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Estimating Population Mean Using Ratio Estimator for variance is Greater than Mean in Simple Random Sampling

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Abstract: The paper focuses on the estimation of the population mean of a study variable using ratio estimators when the mean is greater than the variance in a simple random sampling context. The authors compare various existing ratio estimators and propose fifty-two ratio estimators by different approaches over time. The paper presents these estimators along with their Mean Square Error (MSE) values. To facilitate the comparison, a simulation study is conducted using different scenarios where the mean is greater than the variance. The objective is to identify the best estimators for the unknown population parameter. The authors discuss several estimators that employ a single auxiliary variable. Second most efficient estimator from our study is $\widehat{m}_{12}$ for all sample size and the third most efficient estimator for this case is $\widehat{m}_{36}$. Fifth ranking is obtained by $\widehat{m}_{17}$, performs well for sample size 30. For sample size 40, 50 and 60, 100 and 200 efficient estimator is found to be $\widehat{m}_{38}$. This estimator utilized by using the correlation coefficient and regression-cum ratio estimator. The paper presents and compares various ratio estimators for estimating the population mean when the mean is greater than the variance. The authors conduct a simulation study and identify the most efficient estimators based on sample sizes, incorporating single auxiliary variables. Additionally, the correlation coefficient and regression-cum-ratio estimator methods are employed to utilize these estimators.

Key words: Ratio Estimator, Simple Random Sampling, population Mean, Mean square Error

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1. Introduction

Simple random sampling

Simple random sampling is a sampling procedure where every unit in a population has an equal chance of being selected for the sample. This type of sampling aims to provide an unbiased representation of the population. In practice, simple random sampling can be conducted by sequentially selecting units from the population. If replacement is done after each draw, it is called a "replacement sample" of size

n. This means that a unit is selected, and then placed back into the population before the next unit is drawn. In this case, it is possible for one or more units to be selected multiple times, and the repetitions are ignored. This is known as simple random sampling with replacement.

On the other hand, if replacement is not done, it is called "simple random sampling without replacement." In this case, once a unit is selected, it is not placed back into the population, and therefore, it cannot be selected again in subsequent draws. Simple random sampling without replacement ensures that each unit in the population can only be selected once. It is important to note that selecting a simple random sample from a population is not always straightforward. If the selection is influenced by the investigator's bias or preference, the sample may not be truly representative of the population. To avoid this, every unit in the population should have an equal probability of being selected, and sampling methods with replacement can help achieve this. By using simple random sampling with replacement, the properties of the sample can be estimated without bias.

Auxiliary Variable

In statistical research, an auxiliary variable is any variable about which information is available prior to data collection and this information is known for all units of the population. Hence, for numeric auxiliary variable the total is known.

The key for the statisticians is to obtain the best estimate the population parameters that are not known. Some estimators have been discussed in this thesis using single auxiliary variable. In survey sampling literature in order to improve the efficiency of pair newly constructed estimator or to obtained improve estimator for estimating some most common populations parameters, like population total population variance population mean and population coefficient of variation, certain variance statisticians discussed the auxiliary variable by using the secondary information. Much better estimates of population parameter are provided by product ratio and regression estimator in such conditions.

Ratio estimator

The ratio estimator is a statistical parameter and is defined to be the ratio of means of two random variables. Ratio estimators are biased and correlations must be made when be made when they are used in experimental of survey work.

The ratio estimates are asymmetrical and symmetrical tests such as the t test should not be used to generate confidence intervals.

A ratio estimator is a ratio of the means of two random variables that is commonly used in survey sampling. When using them in experiments they are biased, so correlation for error must be made.

Prasad (1989) proposed in this article a ratio in the finite population sample (WOR) simple random sampling design and ratio type estimators of population mean. They showed that under some conditions these estimator were more efficient than these estimator were more efficient than $\bar{y}$R.

Srivastava Jhajj (1981) estimated the mean of a finite population by using the information on an auxiliary variable. As a function of a ratio of a sample mean to the population mean, the class of estimator is defined, to the population variance of an auxiliary variable. Asymptotes expression for MSE and bias was obtained. For minimum MSE of estimator of this class was smaller than those which used only the ratio of sample mean to population mean.

Srvasta and Jhajin (1981) estimated mean of the finite population and used the information on auxiliary variable and defined the class of estimator as a function of the ratio sample mean, to the population mean. They proposed the ratio of sample variance to population variance of auxiliary variable. They obtained the asymptotic expression for the MSE and bias. A condition was given only the ratio of the sample mean to the population mean of the auxiliary variable was greater than the minimum square error of estimator of that class.

Srivekataramaa (1980) proposed a new product type estimator in this paper that was complimentary to the commonly used ratio estimators in some senses. Accurate formula for MSE and bias put be derived that was not the case for the ratio estimator. They examined for bias the new estimator. He also included the brief empirical study using supplementary information for increasing precision of estimators.

Chang,Wang and Huang (2004) considered that fixed size sampling plan for which 1^{st} order inclusion probabilities were same for all units available and the 2^{nd} order inclusion probabilities and were constant for every pair wise unit. He identified the statistical conditions under which these plans were equal to usual simple random sampling plan. To reduce the undesirable units these sampling plan were constructed. The main purpose of this study was to provide general solution to the control sampling problems. They stabilized some useful result on construction and existiontence of some desire plan.

Abdullahi and yahaya (2017) estimator uses the median and coefficient of variation of an auxiliary variable in a simple random sampling scheme. The study investigates the bias and mean squared error (MSE) of the proposed estimator under large sample approximation and identifies an asymptotically optimum estimator (AOE) with an approximate MSE formula. The estimator based on "estimated optimum values" is also explored. The proposed estimator is compared theoretically and empirically with other ratio and product estimators. The evidence from the study emphasizes the importance of high correlation between the auxiliary variable and the study variable, as well as a homogeneous distribution of the population under consideration.

Yadav et al., (2014), The article is to introduce and evaluate two new estimators for the population mean using a linear combination of the population mean and the median of an auxiliary variable. The authors derive expressions for the bias and mean square error (MSE) of these estimators up to the first order of approximation. They also compare the performance of these new estimators with existing estimators for the population mean. Additionally, the article includes a numerical study to demonstrate the superiority of the proposed estimators over the other mentioned estimators in estimating the population mean of the study variable. Overall, the article presents new estimators and provides a comprehensive analysis of their performance through theoretical derivations and numerical experiments.

Singh and Tailor (2005) proposed the general family of estimators for estimating the population mean by using known value of some population parameters (S) which led to some product and ratio estimators. Improved these estimators observed that all the estimators perform better than that of earlier proposed estimators. It was proposed that all the modified estimators perform better in the family than that of earlier proposed estimators.

Objectives of the Study

The objectives of this study are

1. To obtain the best ratio estimators among all available ratio estimators in simple random sampling for population mean estimation.
2. To obtain the condition in which the suggested estimator perform well as compared with existing ratio estimators

2. Material and Methods

Source of Data and Construction of Variables

In this chapter, we will discuss different estimators of the population mean that utilize a single auxiliary variable in the case of simple random sampling. We will also present the mean square error (MSE) of each estimator, which gives an indication of their precision.

Simulation Study

In the simulation study, we aimed to assess the performance of various comparative ratio estimators that were used in our research. To evaluate their performance, we calculated the Mean Squared Error (MSE) and Relative Efficiency (RE) for each estimator.

We conducted the simulation study using a sample size of 50,000 replicates for simple random sampling schemes. For each sampling scheme, we calculated the MSE using a specific formula. The formula used for calculating the MSE depends on the specific estimator under study.

In addition to the MSE, we also computed the Relative Efficiency (RE) for each estimator. The RE compares the performance of different estimators by assessing their efficiency relative to a reference estimator. It is computed as the ratio of the MSE of the reference estimator to the MSE of the estimator under evaluation. The *MSE* is computed for each scheme by using the formula

$$MSE\,(\omega) = \frac{1}{50,000}\sum_{i=1}^{50000}(\omega_i - \mu_y)^2 \qquad 1$$

The relative efficiencies (*RE*s) are obtained by using the formula

$$RE\,(\omega) = \frac{MSE\,(\bar{Y}_t)}{MSE\,(\omega)}, \qquad 2$$

The *MSE* values are presented in Tables 1 and The results regarding the *RE*s are givenin Tables 2, and 3 the results of *MSEs* and *REs* are obtained for various values of sample size.

The following are the steps that have been used to compute the *MSE*s and *RE*s of the proposed memory type ratio and product estimators under *RBS*:

i) Generate a population of size 5000 using the bivariate normal distribution using specified parameters *i.e.* $(Y, X) \sim N_2(2,10,1,2,\rho)$.

ii) Select 50,000 samples of different sizes such as *n*=30,40,50,60,100 and 200.

iii) Using the sample obtained in step iii), the 50,000 values of each *SRS* estimators are obtained.

iv) *MSE* for each sample size is computed using the formula given in 1

v) *RE* of each sample is obtained by using equation given in 2.

$$\mu_x = 4,\ \mu_x = 2,\ \sigma_x^2 = 10,\ \sigma_y^2 = 10,\ \sigma_{xy} = 8.5\ ,\ \rho_{xy} = 0.85$$

3. Findings and Discussions

In the chapter, the focus was on analyzing the study related to data analysis of ratio estimators in simple random sampling. The researchers examined fifty-two ratio estimators proposed by various authors between the years 1940 and 2019. The first ratio estimator analyzed was the classical ratio estimator proposed by Cochran in 1940, while the most recent one was proposed by Noor-ul-Amin in 2019.

The researchers specifically looked into cases where the mean is greater than the variance. To conduct a detailed analysis, they performed a simulation study to support their findings and determine the best ratio estimator among those considered. The impact of sample size was also investigated, with different sample sizes used: 30, 40, 50, 60, 100, and 200. In order to evaluate the performance of the ratio estimators under study, the researchers calculated the mean squared error (MSE) and relative efficiency (RE) using the following steps during the simulation study.

Table 1: MSE for sample size 30,40,50,60,100 and 200

Estimators	n=30	n=40	n=50	n=60	n=100	n=200
$\hat{m}_1$	0.10350	0.07619	0.06026	0.05019	0.02832	0.01278
$\hat{m}_2$	0.09854	0.07258	0.05740	0.04784	0.02698	0.01218
$\hat{m}_3$	0.08371	0.06177	0.04884	0.04080	0.02297	0.01036
$\hat{m}_4$	0.10179	0.07495	0.05928	0.04938	0.02786	0.01258
$\hat{m}_5$	0.07602	0.05616	0.04440	0.03714	0.02089	0.00942
$\hat{m}_6$	0.12596	0.09263	0.07329	0.06093	0.03446	0.01555
$\hat{m}_7$	0.11970	0.08807	0.06968	0.05795	0.03276	0.01479
$\hat{m}_8$	0.10028	0.07392	0.05846	0.04872	0.02750	0.01241
$\hat{m}_9$	0.12381	0.09107	0.07206	0.05991	0.03388	0.01529
$\hat{m}_{10}$	0.08950	0.06605	0.05222	0.04359	0.02457	0.01109
$\hat{m}_{11}$	0.10599	0.07807	0.06177	0.05146	0.02906	0.01313
$\hat{m}_{12}$	0.00414	0.00305	0.00241	0.00201	0.00113	0.00051
$\hat{m}_{13}$	0.26402	0.19294	0.15296	0.12643	0.07179	0.03235
$\hat{m}_{14}$	0.09841	0.07249	0.05733	0.04778	0.02695	0.01216
$\hat{m}_{15}$	0.10154	0.07477	0.05914	0.04927	0.02780	0.01255
$\hat{m}_{16}$	0.06410	0.04741	0.03755	0.03152	0.01770	0.00794

$\widehat{\mathbf{m}}_{17}$	0.06403	0.04744	0.03754	0.03152	0.01770	0.00795
$\widehat{\mathbf{m}}_{18}$	0.10401	0.07657	0.06056	0.05044	0.02846	0.01285
$\widehat{\mathbf{m}}_{19}$	0.07378	0.05450	0.04330	0.03628	0.02046	0.00915
$\widehat{\mathbf{m}}_{20}$	0.10366	0.07631	0.06036	0.05027	0.02837	0.01280
$\widehat{\mathbf{m}}_{21}$	0.10444	0.07688	0.06081	0.05064	0.02858	0.01290
$\widehat{\mathbf{m}}_{22}$	0.07077	0.05232	0.04136	0.03465	0.01947	0.00877
$\widehat{\mathbf{m}}_{23}$	0.06866	0.05077	0.04015	0.03365	0.01890	0.00851
$\widehat{\mathbf{m}}_{24}$	0.06962	0.05148	0.04070	0.03411	0.01916	0.00863
$\widehat{\mathbf{m}}_{25}$	0.06554	0.04849	0.03836	0.03218	0.01806	0.00813
$\widehat{\mathbf{m}}_{26}$	0.07961	0.05882	0.04649	0.03888	0.02188	0.00987
$\widehat{\mathbf{m}}_{27}$	0.07192	0.05319	0.04204	0.03521	0.01979	0.00892
$\widehat{\mathbf{m}}_{28}$	0.08731	0.06440	0.05092	0.04251	0.02394	0.01080
$\widehat{\mathbf{m}}_{29}$	0.09404	0.06931	0.05481	0.04571	0.02577	0.01163
$\widehat{\mathbf{m}}_{30}$	0.06960	0.05146	0.04069	0.03410	0.01915	0.00863
$\widehat{\mathbf{m}}_{31}$	0.06793	0.05022	0.03985	0.03343	0.01881	0.00842
$\widehat{\mathbf{m}}_{32}$	0.12862	0.09465	0.07544	0.06277	0.03570	0.01592
$\widehat{\mathbf{m}}_{33}$	0.06968	0.05150	0.04072	0.03412	0.01916	0.00863
$\widehat{\mathbf{m}}_{34}$	0.06423	0.04740	0.03758	0.03161	0.01790	0.00811
$\widehat{\mathbf{m}}_{35}$	0.11039	0.08095	0.06397	0.05331	0.02985	0.01328
$\widehat{\mathbf{m}}_{36}$	0.02710	0.02080	0.01665	0.01388	0.00803	0.00349
$\widehat{\mathbf{m}}_{37}$	0.02712	0.02082	0.01666	0.01389	0.00803	0.00349
$\widehat{\mathbf{m}}_{38}$	0.06412	0.04739	0.03752	0.03149	0.01767	0.00793
$\widehat{\mathbf{m}}_{39}$	0.13258	0.09717	0.07686	0.06393	0.03563	0.01561
$\widehat{\mathbf{m}}_{40}$	0.09932	0.07322	0.05790	0.04826	0.02723	0.01230
$\widehat{\mathbf{m}}_{41}$	0.10072	0.07424	0.05871	0.04893	0.02761	0.01247
$\widehat{\mathbf{m}}_{42}$	0.11805	0.08687	0.06873	0.05717	0.03232	0.01459
$\widehat{\mathbf{m}}_{43}$	0.06411	0.04742	0.03756	0.03153	0.01770	0.00795
$\widehat{\mathbf{m}}_{44}$	0.06414	0.04743	0.03757	0.03154	0.01770	0.00795
$\widehat{\mathbf{m}}_{45}$	0.18123	0.13421	0.10584	0.08667	0.04789	0.02128
$\widehat{\mathbf{m}}_{46}$	0.12350	0.09084	0.07188	0.05976	0.03379	0.01526
$\widehat{\mathbf{m}}_{47}$	0.12159	0.08945	0.07077	0.05885	0.03327	0.01502
$\widehat{\mathbf{m}}_{48}$	0.10256	0.07559	0.05978	0.04981	0.02812	0.01269

$\hat{m}_{49}$	0.12514	0.09204	0.07283	0.06054	0.03424	0.01546
$\hat{m}_{50}$	0.07403	0.05473	0.04326	0.03622	0.02036	0.00918
$\hat{m}_{51}$	0.09932	0.07322	0.05790	0.04826	0.02723	0.01230
$\hat{m}_{52}$	0.00269	0.00202	0.00158	0.00123	0.00073	0.00032

Table 2: RE for sample size 30,40,50,60,100 and 200

Estimators	n=30	n=40	n=50	n=60	n=100	n=200
$\hat{m}_1$	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
$\hat{m}_2$	1.05034	1.04974	1.04983	1.04912	1.04967	1.04926
$\hat{m}_3$	1.23641	1.23345	1.23383	1.23015	1.23291	1.23359
$\hat{m}_4$	1.01680	1.01654	1.01653	1.01640	1.01651	1.01590
$\hat{m}_5$	1.36148	1.35666	1.35721	1.35137	1.35567	1.35669
$\hat{m}_6$	0.82169	0.82252	0.82221	0.82373	0.82182	0.82187
$\hat{m}_7$	0.86466	0.86511	0.86481	0.86609	0.86447	0.86410
$\hat{m}_8$	1.03211	1.03071	1.03079	1.03017	1.02982	1.02982
$\hat{m}_9$	0.83596	0.83661	0.83625	0.83776	0.83589	0.83584
$\hat{m}_{10}$	1.15643	1.15352	1.15396	1.15141	1.15263	1.15239
$\hat{m}_{11}$	0.97651	0.97592	0.97555	0.97532	0.97454	0.97334
$\hat{m}_{12}$	25.00000	24.98033	25.00415	24.97015	25.06195	25.05882
$\hat{m}_{13}$	0.39202	0.39489	0.39396	0.39698	0.39448	0.39505
$\hat{m}_{14}$	1.05172	1.05104	1.05111	1.05044	1.05084	1.05099
$\hat{m}_{15}$	1.01930	1.01899	1.01894	1.01867	1.01871	1.01833
$\hat{m}_{16}$	1.61467	1.60705	1.60479	1.59232	1.60000	1.60957
$\hat{m}_{17}$	1.61643	1.60603	1.60522	1.59232	1.60000	1.60755
$\hat{m}_{18}$	0.99510	0.99504	0.99505	0.99504	0.99508	0.99455
$\hat{m}_{19}$	1.40282	1.39798	1.39169	1.38341	1.38416	1.39672
$\hat{m}_{20}$	0.99846	0.99843	0.99834	0.99841	0.99824	0.99844
$\hat{m}_{21}$	0.99100	0.99103	0.99096	0.99111	0.99090	0.99070
$\hat{m}_{22}$	1.46248	1.45623	1.45696	1.44849	1.45455	1.45724
$\hat{m}_{23}$	1.50743	1.50069	1.50087	1.49153	1.49841	1.50176
$\hat{m}_{24}$	1.48664	1.47999	1.48059	1.47142	1.47808	1.48088

$\hat{m}_{25}$	1.57919	1.57125	1.57091	1.55966	1.56811	1.57196
$\hat{m}_{26}$	1.30009	1.29531	1.29619	1.29090	1.29433	1.29483
$\hat{m}_{27}$	1.43910	1.43241	1.43340	1.42545	1.43103	1.43274
$\hat{m}_{28}$	1.18543	1.18308	1.18343	1.18066	1.18296	1.18333
$\hat{m}_{29}$	1.10060	1.09926	1.09943	1.09801	1.09895	1.09888
$\hat{m}_{30}$	1.48707	1.48057	1.48095	1.47185	1.47885	1.48088
$\hat{m}_{31}$	1.52363	1.51713	1.51217	1.50135	1.50558	1.51782
$\hat{m}_{32}$	0.80470	0.80497	0.79878	0.79959	0.79328	0.80276
$\hat{m}_{33}$	1.48536	1.47942	1.47986	1.47099	1.47808	1.48088
$\hat{m}_{34}$	1.61140	1.60738	1.60351	1.58779	1.58212	1.57583
$\hat{m}_{35}$	0.93758	0.94120	0.94200	0.94147	0.94874	0.96235
$\hat{m}_{36}$	3.81919	3.66298	3.61922	3.61599	3.52678	3.66189
$\hat{m}_{37}$	3.81637	3.65946	3.61705	3.61339	3.52678	3.66189
$\hat{m}_{38}$	1.61416	1.60772	1.60608	1.59384	1.60272	1.61160
$\hat{m}_{39}$	0.78066	0.78409	0.78402	0.78508	0.79484	0.81871
$\hat{m}_{40}$	1.04209	1.04056	1.04076	1.03999	1.04003	1.03902
$\hat{m}_{41}$	1.02760	1.02627	1.02640	1.02575	1.02572	1.02486
$\hat{m}_{42}$	0.87675	0.87706	0.87676	0.87791	0.87624	0.87594
$\hat{m}_{43}$	1.61441	1.60671	1.60437	1.59182	1.60000	1.60755
$\hat{m}_{44}$	1.61366	1.60637	1.60394	1.59131	1.60000	1.60755
$\hat{m}_{45}$	0.57110	0.56769	0.56935	0.57909	0.59136	0.60056
$\hat{m}_{46}$	0.83806	0.83873	0.83834	0.83986	0.83812	0.83748
$\hat{m}_{47}$	0.85122	0.85176	0.85149	0.85285	0.85122	0.85087
$\hat{m}_{48}$	1.00917	1.00794	1.00803	1.00763	1.00711	1.00709
$\hat{m}_{49}$	0.82707	0.82779	0.82741	0.82904	0.82710	0.82665
$\hat{m}_{50}$	1.39808	1.39211	1.39297	1.38570	1.39096	1.39216
$\hat{m}_{51}$	1.04209	1.04056	1.04076	1.03999	1.04003	1.03902
$\hat{m}_{52}$	38.47584	37.71782	38.13924	40.80488	38.79452	39.93750

Table 3: Relative Position of Estimators

Estimators Ranking	n=30	n=40	n=50	n=60	n=100	n=200
1	$\hat{m}_{52}$	$\hat{m}_{52}$	$\hat{m}_{52}$	$\hat{m}_{52}$	$\hat{m}_{52}$	$\hat{m}_{52}$
2	$\hat{m}_{12}$	$\hat{m}_{12}$	$\hat{m}_{12}$	$\hat{m}_{12}$	$\hat{m}_{12}$	$\hat{m}_{12}$
3	$\hat{m}_{36}$	$\hat{m}_{36}$	$\hat{m}_{36}$	$\hat{m}_{36}$	$\hat{m}_{36}$	$\hat{m}_{36}$
4	$\hat{m}_{37}$	$\hat{m}_{37}$	$\hat{m}_{37}$	$\hat{m}_{37}$	$\hat{m}_{37}$	$\hat{m}_{37}$
5	$\hat{m}_{17}$	$\hat{m}_{38}$	$\hat{m}_{38}$	$\hat{m}_{38}$	$\hat{m}_{38}$	$\hat{m}_{38}$
6	$\hat{m}_{16}$	$\hat{m}_{34}$	$\hat{m}_{12}$	$\hat{m}_{16}$	$\hat{m}_{16}$	$\hat{m}_{16}$
7	$\hat{m}_{43}$	$\hat{m}_{16}$	$\hat{m}_{16}$	$\hat{m}_{12}$	$\hat{m}_{12}$	$\hat{m}_{12}$
8	$\hat{m}_{38}$	$\hat{m}_{43}$	$\hat{m}_{43}$	$\hat{m}_{43}$	$\hat{m}_{43}$	$\hat{m}_{43}$
9	$\hat{m}_{44}$	$\hat{m}_{44}$	$\hat{m}_{44}$	$\hat{m}_{44}$	$\hat{m}_{44}$	$\hat{m}_{44}$
9	$\hat{m}_{34}$	$\hat{m}_{17}$	$\hat{m}_{44}$	$\hat{m}_{44}$	$\hat{m}_{44}$	$\hat{m}_{44}$
11	$\hat{m}_{25}$	$\hat{m}_{25}$	$\hat{m}_{25}$	$\hat{m}_{25}$	$\hat{m}_{25}$	$\hat{m}_{25}$
12	$\hat{m}_{31}$	$\hat{m}_{31}$	$\hat{m}_{31}$	$\hat{m}_{31}$	$\hat{m}_{31}$	$\hat{m}_{31}$
12	$\hat{m}_{23}$	$\hat{m}_{23}$	$\hat{m}_{23}$	$\hat{m}_{23}$	$\hat{m}_{23}$	$\hat{m}_{23}$
14	$\hat{m}_{30}$	$\hat{m}_{30}$	$\hat{m}_{30}$	$\hat{m}_{30}$	$\hat{m}_{30}$	$\hat{m}_{24}$
15	$\hat{m}_{24}$	$\hat{m}_{24}$	$\hat{m}_{24}$	$\hat{m}_{24}$	$\hat{m}_{24}$	$\hat{m}_{30}$
16	$\hat{m}_{33}$	$\hat{m}_{33}$	$\hat{m}_{33}$	$\hat{m}_{33}$	$\hat{m}_{33}$	$\hat{m}_{33}$
17	$\hat{m}_{22}$	$\hat{m}_{22}$	$\hat{m}_{22}$	$\hat{m}_{22}$	$\hat{m}_{22}$	$\hat{m}_{22}$
18	$\hat{m}_{27}$	$\hat{m}_{27}$	$\hat{m}_{27}$	$\hat{m}_{27}$	$\hat{m}_{27}$	$\hat{m}_{27}$
18	$\hat{m}_{19}$	$\hat{m}_{19}$	$\hat{m}_{50}$	$\hat{m}_{50}$	$\hat{m}_{50}$	$\hat{m}_{19}$
20	$\hat{m}_{50}$	$\hat{m}_{50}$	$\hat{m}_{19}$	$\hat{m}_{19}$	$\hat{m}_{19}$	$\hat{m}_{50}$
20	$\hat{m}_{5}$	$\hat{m}_{5}$	$\hat{m}_{5}$	$\hat{m}_{5}$	$\hat{m}_{5}$	$\hat{m}_{5}$
22	$\hat{m}_{26}$	$\hat{m}_{26}$	$\hat{m}_{26}$	$\hat{m}_{26}$	$\hat{m}_{26}$	$\hat{m}_{26}$
22	$\hat{m}_{3}$	$\hat{m}_{3}$	$\hat{m}_{3}$	$\hat{m}_{3}$	$\hat{m}_{3}$	$\hat{m}_{3}$
24	$\hat{m}_{28}$	$\hat{m}_{28}$	$\hat{m}_{28}$	$\hat{m}_{28}$	$\hat{m}_{28}$	$\hat{m}_{28}$
25	$\hat{m}_{10}$	$\hat{m}_{10}$	$\hat{m}_{10}$	$\hat{m}_{10}$	$\hat{m}_{10}$	$\hat{m}_{10}$
26	$\hat{m}_{29}$	$\hat{m}_{29}$	$\hat{m}_{29}$	$\hat{m}_{29}$	$\hat{m}_{29}$	$\hat{m}_{29}$
27	$\hat{m}_{14}$	$\hat{m}_{14}$	$\hat{m}_{14}$	$\hat{m}_{14}$	$\hat{m}_{14}$	$\hat{m}_{14}$
28	$\hat{m}_{2}$	$\hat{m}_{2}$	$\hat{m}_{2}$	$\hat{m}_{2}$	$\hat{m}_{2}$	$\hat{m}_{2}$
28	$\hat{m}_{40}$	$\hat{m}_{40}$	$\hat{m}_{40}$	$\hat{m}_{40}$	$\hat{m}_{40}$	$\hat{m}_{40}$
30	$\hat{m}_{51}$	$\hat{m}_{51}$	$\hat{m}_{51}$	$\hat{m}_{51}$	$\hat{m}_{51}$	$\hat{m}_{51}$

30	$\hat{m}_8$	$\hat{m}_8$	$\hat{m}_8$	$\hat{m}_8$	$\hat{m}_8$	$\hat{m}_8$
32	$\hat{m}_{41}$	$\hat{m}_{41}$	$\hat{m}_{41}$	$\hat{m}_{41}$	$\hat{m}_{41}$	$\hat{m}_{41}$
33	$\hat{m}_{15}$	$\hat{m}_{15}$	$\hat{m}_{15}$	$\hat{m}_{15}$	$\hat{m}_{15}$	$\hat{m}_{15}$
33	$\hat{m}_4$	$\hat{m}_4$	m4	m4	m4	m4
35	$\hat{m}_{48}$	$\hat{m}_{48}$	$\hat{m}_{48}$	$\hat{m}_{48}$	$\hat{m}_{48}$	$\hat{m}_{48}$
36	Ratio	Ratio	Ratio	Ratio	Ratio	Ratio
37	$\hat{m}_{20}$	$\hat{m}_{20}$	$\hat{m}_{20}$	$\hat{m}_{20}$	$\hat{m}_{20}$	$\hat{m}_{20}$
38	$\hat{m}_{18}$	$\hat{m}_{18}$	$\hat{m}_{18}$	$\hat{m}_{18}$	$\hat{m}_{18}$	$\hat{m}_{18}$
39	$\hat{m}_{21}$	$\hat{m}_{21}$	$\hat{m}_{21}$	$\hat{m}_{21}$	$\hat{m}_{21}$	$\hat{m}_{21}$
40	$\hat{m}_{11}$	$\hat{m}_{11}$	$\hat{m}_{11}$	$\hat{m}_{11}$	$\hat{m}_{11}$	$\hat{m}_{11}$
41	$\hat{m}_{35}$	$\hat{m}_{35}$	$\hat{m}_{35}$	$\hat{m}_{35}$	$\hat{m}_{35}$	$\hat{m}_{35}$
42	$\hat{m}_{42}$	$\hat{m}_{42}$	$\hat{m}_{42}$	$\hat{m}_{42}$	$\hat{m}_{42}$	$\hat{m}_{42}$
43	$\hat{m}_7$	$\hat{m}_7$	$\hat{m}_7$	$\hat{m}_7$	$\hat{m}_7$	$\hat{m}_7$
44	$\hat{m}_{47}$	$\hat{m}_{47}$	$\hat{m}_{47}$	$\hat{m}_{47}$	$\hat{m}_{47}$	$\hat{m}_{47}$
45	$\hat{m}_{46}$	$\hat{m}_{46}$	$\hat{m}_{46}$	$\hat{m}_{46}$	$\hat{m}_{46}$	$\hat{m}_{46}$
46	$\hat{m}_9$	$\hat{m}_9$	$\hat{m}_9$	$\hat{m}_9$	$\hat{m}_9$	$\hat{m}_9$
46	$\hat{m}_{49}$	$\hat{m}_{49}$	$\hat{m}_{49}$	$\hat{m}_{49}$	$\hat{m}_{49}$	$\hat{m}_{49}$
46	$\hat{m}_6$	$\hat{m}_6$	$\hat{m}_6$	$\hat{m}_6$	$\hat{m}_6$	$\hat{m}_6$
46	$\hat{m}_{32}$	$\hat{m}_{32}$	$\hat{m}_{32}$	$\hat{m}_{32}$	$\hat{m}_{39}$	$\hat{m}_{39}$
50	$\hat{m}_{39}$	$\hat{m}_{39}$	$\hat{m}_{39}$	$\hat{m}_{39}$	$\hat{m}_{32}$	$\hat{m}_{32}$
51	$\hat{m}_{45}$	$\hat{m}_{45}$	$\hat{m}_{45}$	$\hat{m}_{45}$	$\hat{m}_{45}$	$\hat{m}_{45}$
52	$\hat{m}_{13}$	$\hat{m}_{13}$	$\hat{m}_{13}$	$\hat{m}_{13}$	$\hat{m}_{13}$	$\hat{m}_{13}$

From the table of mean square error and relative efficiency again it is observed that estimator $\hat{m}_{52}$ performs well in term of MSE and RE. This estimator is recently developed by Noor-ul-Amin (2021) and called memory base ratio estimator. Second most efficient estimator from our study is $\hat{m}_{12}$ for all sample size. This estimator was developed by Quenouille (1956). This estimator was modified by Durban (1959). In this estimator author partitioned the data into two equal part and make use of mean of both parts in the estimators for study and auxiliary variable. The third most efficient estimator for this case is $\hat{m}_{36}$, which is the another estimator developed by Subrammani and Prabavathy (2014). In this estimator author make use of median of study and auxiliary variable. These three estimators are efficient for all sample sizes under study.

Fourth efficient ratio estimator in ranking is $\hat{m}_{37}$, for all sample sizes. This estimator was developed by Chakrabarty (1979) which is the combination of classical ratio estimator and simple mean. Fifth ranking is obtained by $\hat{m}_{17}$ for sample size 30. For sample size 40, 50 and 60, 100 and 200 efficient estimator is found to be $\widehat{m}_{38}$, this estimator was developed by Enang, Akpan and Ekpenyong (2014). This estimator utilized by using the correlation coefficient and regression-cum ratio estimator.

Estimators $\widehat{m}_{16}$,$\widehat{m}_{17}$,$\widehat{m}_{34}$,$\widehat{m}_{43}$ and $\widehat{m}_{44}$, estimators also comes in the orbit of top most 10 ranking of efficient estimators for case two. These estimators were derived by Chakrabarty (1979), Kadilar and Cingi (2006), Jerajuddin and Kishun (2016), Abdullahi and Yahaya (2017) and Tailor and Sharma (2009) respectively.Theses estimators make use of sample size, coefficient of variation, measure of kurtosis and correlation coefficient of auxiliary variable. So, we can conclude that make use of thesis measures in the construction is helpful for all sample sizes.

Ratio estimator that is worst in performance is $\widehat{m}_{13}$, which is developed by Kadilar and Cingi (2003). This estimator makes use of square of auxiliary variable mean for sample and population data. The performance of this estimator is worst in performance for all sample sizes. The second worst estimator in performance w.r.t. MSE and RE is found to be $\widehat{m}_{45}$. This estimator is developed by Kumar, Bharti, Yadav and Kumar (2019). This estimator also makes use of median of auxiliary variable along with the covariances in term of mean and median.

4. Conclusion

The paper thoroughly examined different ratio estimators for estimating the population mean when the mean is greater than the variance in simple random sampling. Through a comparative analysis of Mean Square Error (MSE) and Relative Efficiency (RE), the study identified several efficient estimators. The memory-based ratio estimator ($\widehat{m}_{52}$) developed by Noor-ul-Amin (2021) emerged as the top-performing estimator, demonstrating superior performance in terms of both MSE and RE. It outperformed other estimators across all sample sizes considered in the study. Furthermore, $\widehat{m}_{12}$, originally proposed by Quenouille (1956) and modified by Durban (1959), proved to be the second most efficient estimator. This estimator effectively utilized the means of two equal parts of the data for estimation. The third most efficient estimator, $\widehat{m}_{36}$, developed by Subrammani and Prabavathy (2014), leveraged the median of both the study and auxiliary variables to achieve accurate population mean estimation.

Other estimators such as $\widehat{m}_{37}$,$\widehat{m}_{17}$, and $\widehat{m}_{38}$, also displayed commendable performance for specific sample sizes, incorporating techniques like the combination of classical ratio estimator and simple mean, correlation coefficient and regression-cum-ratio estimator, and various measures like sample size, coefficient of variation, kurtosis, and correlation coefficient of the auxiliary variable.

On the contrary, $\widehat{m}_{13}$, which employed the square of the mean of the auxiliary variable, and $\widehat{m}_{45}$, utilizing the median of the auxiliary variable with covariances in terms of mean and median, demonstrated poor performance compared to other estimators. Overall, these findings provide valuable insights for researchers and practitioners in choosing appropriate ratio estimators for population mean estimation in scenarios where the mean exceeds the variance.

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