*International Journal of Statistics and Management System*
*Vol. 2 No. 2 (July-December, 2017)*

# Sum of squares orthogonal fractional replicates*

Walter T. Federer †

**Abstract**

A class of saturated sum of squares orthogonal fractional replicate plans is presented to cover a vast number of situations. These results greatly increase the number and types of fractional replicates available for research investigations. The number of factors and levels are diverse enough to accommodate almost any situation encountered in experimentation. Use is made of the sum of squares geometry and factorial theory to construct these fractions from an $r$–row by $c$–column array. Complete sets of $n$ $F$–rectangles are used to construct the fractions. Each $F$–rectangle corresponds to a factor in the fractional replicate with $n$ factors. Detailed accounts for $r$–row by 6-column arrays are presented to illustrate the procedure. Lists of fractions available and number of runs for $r$–row by 5 column arrays, $r$–row by 7 column arrays, $r$–row by 8 column arrays, and $r$–row by 9 column arrays are given. Comments about the general case are presented. Computer codes for constructing complete sets of $F$–rectangles, least squares means of levels of factors, and demonstrating the sum of squares orthogonality for the $F$–rectangles are shown in the appendices. Using the codes given, the construction of the fractional replicate plan and the analysis for the responses obtained are straightforward. Extension to other situations is also straightforward.

---

*Key words and phrases:* Sum of squares orthogonality, combinatorial orthogonality, $F$–rectangle, row(column) frequency $F$–rectangle, row–column array, sum of squares geometry, saturated fractional replicate, factorial arrangement, main effect, interaction.

† *Mailing Address:* Department of Biological Statistics and Computational Biology 1176, Comstock Hall, Cornell University, Ithaca, NY 14853, USA. *E–mail:* wtf1@cornell.edu.