (c) shows its simulation result.

c. The parameter *a* is uncertain and the rests are known.

Using the technique in Section 4.2, the observer is constructed as

$$\begin{cases} \dot{p} = -k(x_2 - x_1)^2 p + k^2 (x_2 - x_1)^2 (-x_1 x_2 + \frac{1}{2} x_1^2) - kx_1(bx_1 - x_2 - x_1 x_3), \\ \hat{a} = p + k(x_1 x_2 - \frac{1}{2} x_1^2). \end{cases}$$

Fig. 1 (d) shows its simulation result.

Fig.1 show the simulated results of parameters identification of Lorenz system. The initial conditions are $[x_1(0), x_2(0), x_3(0), p(0)] = [10, 10, 10, 2]$ and the gain coefficient $k = 0.2$ in all cases.

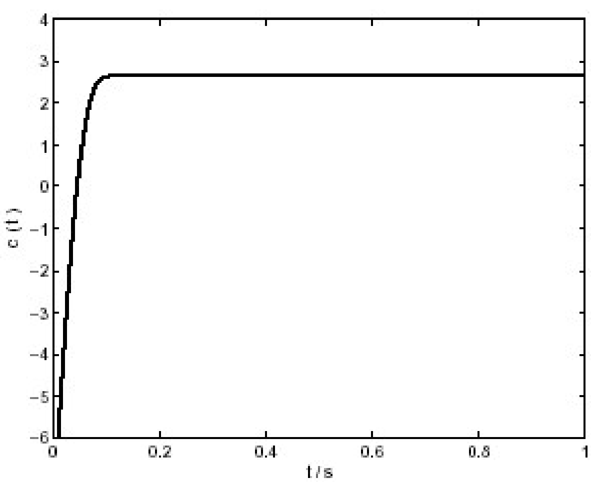

Fig.1(a)

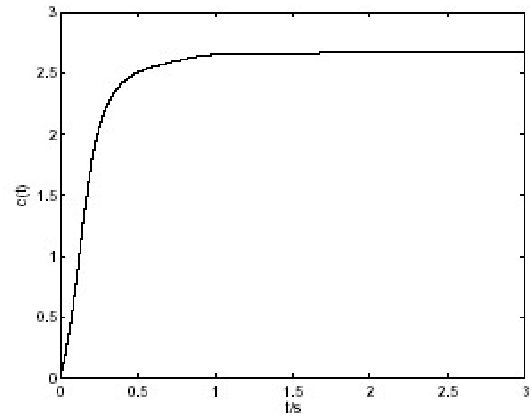

Fig. 1(b)

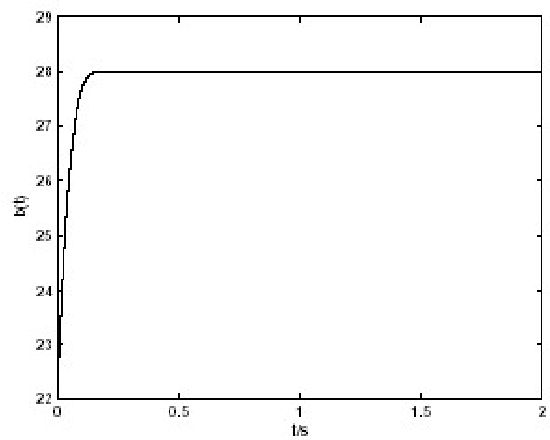

Fig. 1(c)

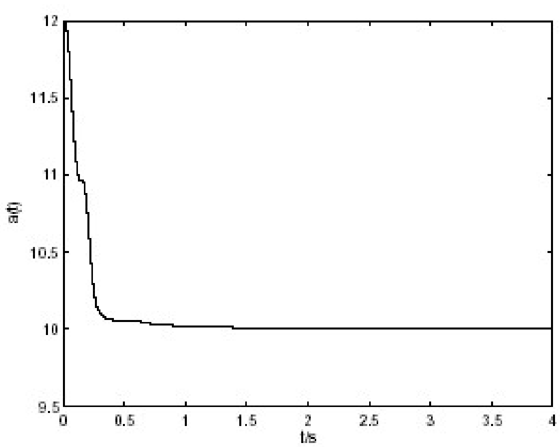

Fig. 1(d)

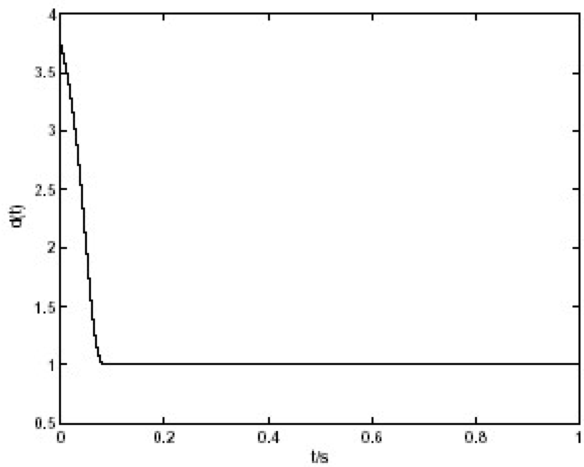

Fig. 2(a)

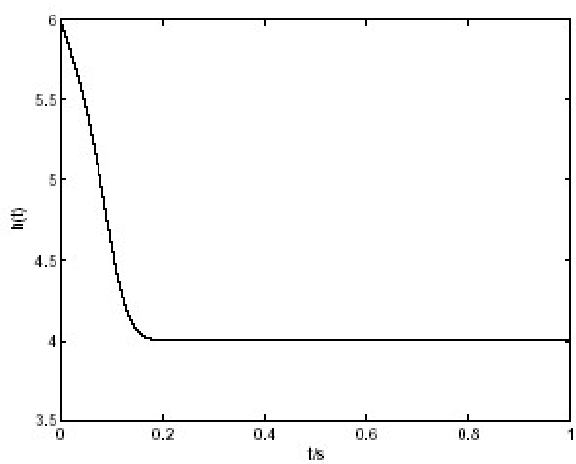

Fig. 2(b)

Examples 2: Identification of a new chaos system [24].

The system is described as

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = bx_1 - dx_1x_3 \, . \\ \dot{x}_3 = cx_3 + hx_1^2 \end{cases}$$

When $a = 10$, $b = 40$, $d = 1$, $c = 2.5$, $h = 4$, the system is chaotic.

a. The parameter d is unknown and the rests are certain.

Adopting the approach in section 5, we can obtain the observer as

$$\begin{cases} \dot{p} = -kx_1^2x_3^2p + kbx_1^2x_3 + k^2x_1^3x_3^3x_2 \\ +kx_2x_3[a(x_2 - x_1)] + kx_1x_2(-cx_3 + hx_1^2), \\ \hat{d} = p - kx_1x_2x_3. \end{cases}$$

Fig. 2 (a) presents its simulation result.

b. The parameter h is unknown and the rests are certain. According to section 5, the observer is designed as

$$\begin{cases} \dot{p} = -kx_1^2p - k^2x_1^2x_3 + kcx_3 \\ \hat{h} = p + kx_3. \end{cases}$$

Fig.2 (b) shows its simulation result.

Fig. 2 show the simulated results of parameters identification of the new chaos system. The initial conditions are $[x_1(0), x_2(0), x_3(0), p(0)] = [5, 5, 5, 5]$ in all cases and (a) $k = 0.01$; (b) $k = 0.2$.

Obviously, the numerical simulations show that the above mentioned observers are effective and feasible.

7. CONCLUSIONS

From Section 3, 4 and 5, we can see that the key to design the unknown parameter observer is to choose an appropriate gain function, which makes the error system Eqs. (6), (19), (27) and (35) asymptotically stable, and construct an auxiliary function that can eliminate the derivative of the state variable in the observers (4), (17), (26) and (34). Summing up these cases in Sections 3, 4, and 5, we can get a general method to choose the gain function and construct the auxiliary function in usual case. Assume the i-th equation of a chaos system is described as $\dot{x}_i = b_i f_{i_1}(x) + f_{i_2}(x)$, where b_i is the unknown parameter to be identified, $f_{i_1}(x)$ is the linear or nonlinear function of the state vector x of the chaos system, $f_{i_2}(x)$ is the rest part. The gain function can usually be chosen as $L = L[f_{i_1}(x)] = [f_{i_1}(x)]^{2m-1} (m = 1,2,\dots)$, and the auxiliary function $\Phi(x)$ satisfies $\frac{\partial \Phi(x)}{\partial x_i} = -L$. Especially, if $f_{i_1}(x) \geq 0$ or $f_{i_1}(x) \leq 0$ for all time, the gain function can be simply chosen as $L = k(\text{or} - k)$ $(k > 0)$, correspondingly, $\Phi = -kx_i(\text{or } kx_i)$. The simulations in Section 6 show the techniques are effective and feasible.

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