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Characterizations of Supra Generalized Preregular Closed Sets

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Abstract: In this paper we extend the study of gpr$^{\mu}$ - closed sets in a supra topological space. In particular we study the relation of gpr$^{\mu}$ - closed sets with various sets in supra toplogical space.

Keywords and Phrases: supra preclosed set, gpr$^{\mu}$ - closed set, supra extremally disconnected space.

1.Introduction: The concept of topological space grew out of the study of the real line and euclidean space and the study of continuous functions on these spaces. In topology the fundamental notion of generalized closed sets was introduced by N. Levine in 1970 [5]. This contribution made the topologists Bhattacharya et al [3], Balachandran et al [2], Maximilian Ganster [7] and Gnanambal et al [6] to investigate new concepts and results in general topology. In 1983, A. S. Mashour et al [9] introduced the supra topological spaces and studied S - continuous maps and S* - continuous maps. The study on supra topological space was explored by several researchers. Devi et al [4], Sayed et al [11], Sayed [10], Ravi et al [8], Arockiarani et al [1] and Vidhya Menon [12] introduced supra α - open sets and Sα - continuous functions, supra b - open sets and supra b - continuity , supra preopen sets and supra pre - continuity on topological spaces, supra sg - closed sets and supra gs - closed sets, supra generalized b - regular closed sets and supra generalized preregular closed sets respectively. In this paper we study the characteristics of gpr$^{\mu}$ - closed sets.

Throughout this paper (X, τ), (Y, σ)and (Z, η) (or simply X , Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated. Let (X, τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ if $\tau \subset \mu$. (X, μ) is called a supra topological space. A subset A of (X,μ)is said to be supra preclosed [10] if cl$^{\mu}$(int$^{\mu}$(A)) $\subseteq$ A. The complement of supra preclosed set is called supra preopen set [10]. The supra pre-closure of A ,denoted by pcl$^{\mu}$(A) is the intersection of the supra preclosed sets containing A. The supra pre-interior of A, denoted by pint$^{\mu}$(A) is the union of the supra pre - open sets contained in A.

2. Preliminaries :

Definition 2.1: A subset A of a supra topological space (X, μ) is called

- supra generalized preclosed (briefly gp$^{\mu}$ - closed) [12] if pcl$^{\mu}$(A) $\subseteq$ U whenever A $\subseteq$ U and U is supra open in (X, μ).

- supra generalized semi - closed (briefly gs$^\mu$ - closed) [1] if scl$^\mu$(A) $\subseteq$ U whenever A $\subseteq$ U and U is supra open in (X, μ).

- supra semi generalized closed (briefly sg$^\mu$ - closed) [1] if scl$^\mu$(A) $\subseteq$ U whenever A $\subseteq$ U and U is supra semi open in (X, μ).

- supra generalized - α closed (briefly gα^μ - closed) [1] if αcl$^\mu$(A) $\subseteq$ U whenever A $\subseteq$ U and U is supra α - open in (X, μ).

- supra generalized semi - preclosed (briefly gsp$^\mu$) - closed) [12] if spcl$^\mu$(A) $\subseteq$ U whenever A $\subseteq$ U and U is supra open in (X, μ).

- supra generalized preregular closed (briefly gpr$^\mu$) - closed) [12] if pcl$^\mu$(A) $\subseteq$ U whenever A $\subseteq$ U and U is supra regular open in (X, μ).

- supra generalized b - closed (briefly g$^\mu$b - closed) [1] if bcl$^\mu$(A) $\subseteq$ U whenever A $\subseteq$ U and U is supra open in (X, μ).

- supra generalized b - regular closed (briefly g$^\mu$br - closed) [1] if bcl$^\mu$(A) $\subseteq$ U whenever A $\subseteq$ U and U is supra regular open in (X, μ).

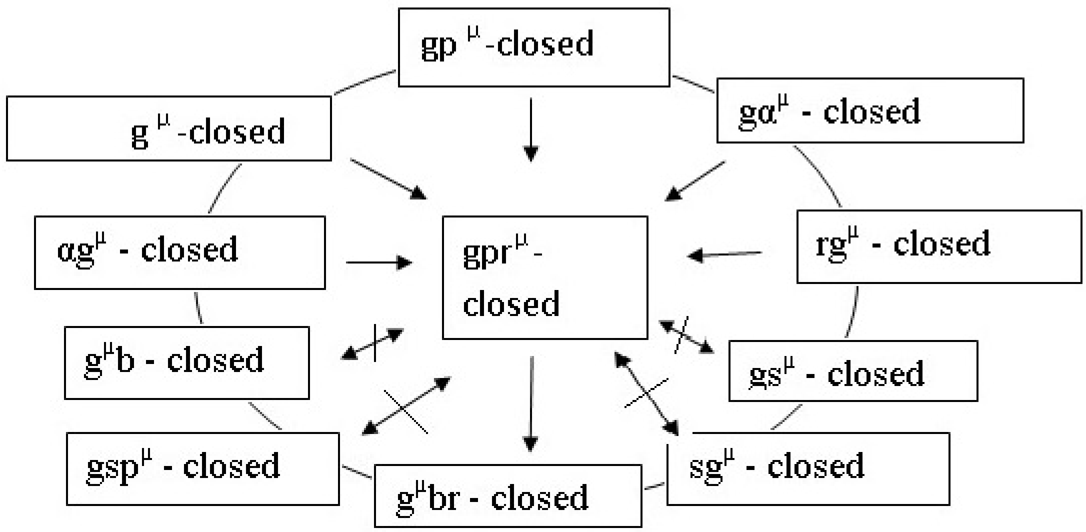
3. gpr$^\mu$ - closed sets

Theorem 3.1: For a supra topological space (X, μ) the following properties hold:
(i) Every gα^μ - closed set is gpr$^\mu$ - closed.
(ii) Every gpr$^\mu$ - closed set is g$^\mu$br - closed.
Proof: Obvious.
However the converse of the above said theorems are not true.
Example 3.2:
(i) Let (X, μ) be a supra topological space where X = {a, b, c, d} and μ = {ϕ, X, {a, c, d}, {a, b}}. Here {a, b} is gpr$^\mu$ - closed but not gα^μ - closed.
(ii) Let (X, μ) be a supra topological space where X = {a, b, c, d} and μ = {ϕ, X, {a, b}, {b, c}, {a, c}, {a, b, c},{a}}. Here {b, c} is g$^\mu$br - closed but not gpr$^\mu$ - closed.
Remark 3.3: If (X, μ) is supra topological space then
(i) gpr$^\mu$ - closed set and g$^\mu$b - closed set are independent of each other.
(ii) gpr$^\mu$ - closed set and gs$^\mu$ - closed set are independent of each other.
(iii) gpr$^\mu$ - closed set and sg$^\mu$ - closed set are independent of each other.
These results are proved by the following examples.
Example 3.4: (i) In example 3.2 (ii) A = {b, c} is g$^\mu$b - closed but not gpr$^\mu$ - closed. Let (X, μ) be a supra topological space where X = {a, b, c} and μ = {ϕ, X, {a, b}, {a, c}}. Consider A = {a, b}. A is gpr$^\mu$ - closed but not g$^\mu$b - closed.
(ii) Let (X, μ) be a supra topological space where X = {a, b, c, d} and μ = {ϕ, X, {a,

c, d}, {a, b}}. Consider A = {a, b}. A is gpr$^\mu$ - closed but not gs$^\mu$ - closed and sg$^\mu$ - closed.

(iii) In example 3.2 (ii) A = {b, c}. A is gs$^\mu$ - closed and sg$^\mu$ - closed but not gpr$^\mu$ - closed.

3.5 From the above results we have the following diagram



Fig(3.5.1)

Theorem 3.6: Let (X, μ) be any supra topological space, then the following are equivalent

(i) Every g$^\mu$br - closed set is gpr$^\mu$ - closed.

(ii) Every supra b - closed set is gpr$^\mu$ - closed.

proof: (i) $\rightarrow$ (ii) is obvious as every supra b - closed set is g$^\mu$br - closed.

(ii) $\rightarrow$ (i) Let A be a g$^\mu$br - closed and A $\subseteq$ U where U is supra regular open. Then bcl$^\mu$(A) is supra b - closed and bcl$^\mu$(A) $\subseteq$ U. Then by (ii), bcl$^\mu$(A) is gpr$^\mu$ - closed. So pcl$^\mu$(A) $\subseteq$ pcl$^\mu$(bcl$^\mu$(A)) $\subseteq$ U. Hence A is gpr$^\mu$ - closed.

Theorem 3.7: Let (X, μ) be a supra topological space. If every supra semiclosed set is supra preclosed in (X, μ), then

(i) Every gs$^\mu$ - closed set is gpr$^\mu$ - closed.

(ii) Every sg$^\mu$ - closed set is gpr$^\mu$ - closed.

Proof: (i) Let A be gs$^\mu$ - closed in (X, μ) and A $\subseteq$ U where U is a supra regular open. Then scl$^\mu$(A) $\subseteq$ U whenever A $\subseteq$ U. Since every supra semiclosed set is supra preclosed (by assumption), pcl$^\mu$(A) $\subseteq$ scl$^\mu$(A) $\subseteq$ U. This implies pcl$^\mu$(A) $\subseteq$ U. Thus A is gpr$^\mu$ - closed.

(ii) Since every sg^μ -closed set is gs^μ -closed, every sg^μ - closed set is gpr^μ - closed by(i).

Theorem 3.8: If every supra open set U is supra regular open in a supra topological space (X, μ) then

(a) Every gpr^μ - closed set is gsp^μ - closed.

(b) The following statements are equivalent:

(i) Every gpr^μ - closed set is αg^μ - closed.

(ii) Every supra preclosed set is αg^μ - closed.

Proof: **(a)** Let A be gpr^μ - closed in (X, μ) and $A \subseteq U$ where U is supra open. By assumption every supra open set is supra regular open, then $pcl^\mu(A) \subseteq U$. Since every supra preclosed set is supra semi - preclosed, $spcl^\mu(A) \subseteq pcl^\mu(A) \subseteq U$.Hence A is gsp^μ - closed set.

(b) (i) $\rightarrow$ (ii). Every supra preclosed is gpr^μ - closed. This implies every supra preclosed set is αg^μ - closed.

(ii) $\rightarrow$ (i). Let A be gpr^μ - closed in (X, μ) and $A \subseteq U$ where U is supra open. If $B = pcl^\mu(A)$ then $B \subseteq U$. By assumption B is αg^μ - closed. Thus α - $cl^\mu(A) \subseteq \alpha$ - $cl^\mu(B) \subseteq$ U. Hence A is αg^μ - closed.

Remark 3.9: $pcl^\mu(A) = A \bigcup cl^\mu(int^\mu(A))$

Proposition 3.10: Let A be a subset of a supra topological space (X, μ). If $A \in SO^\mu$ (X, μ) then $pcl^\mu(A) = cl^\mu(A)$.

proof: We know that every supra closed set is supra preclosed $pcl^\mu(A) \subseteq cl^\mu(A)$. Now given $A \in SO^\mu$ (X, μ), that is $A \subseteq cl^\mu(int^\mu(A))$. Thus $cl^\mu(A) \subseteq cl^\mu(cl^\mu(int^\mu(A))) \Rightarrow cl^\mu(A) \subseteq (cl^\mu(int^\mu(A))) \Rightarrow cl^\mu(A) \bigcup A \subseteq cl^\mu(int^\mu(A)) \bigcup A \Rightarrow cl^\mu(A) \subseteq cl^\mu(int^\mu(A)) \bigcup A \Rightarrow cl^\mu(A) \subseteq pcl^\mu(A)$. Hence $pcl^\mu(A) = cl^\mu(A)$.

Definition 3.11: A space (X, μ)is supra extremally disconnected if the supra closure of every supra open subset of X in (X, μ) is supra open or equivalently if every supra regular closed subset of X in (X, μ) is supra open.

Theorem 3.12: For a space (X, μ) the following are equivalent:

(i) Every gsp^μ - closed subset of X is gpr^μ - closed

(ii) Every supra semi - preclosed subset of X is gpr^μ - closed

(iii) The space (X, μ) is supra extremally disconnected.

Proof:(i) $\rightarrow$ (ii) is obvious as every supra semi - preclosed set is gsp^μ - closed, every supra semi - preclosed subset of X is gpr^μ - closed.

(ii) $\rightarrow$ (iii) Let A be a supra regular open subset of X . Then $A = int^\mu(cl^\mu(A))$.This implies $int^\mu(cl^\mu(int^\mu(A))) \subseteq int^\mu(cl^\mu(A)) = A$. Thus A is supra semi - preclosed. By hypothesis A is gpr^μ - closed and so $pcl^\mu(A) \subseteq A$. Hence $cl^\mu(int^\mu(A)) = A$. Therefore A is supra regular closed, i.e A is supra closed. Hence the space (X, μ) is supra extremally disconnected.

(iii) $\rightarrow$ (i) Let A be a gsp$^{\mu}$ - closed subset of X and let U $\subseteq$ X be supra regular open with A $\subseteq$ U. If B = spcl$^{\mu}$(A), then by assumption A $\subseteq$ B $\subseteq$ U. Since B is supra semi - preclosed, B = spcl$^{\mu}$(B) = B $\bigcup$ int$^{\mu}$(cl$^{\mu}$(int$^{\mu}$(B))). By assumption cl$^{\mu}$(int$^{\mu}$(B)) = int$^{\mu}$(cl$^{\mu}$(int$^{\mu}$(B))). Thus B = pcl$^{\mu}$(B), i.e B is supra preclosed. Thus pcl$^{\mu}$(A) $\subseteq$ pcl$^{\mu}$(B) = B $\subseteq$ U, i.e A is gpr$^{\mu}$ - closed.

Theorem 3.13: For a space (X, μ) the following are equivalent:

(i) Every gs$^{\mu}$ - closed subset of X is gpr$^{\mu}$ - closed.

(ii) Every sg$^{\mu}$ - closed subset of X is gpr$^{\mu}$ - closed.

(iii) The space (X, μ) is supra extremally disconnected.

Proof: (i) $\rightarrow$ (ii) is obvious.

(ii) $\rightarrow$ (iii) Let A be a supra regular open subset of X. Then A = int$^{\mu}$(cl$^{\mu}$(A)). That is int$^{\mu}$(cl$^{\mu}$(A)) $\subseteq$ A, A is supra semi - closed and hence sg$^{\mu}$ - closed. By assumption A is gpr$^{\mu}$ - closed, which implies pcl$^{\mu}$(A) $\subseteq$ A. Thus A = pcl$^{\mu}$(A), so A is supra preclosed. Hence cl$^{\mu}$(int$^{\mu}$(A)) $\subseteq$ A $\Rightarrow$ cl$^{\mu}$(A) $\subseteq$ A as A is supra regular open. Therefore pcl$^{\mu}$(A) $\subseteq$ cl$^{\mu}$(A) $\subseteq$ A, i.e A is supra closed and thus the space (X, μ) is supra extremally disconnected.

(iii) $\rightarrow$ (i) Let U be supra regular open and A be gs$^{\mu}$ - closed with A $\subseteq$ U. Then scl$^{\mu}$(A) = A $\bigcup$ int$^{\mu}$(cl$^{\mu}$(A)) $\subseteq$ U. By assumption int$^{\mu}$(cl$^{\mu}$(A)) is supra closed and so pcl$^{\mu}$(A) = A $\bigcup$ cl$^{\mu}$(int$^{\mu}$(A)) $\subseteq$ U, i.e A is gpr$^{\mu}$ - closed.

Theorem 3.14: A space is supra extremally disconnected iff every g$^{\mu}$br - closed subset of (X, μ) is gpr$^{\mu}$ - closed.

proof: Suppose that the space(X, μ) is supra extremally disconnected. Let A be g$^{\mu}$br - closed and let U be supra regular open containing A. Then bcl$^{\mu}$(A) = A $\bigcup$ [int$^{\mu}$(cl$^{\mu}$(A)) $\bigcap$ cl$^{\mu}$(int$^{\mu}$(A))] $\subseteq$ U. Since int$^{\mu}$(cl$^{\mu}$(A)) is supra closed, we have cl$^{\mu}$(int$^{\mu}$(A)) $\subseteq$ cl$^{\mu}$[int$^{\mu}$(cl$^{\mu}$(A)) $\bigcap$ int$^{\mu}$(A)] $\subseteq$ [cl$^{\mu}$(int$^{\mu}$(cl$^{\mu}$(A))) $\bigcap$ cl$^{\mu}$(int$^{\mu}$(A))] $\subseteq$ U. Hence pcl$^{\mu}$(A) = A $\bigcup$ cl$^{\mu}$(int$^{\mu}$(A)) $\subseteq$ U. This implies A is gpr$^{\mu}$ - closed. Conversely , let every g$^{\mu}$br - closed subset of (X, μ) be gpr$^{\mu}$ - closed. Let A be a supra regular open subset of X in (X, μ). Then bcl$^{\mu}$(A) = A $\bigcup$ [int$^{\mu}$(cl$^{\mu}$(A)) $\bigcap$ cl$^{\mu}$(int$^{\mu}$(A))] = A $\bigcup$ [A $\bigcap$ cl$^{\mu}$(int$^{\mu}$(A))] $\subseteq$ A. By hypothesis A is g$^{\mu}$br - closed and so A is gpr$^{\mu}$ - closed. Since every supra regular open set is supra semi open set , by the proposition 3.10 and A is gpr$^{\mu}$ - closed we have pcl$^{\mu}$(A) = cl$^{\mu}$ $\subseteq$ A . Therefore A is supra closed and (X, μ) is supra extremally disconnected.

Theorem 3.15: Let F be a family of supra regular closed sets in(X, μ). Then the following are equivalent:

(i) μ = F

(ii) Every subset of (X, μ) is gpr$^{\mu}$ - closed set.

proof: (i) $\rightarrow$ (ii). Letμ = F and let A $\subseteq$ G and G be be supra regular open set in (X,

μ). Then $pcl^{\mu}(A) \subseteq pcl^{\mu}(G) = G$, since G is supra regular closed. This implies that $pcl^{\mu}(A) \subseteq G$. That is A is gpr^{μ} - closed.

(ii) $\rightarrow$ (i) Suppose every subset of (X, μ) is gpr^{μ} - closed set. Let G be supra regular open. Since $G \subseteq G$ and G is gpr^{μ} - closed, it follows that $pcl^{\mu}(G) \subseteq G$ and so $G \in F$. Thus $\mu \subseteq F$. On the other hand if $H \in F$, then $X\text{-}H \in \mu \subseteq F$ and so $H \in \mu$. Thus $\mu =$ F.

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